PHYSICS 357S - Problem Set #2 - January 2015

Distributed **26th January** due to be handed in by **9th February** before 17:00. **Please** have a look at the problem set when it comes out. Decide whether it is going to cause you trouble or not…. And ask questions well before the due date. The problem sets are supposed to give you an opportunity to ask questions. There are SIX questions. *As usual, keep an eye out for typos! I am not a very good typist.*

The first two questions are just standard bookwork. BUT make sure you understand them!

1) In class we discussed how some of the kinetic energy of a *beam* particle colliding with a *target* particle can be transformed into the masses of new particles in the final state. Assume that a beam particle A of total energy E collides with a target particle B *(remember the LAB is defined as the frame where the target is at rest.)* New particles $C_1, C_2 ...$ are produced in the final state. We write this according to the notation:

$$A + B \rightarrow C_1 + C_2 + ... + C_n$$

a) Show that the minimum energy E for A is

$$E = \frac{M^2 - m_A^2 - m_B^2}{2m_B} c^2 \text{, where } M \equiv m_1 + m_2 + ... + m_n$$

This minimum energy is known as the *Threshold Energy* for producing the final state $C_1 + C_2 + ... + C_n$.

b) Imagine an experiment to produce a particle called the $\Upsilon(1S)$. This particle is a bound state made of a b-quark and its anti-particle (the $\bar{b}$ -quark ... we will discuss all these concepts later). The $\Upsilon(1S)$ has a mass of $9.460\,\mathrm{GeV/c^2}$. Our experiment consists of firing a beam of positrons at a target containing stationary electrons. What energy does the positron beam have to have? *(Assume that the final state consists of a single $\Upsilon(1S)$.)* Say we made a machine which collided electron and positron beams head on, with equal and opposite momentum. What would have to be the momentum of each beam?

c) Calculate the minimum π^- momentum in the laboratory necessary for the reaction

$$\pi^- p \rightarrow \Lambda^0 K^0$$

to occur. This experiment consists of firing a *pion* beam in to a liquid hydrogen target. ($m_p = 0.981\,\mathrm{GeV/c^2}$, $m_{\pi^-} = 0.140\,\mathrm{GeV/c^2}$, $m_{\Lambda^0} = 1.116\,\mathrm{GeV/c^2}$, $m_{K^0} = 497.6\,\mathrm{MeV/c^2}$)

2) The new particles produced in these experiments are often unstable, and rapidly decay. Consider a particle A at rest *(i.e. consider the particle in its rest-frame, or CM {centre-of-mass, or centre-of-momentum} frame)* decaying according to the scheme:

$$A \to BC$$

Show that the energy of B is:

$$E_B = \frac{m_A^2 + m_B^2 - m_C^2}{2m_A} c^2$$

and also show that the outgoing momenta are given by

$$\left|\vec{p}_b\right| = \left|\vec{p}_c\right| = \frac{\sqrt{\lambda\left(m_A^2, m_B^2, m_C^2\right)}}{2m_A} c$$

where

$$\lambda(x, y, z) \equiv x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$$

Use these results to find the energy in the CM frame of each decay product in the following reactions.

a) $W^+ \to \mu^+ \nu_\mu$

b) $\pi^0 \to \gamma\gamma$

c) $B^0 \to \pi^- \pi^+$

d) $\Xi \to \Lambda \pi^0$

c) $\Omega^- \to \Lambda K^-$

The W^+ has a mass of $80.425\, GeV/c^2$

The photon γ is the quantum of light, and is massless.

The ν_μ is massless (well…. You can assume it is massless, it has a tiny mass.)

The B particle is a bound state of a *b*-quark and a *d-quark.* It has a mass of $5279\, MeV/c^2$

The Λ particle is in some way like a heavy neutral proton. One of the *u*-quarks in the proton is substituted by an *s*-quark. It only appears with no electric charge, so the superscript is omitted. The mass is $1116\, MeV/c^2$. The Λ is distinguished from the proton by carrying one unit of a quantum number called *strangeness*; due to the *s*-quark. The Ξ has two *s*-quarks and an *u*-quark; its mass is $1314.8\, MeV/c^2$. The Ω^- consists of three s-quarks and so carries three units of this strangeness quantum number. It has a mass of $1672\, MeV/c^2$.

3) **(a)** Consider the process, in the LAB

$$K^+ p \to K^+ \pi^0 p .$$

The final state K^+ has a momentum of $5 GeV/c$ and the π^0 has a momentum of $4 GeV/c$, and the angle between them is 7.3°. Calculate the invariant mass of the $K^+\pi^0$. These momenta and the angle are measured with finite experimental errors, so the invariant mass may be of order 1% away from the "true" invariant mass. Now go to http://pdg8.lbl.gov/rpp2014v1/pdgLive/ParticleGroup.action;jsessionid=3564179CD97FD90E46E69FD74C51584B?node=MXXX020

and check if there is a particle that might have a rest mass corresponding to the invariant mass of this combination.

(Take the mass of a K^+ *to be* $493.677\, MeV/c^2$ *, The mass of the* π *to be* $139.57\, MeV/c^2$*.)*

(b) OK. Say there IS such a particle on that web page, what interaction do you think causes the decay of this particle? Base your answer on the mean lifetime of the particle. You can calculate that from the total width given on the web page.

(c) Assume that the $K^+\pi^0$ comes from an intermediate state, and that the beam momentum in this experiment is $20 GeV/c$. Calculate the momentum of the final state proton in the CM frame.

(d) Say you take many events of this type. Sketch the invariant mass distribution of the $K^+\pi^0$ combination for the cases where the intermediate state particle exists, and where it does not. In the former case, indicate the scale of the x-axis in units of MeV/c^2

4) The ρ^0 meson is like an excited π^0. It has all the same quark content as the pion; but the quarks are in a state with one unit of relative angular momentum… hence the mass is larger than the pion, viz. $769\, MeV/c^2$. It can be produced by colliding a pion beam with a liquid hydrogen target, in the reaction,

$$\pi^- p \to \rho^0 n .$$

It decays in about 10^{-24} s to π^- and π^+, and the decay width is $154\, MeV/c^2$.

(a) What is the lifetime and mean decay distance of a $5\, GeV/c$ ρ^0 in the LAB frame?

(b) What is the π^- threshold energy in the LAB frame for producing ρ^0 in the reaction above?

(c) Assume in the above reaction that the ρ^0 is produced travelling in the same direction as the initial π^- in the LAB frame. What is the minimum and maximum opening angle of the two pions from the rho decay in the LAB frame?

5) In our discussion of colliding beam machines, I discussed colliding particles of equal masses and equal and opposite momenta head on. The Large Hadron Collider at CERN is such a machine; a proton synchrotron. This year it will accelerate counter rotating beams of protons to an energy of 6.5 TeV per beam.

a) If the bending magnets in the LHC have a field of 8.7 T, what is the radius of the machine? *Assume that the machine is circular, and that the tunnel is full of bending magnets.*

b) Why is the machine designed to have a CM energy much greater than the supposed mass of the Higgs? All I want is an explanation in words. *(hint: the Higgs is not produced alone, and it is not at rest)*

c) The (now canceled) Superconducting SuperCollider was designed to be similar to the LHC; but with an energy of 20 TeV per beam. Imagine that you wanted to reach the same centre of mass energy by constructing a giant accelerator to reach the same centre of mass energy as the SSC, but with a stationary proton target. What would be the diameter of this machine if the bending magnets were limited to a field of 6 T?

d) Assume that we want to produce the same CM energy by colliding protons from an accelerator with stationary electrons in a liquid hydrogen target. What would have to be the beam energy of such a machine?

6) *The largest linear accelerator in the world was the SLAC (Stanford Linear Accelerator Center) electron LINAC. The accelerator produced electrons of* $40\,GeV/c$ *momentum. However, plans are afoot to build a linear collider with momentum in each beam of* $500\,GeV/c$ *. The machine will collide electrons with positrons.*

a) What is the mass of the most massive new particle one could produce with this machine? What would the momentum of this particle be in the LAB frame?

b) If we use these electrons from one beam to probe for structures inside the proton, what structure size could we resolve? How does that compare with the present limits on the size of a quark? Calculate the resolving power of the SLAC electron beam

c) Calculate the velocity of the electrons as they exit from the accelerator.

d) If the electrons experience a constant accelerating force *eE* and are accelerated over 15 km, find the effective accelerating field *E. I mean the strength of the electric field in Volts/metre in the direction of the particles, which accelerates them.*

e) How far would the electrons have to travel in this field in order to reach the same velocity if Newtonian mechanics applied?

f) Estimate the length of the one "arm" of this accelerator as seen in the electron rest frame. *Assume that the electrons are always travelling at approximately the velocity of light.*

Possibly Useful Physical Constants:

Avogadro No:	$6\times10^{23}\,mole^{-1}$
pi	$\pi = 3.1416$
speed of light:	$c = 3.0\times10^{8}\, m/s$
Plank's constant:	$\hbar = 6.6\times10^{-22}\,MeV\cdot s$
	$\hbar c = 197 MeV.fm$
	$(\hbar c)^2 = 0.4\,GeV^2\cdot mb$
	$1\ eV = 1.6\times10^{-19}\,Joules$
	$1\ eV/c^2 = 1.8\times10^{-36}\,kg$
	$1\ fm = 10^{-15}\,m$
	$1\ mb = 10^{-27}\,cm^2$
1 year	1 year $\approx \pi\times10^{7}s$
electron charge:	$e = 1.602\times10^{-19}\,C$
electron magnetic moment:	$\mu_e = 9.3\times10^{-24}\,Joules\cdot Tesla^{-1}$
fine structure constant:	$\alpha = e^2/(\hbar c) = 1/137.0360$
strong coupling constant:	$\alpha_s(M_Z) = 0.116\pm0.005$
Fermi coupling constant:	$G_F = 1.166\times10^{-5}\ GeV^{-2}$
Cabibbo angle:	$\sin\theta_C = 0.22$
Weak mixing angle:	$\sin^2\theta_W(M_Z) = 0.2319\pm0.0005$
Branching Ratios	$BR(Z\to e^+e^-) = 3.21\pm0.07\%$
	$BR(Z\to hadrons) = 71\pm1\%$

Particle Properties

Boson	Mass $\left(GeV/c^2\right)$
γ	$< 3\times10^{-36}$
$gluon$	~ 0
$W^{\pm}$	80.22
Z^0	91.187
H^0	~ 125

Lepton	Mass $\left(MeV/c^2\right)$
ν_e	$< 10^{-5}$
e	0.510999
ν_μ	< 0.27
μ	105.658
ν_τ	< 10
τ	1777

Hadron	Quark Content	Mass $\left(MeV/c^2\right)$	$\mathbf{I}\left(\mathbf{J}^{\mathbf{PC}}\right)$
π^+, π^0, π^-	$u\bar{d}, \left(u\bar{u} - d\bar{d}\right)/\sqrt{2}, d\bar{u}$	139.57,134.97, 139.57	$1\left(0^{-+}\right)$
K^+, K^-	$u\bar{s}, s\bar{u}$	493.65	$\frac{1}{2}\left(0^-\right)$
$K^0, \bar{K}^0$	$d\bar{s}, s\bar{d}$	497.67	$\frac{1}{2}\left(0^-\right)$
ρ^+, ρ^0, ρ^-	$u\bar{d}, \left(u\bar{u} + d\bar{d}\right)/\sqrt{2}, \bar{u}d$	775.7	$1\left(1^{--}\right)$
p, n	uud, udd	938.27, 939.57	$\frac{1}{2}\left(\frac{1}{2}^+\right)$
$\Delta^-, \Delta^0, \Delta^+, \Delta^{++}$	ddd, udd, uud, uuu	1232	$\frac{3}{2}\left(\frac{3}{2}^+\right)$
Λ^0	uds	1115.6	$0\left(\frac{1}{2}^+\right)$
$\bar{D}^0, D^0$	$u\bar{c}, c\bar{u}$	1863	$\frac{1}{2}\left(0^-\right)$
D^-, D^+	$d\bar{c}, c\bar{d}$	1869	$\frac{1}{2}\left(0^-\right)$
D_S^+, D_S^-	$c\bar{s}, \bar{c}s$	1968	$0\left(0^-\right)$
B^+, B^-	$u\bar{b}, \bar{u}b$	5279	$\frac{1}{2}\left(0^-\right)$
Λ_c^+	udc	2285	$0\left(\frac{1}{2}^+\right)$
$\Sigma^+, \Sigma^0, \Sigma^-$	uus, uds, dds	1189	$1\left(\frac{1}{2}^+\right)$
Ξ^0, Ξ^-	uss, dss	1315	$\frac{1}{2}\left(\frac{1}{2}^+\right)$
Ω^-	sss	1672	$0\left(\frac{3}{2}^+\right)$
Λ_b	udb	5624	$0\left(\frac{1}{2}^+\right)$