PHYSICS 357S - Problem Set #5 - March 2011

Distributed **23rd March**. Due to be handed in by **6th April** in my office.

This problem set counts for 10% of the grade. It has 8 questions and 5 pages. If you don't understand a question ask me about it. If you think there is a bug (error, typo, etc) in a question..tell me. You might be right!

Please check the Web page and your email for any important messages as we approach the end of term. If you have any question in the period leading up to the exam, please feel free to just pass by my office and talk to me. It's usually best to email or phone in advance.

(1) Examine the following processes, and state for each one whether it is *possible* or *impossible* according to the standard model (which does not include exotic extensions like GUTS, with their potential for the non-conservation of lepton flavour and baryon number). If the interaction is possible state which interaction mediates it, i.e. strong, electromagnetic, or weak. If the reaction is impossible cite a conservation law which prevents it from occurring.

In addition to the obvious like electric charge conservation, you need to remember that experimentally baryon conservation holds, and that the strong (color) force conserves quark flavor, while the weak force conserves lepton flavor, but not quark flavor. A photon can only come from the electromagnetic interaction. If an interaction only involves hadrons, and could only happen via the weak force, then it will never happen; because the strong force will intervene and make something else happen. At lowest order the weak for can only change the quark flavour by one unit. Think about that. Only say a process is possible if it happens at leading order.

$p\bar{p} \to \pi^+\pi^0$	$\eta \to \gamma\gamma$
$\Sigma^0 \to \Lambda^0\pi^0$	$\Sigma^- \to n\pi^-$
$e^+e^- \to \mu^+\mu^-$	$\mu^- \to e^-\bar{\nu}_e$
$\Delta^+ \to p\pi^0$	$\bar{\nu}_e p \to ne^+$
$ep \to \nu_e\pi^0$	$pp \to \Sigma^+ n\, K^0\pi^+\pi^0$
$p \to e^+\gamma$	$pp \to ppp\bar{p}$
$n\bar{n} \to \pi^+\pi^-\pi^0$	$\pi^+ n \to \pi^- p$
$K^- \to \pi^-\pi^0$	$\Sigma^+ n \to \Sigma^- p$
$\Sigma^0 \to \Lambda^0\gamma$	$\Xi^- \to \Lambda^0\pi^-$
$\Xi^0 \to p\pi^-$	$\pi p \to \Lambda^0 K^0$
$\pi^0 \to \gamma\gamma$	$\Sigma \to n\, e^-\bar{\nu}_e$

(2) Given that K and π mesons have spin 0 show that one of the weak decay processes $K^+ \to \pi^+\pi^0$ and $K^+ \to \pi^+\pi^-\pi^+$ must violate parity conservation.

(3) (a) The Δ^{++} has $J^P = \frac{3}{2}^+$. It decays to p and a π^+ via the strong force. If the p has $J^P = \frac{1}{2}^+$, and the spin *0* π^+ is found to be in an *l=1* orbital state . What does that tell you about the parity of the pion?

(b) The major decay mode of the Ξ^0 is $\Xi^0 \to \Lambda\pi^0$. The Λ has the same spin and parity as the proton. Do you think this decay mode can be used to determine the spin and parity of the π^0? Why? Do you think that the Ξ^0 as a definite spin and parity? Why?

(4) Which of the following decays are forbidden by charge conjugation invariance?

$$\omega^0 \to \pi^0\gamma$$
$$\eta' \to \rho^0\gamma$$
$$\pi^0 \to \gamma\gamma\gamma$$
$$J/\psi \to p\bar{p}$$
$$\rho^0 \to \gamma\gamma$$

Note that the π^0, η' have positive charge conjugation, while the $\omega, \rho, J/\psi$ are all negative. The J/ψ is different from the others in that is a bound state of a $c-quark$ and a $\bar{c}-quark$.

(5) What is the charge conjugate reaction corresponding to $K^- + p \to \bar{K}^0 + n$? Can a K^-p system be an eigenstate of the charge conjugation operator?. Give a similar discussion of the reaction $\bar{p}p \to \pi^+\pi^-$.

(6) (a) Show that

$$Y_0^0(\theta,\varphi) = (4\pi)^{-\frac{1}{2}}$$

$$Y_1^0(\theta,\varphi) = \frac{1}{2}\left(\frac{3}{\pi}\right)^{\frac{1}{2}}\cos\theta$$

$$Y_1^{\pm 1}(\theta,\varphi) = \pm\frac{1}{2}\left(\frac{3}{2\pi}\right)^{\frac{1}{2}}\sin\theta\exp(\pm i\varphi)$$

are eigenfunctions of *Parity*. Find the eigenvalues in each case.

(b) As well as being eigenfunctions of Parity, what other operator are these eigenfunctions of?

(c)) The spherical harmonics $Y_m^l(\theta,\varphi)$ are defined in terms of the associated Legendre polynomials $P_m^l(\cos\theta)$ through the equation

$$Y_m^l(\theta,\varphi) = \left[\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}\right]^{1/2} P_m^l(\cos\theta)\exp(im\varphi)\,.$$

Given that the associated Legendre polynomials are polynomials in even (odd) powers of $\cos\theta$ if $l-|m|$ is even (odd), show that the parities of the spherical harmonics are $(-1)^l$.

(7) (a) Show that if right-handed (helicity +1) anti-neutrinos are emitted in β^- decay of a neutron, then left-handed neutrinos are emitted in β^+ decays of an anti-neutron. *(Think about the invariance (to a good approximation) of the weak interaction under CP. Just sketch the different configurations, and decide whether they are possible or not.)*

(b) Show that, if an unpolarized source emits electrons which are spin-polarized with helicity = -1, then parity is not conserved in the decay *(think about the mirror image of this source, and whether the source and its mirror image have equal transition rates to produce electrons)*

(c) If you observed such a source, could the electrons be coming from an electromagnetic process?

(8) In *Grand Unified Theories*, where the weak, electromagnetic, and strong forces come together at a very high energy scale called the GUT scale (it is close to the Plank scale --- if it exists) it is possible for the proton to decay. This is mediated by *Leptoquarks*. See below for the vertex diagrams, and possible processes.

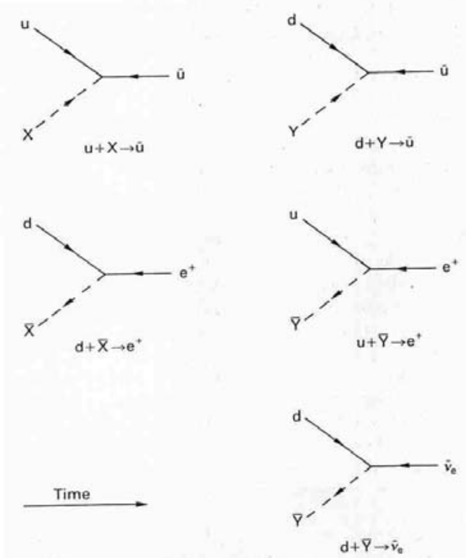

Fig. 13.2 Feynman vertices for the interaction of the predicted X (charge $-\frac{4}{3}$) and Y (charge $-\frac{1}{3}$) leptoquarks of *SU*(5). Other diagrams of time-ordered processes may be made by changing the orientation of the arms and interpreting particles with arrows pointing back as antiparticles going forward in time, as usual. Each diagram above has a charge conjugate (reverse all arrows on each arm of a vertex). Colour labels have not been given: colour is conserved so that such labelling needs the understanding that $R \rightarrow \overline{G}\overline{B}$, for example, is allowed.

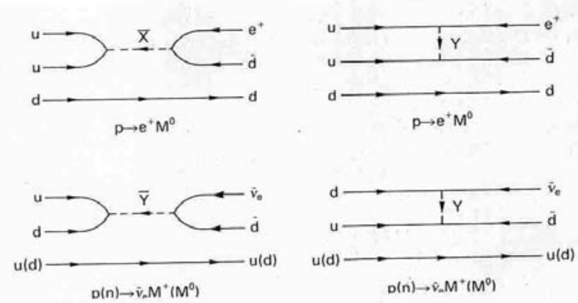

Fig. 13.3 Feynman diagrams for proton or bound neutron decay involving leptoquarks. The M^+ or M^0 represents a meson such as π^+ or π^0, or other states with the correct charge ($\rho, \omega, \ldots$). The usual fragmentation process suggests that additional mesons can be produced, within the limitations of energy and charge conservation.

correct formula we need five powers of an energy. The only one available is the proton rest mass (M_p) energy. Putting these things together gives a formula for the transition rate ω which looks like

$$\hbar\omega \sim \frac{g_s^2}{4\pi}\frac{M_p^5 c^2}{M_X^4}.$$

Then

$$\omega \sim 9 \times 10^{-37}\,\mathrm{s}^{-1},$$

or a mean life $\tau = \frac{1}{\omega} \sim 4 \times 10^{28}$ years.

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(a) Draw Feynman diagrams for the possible proton decay processes,

See Over

$$p \to \mu^+ K^0$$
$$p \to \bar{\nu}_e \pi^+$$

(b) Assume that the proton lifetime is 10^{34} years. Consider a proton decay search experiment which consists of a large cubical tank of water. What would be the dimensions of this tank if one required 10 nucleon decays per year? Use the formula above. The $SU(5)$ coupling constant $g_S^2 \sim 1/50$.

That's All, Folks!
