PHYSICS 357S - Problem Set #4 - March 2003

Distributed **26th March**. Due to be handed in by **9th April** at class. After this date it should be handed to Stan Lai. Please be careful handing work in. Try to give it to Stan personally. Lost work cannot be given credit. Please talk to me or Stan if you have difficulties. We're here to help you.

This problem set counts for 10% of the grade. For the numerical values of constants, such as masses, (that I may have forgotten to give you!), you should use the Appendix A at end of the text book starting on page 377. Or you can look in the Review of Particle Physics;There should be a copy in the library. Or you can look on the Web at http://pdg/lbl.gov If you don't understand a question ask me about it. If you think there is a bug (error, typo, etc) in a question..tell me. You might be right!

1) Determine the weak isospin and weak hypercharge currents for the second generation of leptons and quarks, ν_μ, μ^-, c and s'. s' is the weak eigenstate of the s quark.

2) In the standard model the partial width for the decay of the Z^0 to a fermion-antifermion pair is

$$\Gamma\left(Z^0 \to f\bar{f}\right) = 2\left[\left(c_V^f\right)^2 + \left(c_A^f\right)^2\right]\frac{GM_Z^3}{12\pi\sqrt{2}}$$

where c_V^f and c_A^f are the fermion couplings to the Z^0 given in Table 8.1 of Perkins. Assuming three generations of fermions calculate the total width of the Z^0 if $G = 1.17\times10^{-5}\,GeV^{-2}$, $M_Z = 91.2GeV$, and $\sin^2\theta_W = 0.23$. Because each quark comes in three colours, the decays can occur in three times as many ways as those for the uncoloured leptons. Allow for this.

3) Problem 8.4 from Perkins (page 274)

4) For two particles to interconvert (mix), $A \Leftrightarrow B$, it is necessary that they have the same mass (Why?), the same charge, and the same baryon and lepton numbers. In practice, this means that they have to be antiparticles of one another. In the standard model, with the usual three generations, show that *A* and *B* have to be neutral mesons, and identify all the possible quark contents. Figure out which of these particles have been observed so far. Why does a neutron not mix with an antineutron, in the same way that the K^0 and $\overline{K}^0$ to produce the K_1 and K_2? Why does one not see mixing in the *vector (i.e. spin 1, with quark spins aligned)* strange mesons K^{0*} and $\overline{K}^{0*}$?

5) ***Stan covered this in a tutorial. If you weren't there, you should be able to figure it out from Perkins... or maybe you did it in QM.***

The Clebsch Gordan coefficients involved in the addition of angular momenta are usually written in tabular form as follows

		j	j
		m	m
m_1	m_2	**Clebsch**	**Gordan**
m_1	m_2	**Coeffs**	

Where *j, m* are the total angular momentum quantum number, and z-component quantum number, for some total angular momentum state. The z-components of the angular momentum states being added are given by m_1, and m_2.

For the addition of angular momentum 1 and angular momentum of $\frac{1}{2}$ (units of $\hbar$ assumed), the relevant tables are

		$3/2$
		$+3/2$
$+1$	$+1/2$	**1**

		$3/2$	$1/2$
		$+1/2$	$+1/2$
$+1$	$-1/2$	**1/3**	**2/3**
0	$+1/2$	**2/3**	**-1/3**

		$3/2$	$1/2$
		$-1/2$	$-1/2$
0	$-1/2$	**2/3**	**1/3**
-1	$+1/2$	**1/3**	**-2/3**

$3/2$

$-3/2$

$-1 \quad -1/2 \quad$ **1**

(a) I denote an angular momentum state by $|j,m\rangle$. What is the probability of :

i) A $|1,0\rangle$ state and a $|1/2,-1/2\rangle$ state combining to give a $|1/2,-1/2\rangle$ state. *In this and the next part, write out the expansion so that I can see where your result comes from.*

ii) A $|1,-1\rangle$ state and a $|1/2,+1/2\rangle$ state combining to give a $|3/2,-1/2\rangle$ state.

(b) Consider a particle which has intrinsic angular momentum of $\hbar/2$. The operator for the square of the x-component of the intrinsic angular momentum is

$$\hat{S}_x^2 = \frac{\hbar^2}{4}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Look at the action of this operator upon the arbitrary spinor state $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$, and use the result to explain why the operator for the square of the intrinsic angular momentum, $\hat{S}^2$, always has a corresponding eigenvalue of $3\hbar^2/4$.

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6) *This problem is from Chapter 12 of the book by Frauenfelder & Henley. You might want to look at section 12.1 to clarify the notiation.*

(a) The Schroedinger equation is

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi$$

Show that with the Hamiltonian

$$H = \frac{1}{2m}\left(\overline{p} - \frac{q}{c}\overline{A}\right)^2 + qA_0\,,$$

the Schroedinger equation is invariant under the local gauge transformations,

$$\psi'_q = \exp\left(iQ\alpha\left[\overline{x},t\right]\right)\psi_q$$
$$A'_0 = A_0 - \hbar\frac{\partial\alpha}{\partial t}$$
$$\overline{A} = \overline{A} + \hbar c\nabla\alpha$$

(b) I said in class that *"gauge bosons must be massless to have a gauge invariant theory"*. If the photon had a non-zero mass m_γ, Maxwell's equations would be

$$\frac{1}{c^2}\frac{\partial^2 A_0}{\partial t^2} - \nabla^2 A_0 + \frac{m_\gamma^2 c^2 A_0}{\hbar^2} = \rho$$
$$\frac{1}{c^2}\frac{\partial^2 \overline{A}}{\partial t^2} - \nabla^2 \overline{A} + \frac{m_\gamma^2 c^2 \overline{A}}{\hbar^2} = \frac{\overline{j}}{c}$$

Show that Maxwell's equations are gauge invariant only if the photon has zero mass.