

PHYSICS 357S - Problem Set #1 - January 2014

Distributed **13th January**. Due to be handed in by **27th January** at class. After this date it should be handed to Catalina Gomez. Please be careful handing work in. Try to give it to myself or the TA personally. Lost work cannot be given credit.

This problem set counts for 10% of the grade. There are SIX questions. For the numerical values of constants, such as masses, (that I may have forgotten to give you!), you should look at http://www.iop.org/EJ/article/0954-3899/33/1/001/g_33_1_001.html There is also an appendix with some useful info at the end of this set. If you don't understand a question ask me about it. If you think there is a bug (error, typo, etc) in a question..tell me. You might be right! For an elementary discussion of drawing Feynman diagrams see

<http://www.iop.org/EJ/abstract/0031-9120/36/5/301>

(This has a typo corrected in problem#2..tell me if you find any more)

1) The η^0 particle has a rest mass (often just called *mass*) of $549 \text{ MeV}/c^2$. We can define the Kinetic Energy, T , by the relativistic equation,

$$T = E - mc^2.$$

E is the total relativistic energy, and m is the rest mass.

Calculate the momentum, in MeV/c , of a η^0 with a kinetic energy of **(a)** 1MeV, **(b)** 200 MeV, **(c)** 1 GeV.

2) A Kaon (K) is an example of a type of particle known as a meson. It actually consists of a quark and an anti-quark bound together by the strong, or colour, force. The μ is a lepton, as far as we know it is a point particle with no internal structure; just like a quark, but it does not see the color force, just the electromagnetic and weak forces. The μ (muon) can carry either positive or negative electric charge of the same magnitude as the charge carried by the electron (which is also a lepton). The neutrino ν_μ is also a lepton, but it is electrically neutral. Notice the subscript μ . This shows that the neutrino we are talking about is one which is associated with the μ . Each *flavour* of lepton (e, μ, τ) has its own kind of neutrino associated with it. So sometimes a K can decay into a muon and a neutrino. We write this according to the following symbolic form.

$$\text{Initial State} \rightarrow \text{Final state}$$

in this specific case:

$$K^+ \rightarrow \mu^+ \nu_\mu$$

Note that electric charge is conserved in this decay process. The superscript on the particle symbol gives the electric charge of the particle.

a) Assume that a K traveling at a speed v decays this way. If the ν_μ emerges at 90° to the original K direction, show that the μ comes off at an angle of

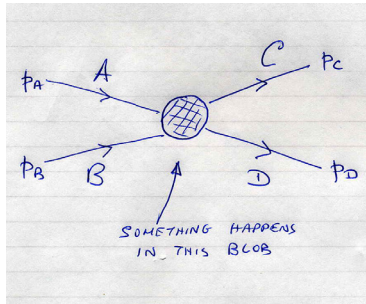
$$\tan \theta = \left(1 - m_\mu^2 / m_K^2\right) / 2\beta\gamma^2$$

This problem is described in the LAB frame. m_K and m_μ are the rest masses of the K and muon, while the neutrino is massless. (*Neutrinos are not massless actually. They have tiny masses, as discovered first at Kamioka in Japan using neutrinos produced by cosmic ray interactions in the earth's atmosphere, and then at Sudbury (Ontario) Solar Neutrino Observatory, using neutrinos from the sun.*)

b) If the K has a β of 0.8 calculate the decay angle for the muon. Then do the same calculation if the particle decay process is $\pi^+ \rightarrow \mu^+ \nu_\mu$. (Take the mass of the π to be $140 \text{ MeV}/c^2$, and the mass of the K^+ to be $493 \text{ MeV}/c^2$.)

c) Suppose the pions have a momentum of $200 \text{ GeV}/c$ when they enter a long evacuated tunnel. Calculate what proportion of them decay in the tunnel if it is 340 m long. (*This is how neutrino beams are produced. You can find more details starting at page 367 of the textbook, the mean lifetime of the pion is $2.6 \times 10^{-8} \text{ s}$*)

3)



In the figure above, two particles with 4-momenta p_A and p_B undergo some interaction in the blob. It could be electromagnetic, strong, weak, or something we have never thought of. The nature of this interaction is the *dynamics* of this interaction. Whatever the nature of this interaction, the *kinematics* is the same. That is, for example, the values of the outgoing 4-momenta p_C and p_D will be constrained by 4-momentum conservation. Also there will be various Lorentz invariant combinations of the 4-momenta. One set of invariants often used is:

$$s \equiv (p_A + p_B)^2$$

$$t \equiv (p_A - p_C)^2$$

$$u \equiv (p_A - p_D)^2$$

a) Why do I know that these are Lorentz invariant?

Now imagine an elastic scattering event between two particles having the same mass m and in the CM frame they have 3-momentum $|\vec{p}|$. After this interaction the particles have the same $|\vec{p}|$, that's what we mean by elastic, and they have scattered through an angle θ in the CM frame. Just to clarify your thoughts sketch this interaction in the CM before and after the interaction. Since these are Lorentz invariant quantities, you can calculate them in any frame you like... and believe me, the CM frame is the one to use!

b) Show that, for this special case of elastic 2-body scattering,

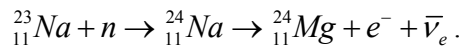
$$\begin{aligned} s &= 4(\vec{p}^2 + m^2) \\ t &= -2\vec{p}^2(1 - \cos\theta) \\ u &= -2\vec{p}^2(1 + \cos\theta) \end{aligned}$$

Note that I am using $c=1$, put in the c 's explicitly if you like.

These Lorentz invariants are called the *Mandelstam Variables*.

c) Explain why the 4-momentum transfer for what I call a *time-like process* is negative, and that of a *space-like process* has to be positive.

4) A nuclear reactor is a powerful source of neutrons. One way of making radioactive materials is by exposing (in this case sodium) to an intense beam of neutrons. This produces an unstable *isotope* of sodium. That is, a nucleus with one more neutron, and the same number of protons (so its atoms have the same number of electrons, and thus the same chemical properties). This unstable isotope can decay via the "decay" of one of the neutrons into a proton, and electron, and a neutrino (known as β -decay). Now we have a nucleus of a different element, e.g.



In a situation like this, the depletion of the target material is negligible, so the rate of production of ${}_{11}^{24}\text{Na}$ is constant. However, the ${}_{11}^{24}\text{Na}$ does decay. If the production rate is p and the decay constant ω , the number of N of ${}_{11}^{24}\text{Na}$, satisfies;

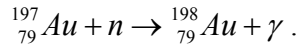
$$\frac{dN}{dt} = p - \omega N.$$

If $N=0$ at $t=0$, the solution of this equations is,

$$N(t) = \frac{p}{\omega}(1 - e^{-\omega t}).$$

Clearly there will be a saturation point when the rate of production is equal to the rate of decay, ωN .

OK.... A sample of Gold is exposed to a neutron beam of constant intensity, such that 10^{10} neutrons per second are absorbed in the reaction



The nucleus ${}_{79}^{198}\text{Au}$ undergoes β -decay to ${}_{80}^{198}\text{Hg}$ (look at the number of protons and neutrons!) with a mean lifetime of 3.89 days. How many nuclei of ${}_{79}^{198}\text{Au}$ will be present after 6 days of irradiation? How many nuclei of ${}_{80}^{198}\text{Hg}$ will be present at that time, assuming that the ${}_{80}^{198}\text{Hg}$ is unaffected by the neutron beam. What is the equilibrium number of ${}_{79}^{198}\text{Au}$ nuclei.

5) If a **parent nucleus (1)** decays to a **daughter nucleus (2)**, which is also radioactive, then the equations governing the growths and decays are

$$\frac{dN_1}{dt} = -\omega_1 N_1 ,$$

$$\frac{dN_2}{dt} = -\omega_2 N_2 + \frac{dN_1}{dt} .$$

In any situation, the outcome depends on the relative decay rates. However, take ${}_{83}^{210}\text{Bi}$ which has a mean lifetime of 7.2 days, and is a β -emitter, giving ${}_{84}^{210}\text{Po}$ (mean life 200 days), which in turn decays by α -emission to ${}_{82}^{206}\text{Pb}$ (again, look at the number of protons and neutrons). ${}_{82}^{206}\text{Pb}$ is stable. If the source initially only contains ${}_{83}^{210}\text{Bi}$, after how long will the rate of α -emission be a maximum?

6) Here are some questions about Feynman diagrams. When you draw them, use the ones I did as examples. Remember to put in little arrows showing the direction of particles and antiparticles, also label each line. Also remember that electric charge is conserved at each vertex of a Feynman diagram. Make sure that you understand what is a virtual particle line, and what is a freely propagating particle line. Virtual lines do not have arrows on them why is that?

a) Draw all the leading order (2 vertices) and next-to-leading order (4 vertices) for electron-positron elastic scattering. I count two leading order and ten next-leading order. *Remember that all the lines have to be connected; except the ends of the incoming and outgoing particles.*

b) Elastic scattering between electrons and positrons, $e^+e^- \rightarrow e^+e^-$, is known as *Bhabha* scattering after the Indian physicist of that name. Determine the mass of the virtual photon in Bhabha scattering, assuming that the electron and positron are at rest. What is the velocity of the virtual photon? Is that possible for *real* photons?

c) In the introductory lecture I mentioned that the Z^0 was a mediator of the weak interaction. So, it couples weakly to both e^+, e^- and to the charged mediators of the weak interaction, W^+, W^- . I also

said that the weak and electromagnetic interactions were really the *same interaction*. That means that a process taking place via virtual photon exchange, can equally well take place through virtual Z^0 exchange. Now, draw all the leading order diagrams contributing to the process $e^+e^- \rightarrow W^+W^-$; assuming that the weak and electromagnetic interactions have equal strength. The Large Electron Positron collider (LEP) at CERN studied this process, as a precision test of the electroweak model.

e) Draw the Feynman diagram for pair production,

$$\gamma \rightarrow e^+e^-$$

and for the process of Bremsstrahlung, which is when an electron radiates a photon, in the field of a heavy nucleus.

$$e^+ \rightarrow \gamma e^+$$

Why do neither of these processes occur in free space, i.e. why do they only occur in the field of a nucleus? Demonstrate this using 4-vector notation

Hint: This is really simple if you use 4-vector notation. Write down the total 4-momentum squared for the initial state and the final state, remembering that the photon has zero mass. Do this in the rest frame of the photon. Remember that 4-momentum is conserved, as is 3-momentum.

Possibly Useful Physical Constants:

Avogadro No:	$6 \times 10^{23} \text{ mole}^{-1}$
pi	$\pi = 3.1416$
speed of light:	$c = 3.0 \times 10^8 \text{ m/s}$
Plank's constant:	$\hbar = 6.6 \times 10^{-22} \text{ MeV} \cdot \text{s}$ $\hbar c = 197 \text{ MeV} \cdot \text{fm}$ $(\hbar c)^2 = 0.4 \text{ GeV}^2 \cdot \text{mb}$
	$1 \text{ eV} = 1.6 \times 10^{-19} \text{ Joules}$
	$1 \text{ eV}/c^2 = 1.8 \times 10^{-36} \text{ kg}$
	$1 \text{ fm} = 10^{-15} \text{ m}$
	$1 \text{ mb} = 10^{-27} \text{ cm}^2$
1 year	$1 \text{ year} \approx \pi \times 10^7 \text{ s}$
electron charge:	$e = 1.602 \times 10^{-19} \text{ C}$
electron magnetic moment:	$\mu_e = 9.3 \times 10^{-24} \text{ Joules} \cdot \text{Tesla}^{-1}$
fine structure constant:	$\alpha = e^2/(\hbar c) = 1/137.0360$
strong coupling constant:	$\alpha_s(M_Z) = 0.116 \pm 0.005$
Fermi coupling constant:	$G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$
Cabibbo angle:	$\sin \theta_C = 0.22$
Weak mixing angle:	$\sin^2 \theta_W(M_Z) = 0.2319 \pm 0.0005$
Branching Ratios	$BR(Z \rightarrow e^+ e^-) = 3.21 \pm 0.07\%$ $BR(Z \rightarrow \text{hadrons}) = 71 \pm 1\%$

Particle Properties

Boson	Mass (GeV/c^2)
γ	$< 3 \times 10^{-36}$
<i>gluon</i>	~ 0
W^\pm	80.22
Z^0	91.187
H^0	> 116

Lepton	Mass (MeV/c^2)
ν_e	$< 10^{-5}$
e	0.510999
ν_μ	< 0.27
μ	105.658
ν_τ	< 10
τ	1777

Hadron	Quark Content	Mass (MeV/c^2)	$\mathbf{I}(\mathbf{J}^{\mathbf{PC}})$
π^+, π^0, π^-	$u\bar{d}, (u\bar{u} - d\bar{d})/\sqrt{2}, d\bar{u}$	139.57, 134.97, 139.57	$1(0^{+-})$
K^+, K^-	$u\bar{s}, s\bar{u}$	493.65	$\frac{1}{2}(0^-)$
K^0, \bar{K}^0	$d\bar{s}, s\bar{d}$	497.67	$\frac{1}{2}(0^-)$
ρ^+, ρ^0, ρ^-	$u\bar{d}, (u\bar{u} + d\bar{d})/\sqrt{2}, \bar{u}d$	775.7	$1(1^{--})$
p, n	uud, udd	938.27, 939.57	$\frac{1}{2}\left(\frac{1}{2}^+\right)$
$\Delta^-, \Delta^0, \Delta^+, \Delta^{++}$	ddd, udd, uud, uuu	1232	$\frac{3}{2}\left(\frac{3}{2}^+\right)$
Λ^0	uds	1115.6	$0\left(\frac{1}{2}^+\right)$
\bar{D}^0, D^0	$u\bar{c}, c\bar{u}$	1863	$\frac{1}{2}(0^-)$
D^-, D^+	$d\bar{c}, c\bar{d}$	1869	$\frac{1}{2}(0^-)$
D_s^+, D_s^-	$c\bar{s}, \bar{c}s$	1968	$0(0^-)$
B^+, B^-	$u\bar{b}, \bar{u}b$	5279	$\frac{1}{2}(0^-)$
Λ_c^+	udc	2285	$0\left(\frac{1}{2}^+\right)$
$\Sigma^+, \Sigma^0, \Sigma^-$	uus, uds, dds	1189	$1\left(\frac{1}{2}^+\right)$
Ξ^0, Ξ^-	uss, dss	1315	$\frac{1}{2}\left(\frac{1}{2}^+\right)$
Ω^-	sss	1672	$0\left(\frac{3}{2}^-\right)$
Λ_b	udb	5624	$0\left(\frac{1}{2}^+\right)$

