

PHYSICS 357S - Problem Set #3 - February 2015

Distributed **9th Feb** and be handed in by **25th February** before 17:00. **Please** have a look at the problem set when it comes out. Decide whether it is going to cause you trouble or not.... And ask questions well before the due date. The problem sets are supposed to give you an opportunity to ask questions. There are 7 questions. *As usual, keep an eye out for typos! I am not a very good typist.*

(1) Baryons like the proton, or neutron, are bound states of three quarks. Since we can have all sorts of orbital and spin angular momentum combinations, the spin (by spin I mean the modulus of the maximum z-component of the angular momentum. An electron is spin $\frac{1}{2}$) The spin spectrum of baryons is much more complex than mesons.

a) What are the possible spin states of a baryon in which the quarks have no relative angular momentum?

b) What are the possible spin states for a baryon in which the quarks can have one unit of orbital angular momentum?

c) An electron in a hydrogen atom is in a state with orbital angular momentum quantum number $l=1$. If the total angular momentum quantum number j is $\frac{3}{2}$, and the z component of the total angular momentum is $\frac{1}{2}\hbar$, what is the probability of finding the electron with $m_s = +\frac{1}{2}$?

d) Suppose you had two particles of spin 2 each in the states $|2,+1\rangle$. They collide and form a system (bound state) of zero orbital angular momentum. What values could you get for the total angular momentum of this system? What is the probability of each value. You get some value with probability 1, so check that the sum of your probabilities is unity.

e) A particle has intrinsic spin 1, and is in a $|1,-1\rangle$ state. It decays to a spin 1/2 particle and a spin 3/2 particle, which have no relative orbital angular momentum. What is the probability of finding the spin 1/2 particle in the state $|\frac{1}{2},+\frac{1}{2}\rangle$? In this case, what is the spin state of the 3/2 particle?

(2) A tungsten target of thickness 0.1 cm intercepts a particle beam of 3 cm^2 area. Nuclear scattering occurs between the beam particles and the W nuclei. W has a density of 19.3 gm/cm^3 and an atomic weight of 183.4.

(a) Calculate the number of scattering centers intercepted by the beam.

(b) Assuming a total cross section of 0.1 mb for the interaction, what fraction of the incident beam is scattered?

(3) A beam of neutrons of kinetic energy 0.29 eV , and intensity 10^5 s^{-1} traverses normally a foil of ${}^{235}_{92}\text{U}$, thickness 10^{-1} kg m^{-2} . Any neutron-nucleus collision can have one of three possible results:

a) elastic scattering of neutrons: $\sigma_e = 2 \times 10^{-30}\text{ m}^2$

b) Capture of a neutron followed by the emission of a γ -ray by the nucleus:

$$\sigma_c = 7 \times 10^{-27}\text{ m}^2$$

c) capture of the neutron followed by the splitting of the nucleus into two almost equal parts (fission): $\sigma_f = 2 \times 10^{-26}\text{ m}^2$

Calculate :

i) the attenuation of the beam by the foil;

ii) the number of fission reactions occurring per second in the foil, caused by the incident beam;

iii) the flux of elastically scattered neutrons at a point 10m from the foil and out of the incident beam, assuming isotropic distribution of the scattered neutrons. Isotropic means that the scattered flux is independent of the scattering angle

(4) (a) The typical cross section for the absorption of antineutrinos with the typical energies emitted from a reactor is 10^{-43} cm^2 . Compute the thickness of water absorber needed to reduce the intensity of an antineutrino beam by a factor of 2.

(b) Since neutrinos are very penetrating particles, they are difficult to detect. A liquid scintillator is an organic liquid which emits small flashes of light when traversed by a charged particle. Typically liquid scintillators are some mixture of Hydrogen and Carbon, with an atomic ratio of $H/C = 1.10$. The density of a typical scintillator might be about 0.95 g/cm^3 . Consider an experiment where a tank containing 10^3 litres of liquid scintillator intercepts a beam of antineutrinos from a reactor. The flux of antineutrinos is $10^{13}\text{ } \bar{\nu}/\text{cm}^2\text{ sec}$. How many events of the type $\bar{\nu}_e p \rightarrow e^+ n$ will occur per day, if the cross section is 10^{-43} cm^2 ?

(c) What property of the final state particles would one have to observe in order to be sure that the initial particle was a neutrino and not an antineutrino? Write down the analogous reaction which contains an initial state neutrino.

(5) a) At the end of the lecture on “particle Classification & Lepton Number”. I drew diagrams showing the flow of the quarks and force bosons for the interaction

$$K^- p \rightarrow \Lambda^0 \pi^0.$$

Draw the same kind of diagrams for the interaction $\pi^+ n \rightarrow K^+ \Lambda^0$, followed by $K^+ \rightarrow \pi^0 \pi^+$ and $\Lambda^0 \rightarrow p \pi^-$. There is an \bar{s} quark in the K^+ , and an s quark in the Λ^0 . You have to produce them from a *gluon*, in a manner analogous to the production of the $\bar{u}u$ in the $K^- p \rightarrow \Lambda^0 \pi^0$. This is an example of the colour force *conserving strangeness, or quark flavour*. Also note that the weak interaction can change an s quark into a u quark or an \bar{s} into a \bar{u} . This is an example of the weak force *not conserving quark flavour*. In each of these quark flow diagrams, explicitly state which force is acting in each decay.

b) At the end of the lecture on “Scattering and Feynman Diagrams”, I showed the quark flow diagrams for various weak processes. By analogy, draw the quark flow diagrams for the weak interaction processes,

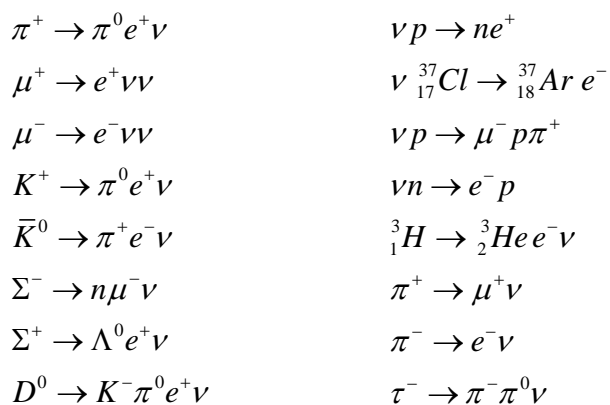
$$\Xi^0(uss) \rightarrow \Lambda^0 \pi^0$$

$$\bar{\nu}_e p \rightarrow e^+ n$$

$$\tau^- \rightarrow \nu_\mu \bar{\nu}_\mu \mu^-$$

(6) (a) The Δ^{++} has spin $\frac{3}{2}$. What does that tell you about the angular momentum state of the quarks in it? How does that make you conclude that the quarks must carry an additional quantum number, as well as spin, electric charge, and mass? Think about the spin-statistics of spin $\frac{1}{2}$. What is that quantum number? If you can't figure this out, it is a well know argument, you'll find it by Googling Δ^{++}

(b) Assign the lepton generation (*this is the same as lepton flavour*) subscript and distinguish antineutrinos from neutrinos in the reactions and decays and decays on the next page. Use the symbols $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$. Draw the lowest order Feynman diagram for each process. Where there are mesons or baryons in both sides of the process, just show how the quarks flow through the interaction; but remember to label them with their flavours and charges. The chart at the end of the problem set will help you with this. It is from the book Nuclear and Particle Physics by W.S.C. Williams



(7) The formula I gave for the luminosity of a colliding beam accelerator works for circular colliders or linear colliders. The luminosity of the proposed International Linear Collider is given by

$$\mathcal{L} = \frac{N_1 N_2}{4\pi\sigma_x \sigma_y} f n_b.$$

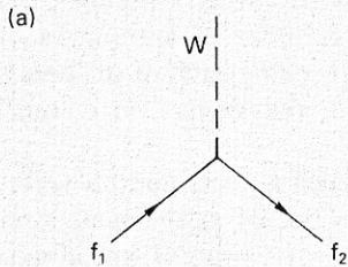
N_1 and N_2 are the number of particles per bunch in each of the colliding beams, f is the repetition frequency of pulses from the machine, n_b is the number of bunches per pulse, and σ_x and σ_y are the r.m.s. size of the beams in the x and y directions. The beams are supposed to have a Gaussian profile. The relevant parameters of the ILC are:

Beam Parameters for 500 GeV in CMS (Global Design Group Feb 2007)	
Repetition Rate, f Hz	5
Number of Particles per Bunch N	2×10^{10}
Number of bunches per pulse n_b	2625
R.M.S. beam size σ_x (nanometers)	639
R.M.S. beam size σ_y (nanometers)	5.7

Notice the tiny cross section of the beams. How do you get them to collide when the accelerators are 15 km long? Do you have to have an accuracy of 6 nanometers over 15 km? I don't expect you to answer that! Just be amazed.

- what is the luminosity?
- If the cross section for collisions is 10 picobarns, determine the total number of scattering events per second.
- Find the *average* flux of electrons.

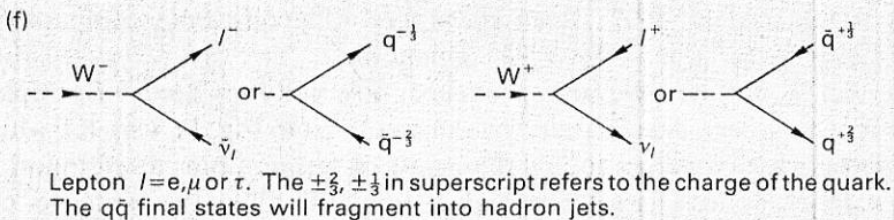
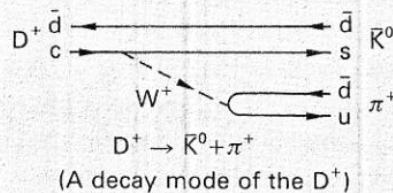
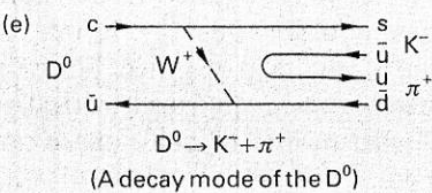
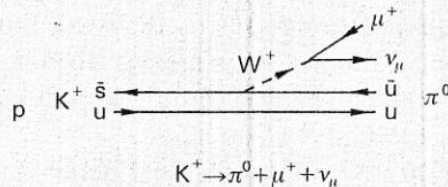
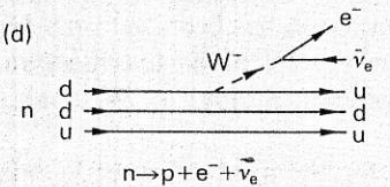
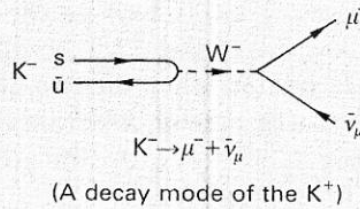
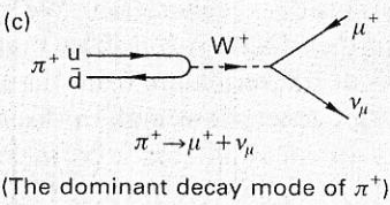
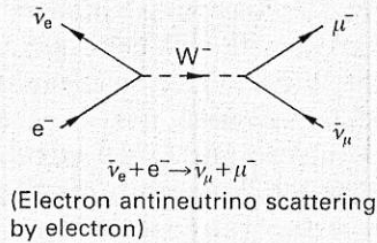
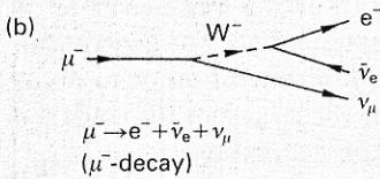
(d) If the beam of electrons is extracted from the machine and allowed to scatter from a stationary target of liquid hydrogen (density = 0.1 g.cm^{-3}) 2 m. long, find the number of scattering events per second and compare to the answer in (b). You have to assume that the scattering cross section in this case is the same as in the colliding beam case. This is unrealistic..... but there we are.



Rules for the Wf_1f_2 vertex

1. Electric charge conserved and, from f_1 to f_2
2. Lepton (generation) number conserved
3. Quark number conserved
4. Quark flavour is not conserved.
5. Quark colour is conserved.

Examples follow in (b) to (f). Time from left to right



Gluon exchange is omitted in the examples involving hadrons. The $u\bar{u}$ pair in $D^0 \rightarrow K^- \pi^+$ is created by gluon exchange with the other quarks.

Possibly Useful Physical Constants:

Avogadro No:	$6 \times 10^{23} \text{ mole}^{-1}$
pi	$\pi = 3.1416$
speed of light:	$c = 3.0 \times 10^8 \text{ m/s}$
Plank's constant:	$\hbar = 6.6 \times 10^{-22} \text{ MeV} \cdot \text{s}$ $\hbar c = 197 \text{ MeV} \cdot \text{fm}$ $(\hbar c)^2 = 0.4 \text{ GeV}^2 \cdot \text{mb}$
	$1 \text{ eV} = 1.6 \times 10^{-19} \text{ Joules}$
	$1 \text{ eV}/c^2 = 1.8 \times 10^{-36} \text{ kg}$
	$1 \text{ fm} = 10^{-15} \text{ m}$
	$1 \text{ mb} = 10^{-27} \text{ cm}^2$
1 year	$1 \text{ year} \approx \pi \times 10^7 \text{ s}$
electron charge:	$e = 1.602 \times 10^{-19} \text{ C}$
electron magnetic moment:	$\mu_e = 9.3 \times 10^{-24} \text{ Joules} \cdot \text{Tesla}^{-1}$
fine structure constant:	$\alpha = e^2/(\hbar c) = 1/137.0360$
strong coupling constant:	$\alpha_s(M_Z) = 0.116 \pm 0.005$
Fermi coupling constant:	$G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$
Cabibbo angle:	$\sin \theta_C = 0.22$
Weak mixing angle:	$\sin^2 \theta_w(M_Z) = 0.2319 \pm 0.0005$
	$BR(Z \rightarrow e^+ e^-) = 3.21 \pm 0.07\%$
Branching Ratios	$BR(Z \rightarrow \text{hadrons}) = 71 \pm 1\%$

Particle Properties

Boson	Mass (GeV/c^2)
γ	$< 3 \times 10^{-36}$
<i>gluon</i>	~ 0
W^\pm	80.22
Z^0	91.187
H^0	125

Lepton	Mass (MeV/c^2)
ν_e	$< 10^{-5}$
e	0.510999
ν_μ	< 0.27
μ	105.658
ν_τ	< 10
τ	1777

Hadron	Quark Content	Mass (MeV/c^2)	$\mathbf{I}(\mathbf{J}^{\text{PC}})$
π^+, π^0, π^-	$u\bar{d}, (u\bar{u} - d\bar{d})/\sqrt{2}, d\bar{u}$	139.57, 134.97, 139.57	$1(0^{+-})$
K^+, K^-	$u\bar{s}, s\bar{u}$	493.65	$\frac{1}{2}(0^-)$
K^0, \bar{K}^0	$d\bar{s}, s\bar{d}$	497.67	$\frac{1}{2}(0^-)$
ρ^+, ρ^0, ρ^-	$u\bar{d}, (u\bar{u} + d\bar{d})/\sqrt{2}, \bar{u}d$	775.7	$1(1^{--})$
p, n	uud, udd	938.27, 939.57	$\frac{1}{2}\left(\frac{1}{2}^+\right)$
$\Delta^-, \Delta^0, \Delta^+, \Delta^{++}$	ddd, udd, uud, uuu	1232	$\frac{3}{2}\left(\frac{3}{2}^+\right)$
Λ^0	uds	1115.6	$0\left(\frac{1}{2}^+\right)$
\bar{D}^0, D^0	$u\bar{c}, c\bar{u}$	1863	$\frac{1}{2}(0^-)$
D^-, D^+	$d\bar{c}, c\bar{d}$	1869	$\frac{1}{2}(0^-)$
D_s^+, D_s^-	$c\bar{s}, \bar{c}s$	1968	$0(0^-)$
B^+, B^-	$u\bar{b}, \bar{u}b$	5279	$\frac{1}{2}(0^-)$
Λ_c^+	udc	2285	$0\left(\frac{1}{2}^+\right)$
$\Sigma^+, \Sigma^0, \Sigma^-$	uus, uds, dds	1189	$1\left(\frac{1}{2}^+\right)$
Ξ^0, Ξ^-	uss, dss	1315	$\frac{1}{2}\left(\frac{1}{2}^+\right)$
Ω^-	sss	1672	$0\left(\frac{3}{2}^-\right)$
Λ_b	udb	5624	$0\left(\frac{1}{2}^+\right)$