PHYSICS 357S - Problem Set #2 - January 2014

Distributed **27th January** due to be handed in by **10th February** before 17:00. **Please** have a look at the problem set when it comes out. Decide whether it is going to cause you trouble or not…. And ask questions well before the due date. The problem sets are supposed to give you an opportunity to ask questions. There are SIX questions. *As usual, keep an eye out for typos! I am not a very good typist.*

The first two questions are just standard bookwork. BUT make sure you understand them!

1) In class we discussed how some of the kinetic energy of a *beam* particle colliding with a *target* particle can be transformed into the masses of new particles in the final state. Assume that a beam particle A of total energy E collides with a target particle B *(remember the LAB is defined as the frame where the target is at rest.)* New particles $C_1, C_2 ...$ are produced in the final state. We write this according to the notation:

$$A + B \rightarrow C_1 + C_2 + ... + C_n$$

a) Show that the minimum energy E for A is

$$E = \frac{M^2 - m_A^2 - m_B^2}{2m_B} c^2 \text{, where } M \equiv m_1 + m_2 + ... + m_n$$

This minimum energy is known as the *Threshold Energy* for producing the final state $C_1 + C_2 + ... + C_n$.

b) We can imagine producing new particles by firing a beam of protons or pions into a target of liquid hydrogen. A liquid hydrogen target is just a target of stationary protons. Many new bound states of the various quarks were discovered in this way. Use the result above to determine the minimum momentum for the beam particles in the following experiments. Note that I miss out the "+" signs between the lists of particles in the initial and final states.

a) $pp \rightarrow pp\pi^0$

b) $pp \rightarrow pp\pi^+\pi^-$

c) $\pi^- p \rightarrow p\bar{p}n$

d) $\pi^- p \rightarrow K^0\Sigma^0$

e) $pp \rightarrow p\Sigma^+K^0$

You can look particle masses and other properties at http://pdg.lbl.gov/ . I give a summary at the end of this assignment. It's the same table you will get on the exam. To help, I also give them here:

p is the symbol for the proton , mass $= 938\, MeV/c^2$.

n is the symbol for the neutron, mass $= 939.6\, MeV/c^2$.

π^0 is the symbol for the neutral pion, mass = $135\,MeV/c^2$

$\pi^\pm$ is the symbol for a charged pion., mass = $140\,MeV/c^2$

K^0 is the symbol for a neutral Kaon, mass $= 498\,MeV/c^2$. The Kaon is a meson, like the pion.

Σ is the sigma particle, it is like a heavy proton, the mass is $1189\,MeV/c^2$ if it is charged, and $1193\,MeV/c^2$ if it is neutral.

$\bar{p}$ is the anti-particle of the proton, known as the anti-proton. Anti-particles have the same masses as particles, but opposite electric charges. So the $\bar{p}$ has the same mass as the proton, but has a negative electric charge

2) The new particles produced in these experiments are often unstable, and rapidly decay. Consider a particle A at rest *(i.e. consider the particle in its rest-frame, or CM {centre-of-mass, or centre-of-momentum} frame)* decaying according to the scheme:

$$A \to BC$$

Show that the energy of B is:

$$E_B = \frac{m_A^2 + m_B^2 - m_C^2}{2m_A} c^2$$

and also show that the outgoing momentum are given by

$$|\vec{p}_b| = |\vec{p}_c| = \frac{\sqrt{\lambda\left(m_A^2, m_B^2, m_C^2\right)}}{2m_A} c$$

where

$$\lambda(x, y, z) \equiv x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$$

Use these results to find the energy in the CM frame of each decay product in the following reactions.

a) $D^+ \to \mu^+ \nu_\mu$

b) $\pi^0 \to \gamma\gamma$

c) $K^+ \to \pi^0 \pi^+$

d) $\Delta^{++} \to p\pi^+$

c) $\Omega^- \to \Lambda K^+$

The μ has a mass of $106\,MeV/c^2$

The photon γ is the quantum of light, and is massless.

ν_μ is massless (well, it's so small that you can assume that it is zero). The η^0 is in some way a heavy version of the π^0

The Δ^{++} particle is in some way like a heavy proton. It has the *quark structure (uuu)*. In fact it can be considered as an *excited state* of the proton. The mass is $1232\, MeV/c^2$.

The K^+ is distinguished from the π^+ not only by being heavier; but it also carries one unit of a quantum number called *strangeness.* The K^+ and π^+ are mesons; we will learn that this means they are made up of a quark and an anti-quark. A state made of three quarks is called a baryon, The proton is a familiar *baryon.* The Ω^- is also a baryon; but it is a *strange baryon.* It carries three units of this strangeness quantum number. It has a mass of $1672\, MeV/c^2$.

3) The strangeness quantum number is conserved in strong interaction, but can change by one unit in a weak interaction. The following is a strong interaction process producing the Ω^-.

$$K^- p \rightarrow \Omega^- K^+ K^0$$

The K^- has a strangeness of one unit (+1) (it has one s-quark in it). The K^+ has opposite strangeness (-1), as it is the antiparticle of the K^-, it contains an anti-s quark.

The Ω^- has three s-quarks, and so has strangeness of +3. For strangeness to be conserved, this means that the K^0 has to contain an anti-s quark and have strangeness of -1. The sum of the strangeness does not change from the initial to the final state in the reaction above. The K^0 decays to a $\pi^- \pi^+$ pair. There are no final state strange-quarks in this decay… we will learn what happens. Anyway, this question is about the kinematics.

(a) What is the threshold *kinetic* energy for the reaction to occur if the proton is at rest. Write it in terms of the masses of the particles involved.

(b) Suppose the K^0 travels at a speed $0.8\,c$. It decays into two pions. Find the maximum angle in the LAB frame that the pions can make with the K^0 line of flight.

Hint: use a Lorentz transformation to take the pions from the K^0 rest frame to the LAB. *Then get the tan of the angle required and find its maximum.*

4) In relativistic collisions the kinetic energy of the colliding particles can be transformed into the mass of *new particles.* So neither the number of particles, nor the total mass, is necessarily conserved in a relativistic collision. Very short lived particles can be produced, and we detect their presence by looking for a peak in the invariant mass of the products of the decay.

Say we suspected, or predicted, that there was a particle which decayed to an oppositely charged pair of *K* mesons. We could search for this particle by firing pions into a liquid hydrogen target.

$$\pi^+ p \to K^+ K^- \pi^+ p$$

We would then reconstruct the invariant masses of the oppositely charged *K* mesons, and build up a histogram of the invariant mass distribution over many events. If the invariant mass showed an enhancement at some mass, we could conclude that we had discovered the particle. Consider one event, and assume that the K^+ and the K^-, in the final state, are produced so that they have an angle of 6.3° between them , and that they have momenta 10.0 *GeV/c*, and 5.0 *GeV/c* respectively, in the LAB. What is the *invariant mass* of the K^+K^- system?

5) The relationship between the "proper" force $\overline{f}$ and the change in energy and momentum is given in relativistic kinematics by,

$$\frac{d\overline{p}}{dt} = \overline{f} \; ; \; \frac{dE}{dt} = \overline{f}.\overline{v} \, .$$

The velocity in the observer's frame is $\overline{v}$, and the proper force is just the 4-force in the instantaneous rest frame. So,

$$\frac{d\overline{p}}{dt} = \frac{E}{c^2}\frac{d\overline{v}}{dt} + \frac{v}{c^2}\frac{dE}{dt} \, .$$

Show that

$$m\frac{d\overline{v}}{dt} = \left(\overline{f} - \frac{\overline{v}\left(\overline{v}.\overline{f}\right)}{c^2} \right)\gamma^{-1} ,$$

Where I have kept *c* explicitly.

The International Linear Collider will consist of two linear accelerators colliding head-on. Assume that one of them is 15 km long and electrons are subject to a constant force of *eE*, in the LAB frame, and are accelerated to a momentum of 500 TeV/c in the LAB frame.

a) What is the final velocity if the electrons if they start from rest.

b) Find the field strength *E. I mean the strength of the electric field in Volts/metre in the direction of the particles, which accelerates them.*

c) How far would the electrons have to travel in this field in order to reach the same velocity, if Newtonian mechanics applied?

Estimate the length of the accelerator in the electrons' rest frame. *Assume that the electrons are always travelling at approximately the velocity of light*

6) The Large Hadron Collider is a proton collider designed to produce the Higgs boson. The Higgs was observed in 2012 (http://en.wikipedia.org/wiki/Higgs_boson) with a mass of $125\,\mathrm{GeV/c^2}$. While the particle looks like the Higgs, there is still uncertainty about its properties, due to the fact that the colliding quarks inside the protons do not have a well-defined momentum. The International Linear Collider is designed to study the couplings of the Higgs in detail, as the colliding electrons and positrons will have well defined momenta. To go to even higher energies than the LHC, a Muon Collider has been proposed. This would be a synchrotron storage ring colliding μ^+ and μ^- head on.

a) How would you produce the muons necessary to inject into the storage ring? Look at p.367 in the text book.

b) With a site which is 50 km on the side, what is the maximum beam energy, if the machine is built in a circular tunnel full of bending magnets with a field of 5 Tesla?

c) Assume that we want to produce the same CM energy by colliding muons from an accelerator with stationary protons in a liquid hydrogen target. What would have to be the beam energy of such the muons? What would be CM energy be if these muons collided with the atomic electrons in the liquid hydrogen?

d) Use the diagram 11.12 on page of the text book to estimate the energy of the protons you need to use to produce this energy of muons. *Assume the energy spectrum of muons and neutrinos in the decay is the same, scale the peak of the spectrum in the figure.*

e) In the proton machine in section

d) Calculate the relative energy losses if the beams are protons, muons, or electrons. Calculate the power loss for proton and electron beams. Look at page 26 of the textbook.

Possibly Useful Physical Constants:

Avogadro No:	$6\times10^{23}\,mole^{-1}$
pi	$\pi = 3.1416$
speed of light:	$c = 3.0\times10^{8}\,m/s$
Plank's constant:	$\hbar = 6.6\times10^{-22}\,MeV\cdot s$
	$\hbar c = 197\,MeV.fm$
	$(\hbar c)^2 = 0.4\,GeV^2\cdot mb$
	$1\ eV = 1.6\times10^{-19}\,Joules$
	$1\ eV/c^2 = 1.8\times10^{-36}\,kg$
	$1\ fm = 10^{-15}\,m$
	$1\ mb = 10^{-27}\,cm^2$
1 year	1 year $\approx \pi\times10^{7}s$
electron charge:	$e = 1.602\times10^{-19}\,C$
electron magnetic moment:	$\mu_e = 9.3\times10^{-24}\,Joules\cdot Tesla^{-1}$
fine structure constant:	$\alpha = e^2/(\hbar c) = 1/137.0360$
strong coupling constant:	$\alpha_s(M_Z) = 0.116 \pm 0.005$
Fermi coupling constant:	$G_F = 1.166\times10^{-5}\ GeV^{-2}$
Cabibbo angle:	$\sin\theta_C = 0.22$
Weak mixing angle:	$\sin^2\theta_W(M_Z) = 0.2319 \pm 0.0005$
Branching Ratios	$BR(Z\to e^+e^-) = 3.21 \pm 0.07\%$
	$BR(Z\to hadrons) = 71 \pm 1\%$

Particle Properties

Boson	Mass $\left(GeV/c^2\right)$
γ	$<3\times10^{-36}$
$gluon$	~ 0
$W^{\pm}$	80.22
Z^0	91.187
H^0	~ 125

Lepton	Mass $\left(MeV/c^2\right)$
ν_e	$<10^{-5}$
e	0.510999
ν_μ	<0.27
μ	105.658
ν_τ	<10
τ	1777

Hadron	Quark Content	Mass $\left(MeV/c^2\right)$	$\mathbf{I}\left(\mathbf{J}^{\mathbf{PC}}\right)$
π^+,π^0,π^-	$u\bar{d},\left(u\bar{u}-d\bar{d}\right)/\sqrt{2},d\bar{u}$	139.57,134.97, 139.57	$1\left(0^{-+}\right)$
K^+,K^-	$u\bar{s},s\bar{u}$	493.65	$\frac{1}{2}\left(0^-\right)$
$K^0,\bar{K}^0$	$d\bar{s},s\bar{d}$	497.67	$\frac{1}{2}\left(0^-\right)$
ρ^+,ρ^0,ρ^-	$u\bar{d},\left(u\bar{u}+d\bar{d}\right)/\sqrt{2},\bar{u}d$	775.7	$1\left(1^{--}\right)$
p,n	uud,udd	938.27, 939.57	$\frac{1}{2}\left(\frac{1}{2}^+\right)$
$\Delta^-,\Delta^0,\Delta^+,\Delta^{++}$	ddd,udd,uud,uuu	1232	$\frac{3}{2}\left(\frac{3}{2}^+\right)$
Λ^0	uds	1115.6	$0\left(\frac{1}{2}^+\right)$
$\bar{D}^0,D^0$	$u\bar{c},c\bar{u}$	1863	$\frac{1}{2}\left(0^-\right)$
D^-,D^+	$d\bar{c},c\bar{d}$	1869	$\frac{1}{2}\left(0^-\right)$
D_S^+,D_S^-	$c\bar{s},\bar{c}s$	1968	$0\left(0^-\right)$
B^+,B^-	$u\bar{b},\bar{u}b$	5279	$\frac{1}{2}\left(0^-\right)$
Λ_c^+	udc	2285	$0\left(\frac{1}{2}^+\right)$
$\Sigma^+,\Sigma^0,\Sigma^-$	uus,uds,dds	1189	$1\left(\frac{1}{2}^+\right)$
Ξ^0,Ξ^-	uss,dss	1315	$\frac{1}{2}\left(\frac{1}{2}^+\right)$
Ω^-	sss	1672	$0\left(\frac{3}{2}^+\right)$
Λ_b	udb	5624	$0\left(\frac{1}{2}^+\right)$