PHYSICS 357S - Problem Set #4 - March 2015

*There are **6** questions and **10** pages in this problem set.*

Distributed Wednesday **25th February**. Due to be handed in by **11th March** before 17:00. You can give it to me at class, give it to Laurelle, or put it in my mail box in room 804. Please be careful handing work in. Try to give it to one of us personally. Lost work cannot be given credit. **Please** have a look at the problem set when it comes out. Decide whether it is going to cause you trouble or not…. And ask questions well before the due date. The problem sets are supposed to give you an opportunity to ask questions.

There are SIX questions

There is a lot of text, but that is just to enlarge on some things in the lectures, and to try to help. I think the questions are actually quite simple.

As usual, keep an eye out for typos! I am not a very good typist.

(1) (a) Both $W^{\pm}$ and Z^0 exchange diagrams can contribute to interactions. You can consider quark line diagrams as being the Feynman diagrams for the weak interaction part of these processes. I say weak "interaction part " since any quarks will be exchanging gluons, and we are ignoring this. We ignore it because the gluon exchange takes place at a much larger distance scale than the weak interaction part. So these quark line diagrams really correspond to *weak interaction amplitudes.*

Draw the quark line diagrams for the following weak interactions. Include any exchanged virtual *W* or *Z*. One of the processes has TWO possible diagrams. These correspond to two possible amplitudes contributing to the scattering cross section.

$$\nu_e e^- \to \nu_e e^-$$
$$\nu_e n \to \nu_e n$$
$$\nu_\mu n \to \mu^- p$$
$$\pi^- p \to \Lambda \pi^0$$

(b) Draw quark line diagrams for the following process

$$\tau^- \to e^- \bar{\nu}_e \nu_\tau$$
$$K^0 \to \pi^- e^+ \nu_e$$
$$D^+ \to \bar{K}^0 \mu^+ \nu_\mu$$
$$\tau^+ \to \pi^+ \nu_\tau$$
$$\Lambda^0 \to p e^- \bar{\nu}_e$$
$$\Xi^- \to \Sigma^0 \pi^-$$

Note that the Ξ^- is a baryon containing two strange quarks *(dss)*, and the Σ^0 is a baryon containing one strange quark. Notice that while the Z^0 can be involved in scattering (exchange) processes, it does not show up in decays. The chart at the end of the problem set will help you.

(2) Classify the following experimentally observed process into strong, electromagnetic and weak interactions by considering the particles involved and the appropriate selections rules (i.e. quantum numbers conserved)

$$\pi^- + p \to \pi^- + \pi^+ + n$$

$$\gamma + p \to \pi^0 + p$$

$$\bar{\nu}_\mu + p \to \mu^+ + n$$

$$\pi^0 \to e^+ + e^- + e^+ + e^-$$

$$p + \bar{p} \to \pi^+ + \pi^- + \pi^0$$

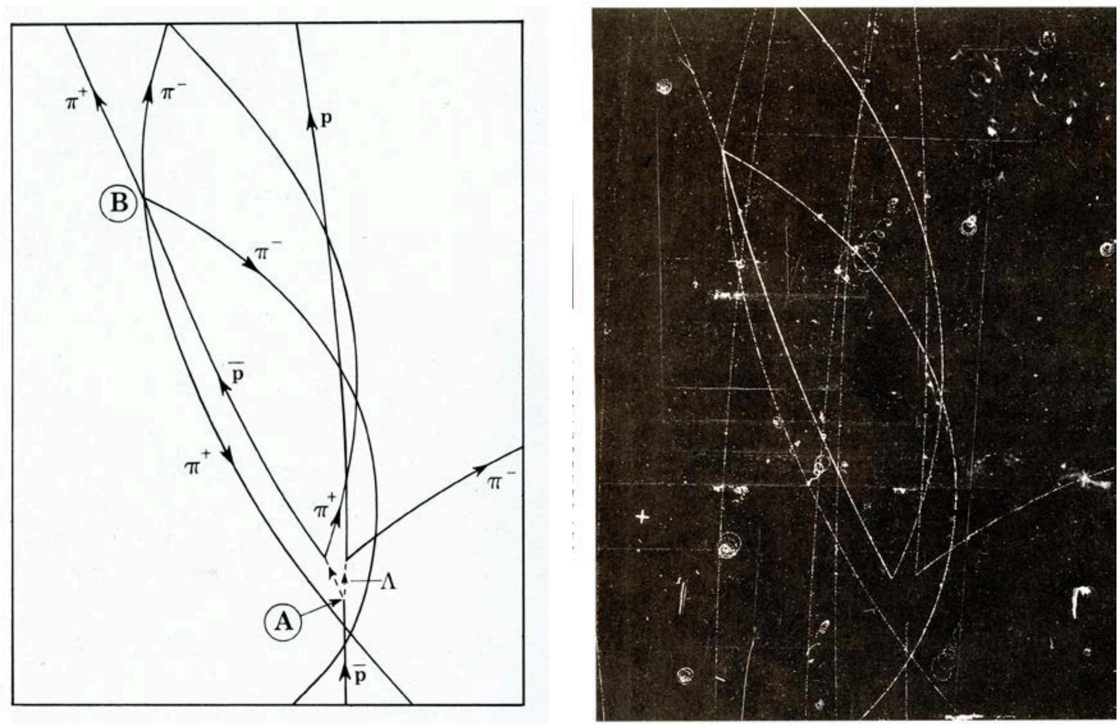
$$\tau^- \to \pi^- + \nu_\tau$$

$$D^- \to K^+ + \pi^- + \pi^-$$

$$\pi^- \to \pi^0 + e^- + \bar{\nu}_e$$

$$\Lambda + p \to K^- + p + p$$

(3)



The above figure is a hydrogen bubble chamber picture and an explanatory sketch. Charged particles leave a track in the chamber, while neutral particles do not. The vertical scale of the picture is about 1 m. The event is produced by the interaction of a $\bar{p}$ at point A.

(a) Which force cases the initial interaction of the $\bar{p}$ in the bubble chamber?

(b) Use the conservation laws for the known quantum numbers to deduce what neutral particle is produced at A in association with the Λ. Explain your conclusion.

(c) Which force causes the decay of the two neutral particles? Explain your conclusion, numerically; given the size of the bubble chamber.

(d) Draw a quark line diagram for the initial interaction of the incoming $\bar{p}$.

(e) Draw a quark line diagram of the decay of the particle you identified in **(b).**

(f) Write down the interaction at B, and list the relevant quantum numbers. Explain any that are conserved or violated. Which force causes that interaction?

(g) How would you measure the rest mass of the Λ?

(h) Say you observed many examples of the decay of a short-lived particle. Sketch the distribution in masses you would expect. Given that the mean lifetime of the Λ is $2.6\times10^{-10}s$, explain numerically why you would, or would not, expect to see such a distribution if you were observing Λ decays? *(in a bubble chamber the precision that you can measure track momenta with is, at best, a few MeV*

(4) (a) Show that for a spherically symmetrical charge distribution, the form factor is

$$F\left(q^2\right)=1-\frac{q^2\left\langle r^2\right\rangle}{6\hbar^2}+O\left(q^4\right).$$

Where $\left\langle r^2\right\rangle$ is the mean of the square of the electric charge distribution. Note that q^2 refers to the 3-momentum throughout this question.

Outline: First you should write down the general expression for the form factor $F\left(q^2\right)$ in terms of the charge density $\rho\left(\overline{r}\right)$. By assuming spherical symmetry, you will end up with an integral with a term $\sin\left(qr/\hbar\right)$ in it. You can expand this as

$$\frac{\sin\left(qr/\hbar\right)}{qr/\hbar}=\left[1-\left(\frac{1}{6}\right)\left(\frac{qr}{\hbar}\right)^2+...\right].$$

b) Show that the form factor corresponding to the charge distribution

$$\rho\left(r\right)=\rho_0 e^{-r/a}/r$$

is

$$F\left(q^2\right)=\frac{1}{\left[1+\left(q^2a^2/\hbar^2\right)\right]}.$$

Note that $\rho\left(r\right)$ is spherically symmetrical, so you can use the integral for $F\left(q^2\right)$ from part **a).** *Remember to normalize the result by dividing through by the integral over the charge density over the volume of the nucleus.*

(c) Now let's look at what the form factor does in numerical terms. An electron of momentum 500 MeV/c is scattered through an angle of 11° by an argon nucleus. Assume that there is no recoil and calculate the *momentum transfer* and also the *reduced de Broglie wavelength* of the electron. Further, calculate the *Mott differential cross section*, which corresponds to the case where the argon nucleus could be considered to be point-like. Finally, calculate by how much the *differential cross section* changes if the argon nucleus is considered to be represented by the spatial distribution in part **(a)** of this question. You should take the value of $r=1.2\times A^{1/3}\ fm$. *Argon has an Atomic Mass Number of 40, and an Atomic Number of 18.*

(5) (a) The Bethe-Weizsäcker, or Semi-empirical, mass formula is based on the idea of the nucleus behaving like a liquid drop. Show explicitly that for fixed *A*, the expression for *M(A,Z)* has a minimum value. Is there any evidence for the "valley of stability" (the valley of stability is the region of stable nuclides, when you plot the number of neutrons, *N*, against the Atomic Number *Z*, see Fig. 4.6 on next page, or Fig 16.3 in the textbook). Which is the most stable nuclide with *A*=16? And which for *A*=208? *You can either differentiate the semi-empirical mass formula, or plot M as a function of Z. Do both, so that you can get a visual impression of the situation. Use the following form (My lecture version was slightly different in how I expressed the last term)*

$$B.E. = -a_1 A + a_2 A^{2/3} + a_3 \frac{Z^2}{A^{1/3}} + a_4 \frac{(N-Z)^2}{A} \pm a_5 A^{-3/4}$$

and

$$M(A,Z)c^2 = (A-Z)m_n c^2 + Zm_p c^2 - a_1 A + a_2 A^{2/3} + a_3 \frac{Z^2}{A^{1/3}} + a_4 \frac{(A-2Z)^2}{A} \pm a_5 A^{-3/4}$$

(b) Using the semi-empirical mass formula (with the Fermi Gas model corrections) compute the total binding energy and *B/A* for $^{8}_{4}\mathrm{Be}$, $^{12}_{6}\mathrm{C}$, $^{56}_{26}\mathrm{Fe}$, and $^{208}_{82}\mathrm{Pb}$. Compare these values with the values calculated from the fact that the atomic masses of these nuclides are, 8.005305 u, 12.000000 u , 55.934939 u, 207.976626 u. The atomic mass of a proton is 1.00794 u and a neutron 1.008665 u

Take $a_V \cong 15.6 MeV$, $a_S \cong 17.2 MeV$, $a_C \cong 0.70 MeV$, $a_A \cong 23.3 MeV$, $a_P \cong 12.0 MeV$

(6) (a) This is a simple problem in electrostatics. Show that the potential energy due to electrostatic forces of a uniformly charged sphere of total charge *Q* and radius *R* is

$$\frac{3Q^2}{20\pi\varepsilon_0 R}$$

Calculate the work done in bringing a charge dq contained in the layer thickness dr. Then integrate from 0 to R.

The Coulomb term in the semi-empirical (liquid drop) mass formula is

$$\frac{a_c Z^2}{A^{1/3}}.$$

(b) Using the result of part **(a)** calculate the value of a_c in MeV/c^2 . Assume as usual that the nuclear radius is given by $R = 1.24 \times A^{1/3}\ fm$, and use the fact that

$$\frac{e^2}{4\pi\varepsilon_0} = \frac{197.3}{137.04} MeV \bullet fm$$

Using the values of a_V, a_S and a_A given in the Table 4.1 below, and the fact that the binding energy of ${}^{181}_{73}Ta$ is 1454 MeV, check your value for a_c.

Now, interestingly enough, the nucleus ${}^{235}_{92}U$ can undergo spontaneous fission, one of the many fission channels is

$${}^{235}_{92}U \to {}^{87}_{36}Br + {}^{145}_{57}La + 3\,neutrons$$

Estimate the energy released in this process.

4.6 The β-stability valley. Filled squares denote the stable nuclei and long-lived nuclei occurring in nature. Neighbouring nuclei are unstable. Those for which data on masses and mean lives are known fill the area bounded by the lines. For the most part these unstable nuclei have been made artificially. (Data taken from *Chart of the Nuclides* (1977), Schenectady: General Electric Company.)

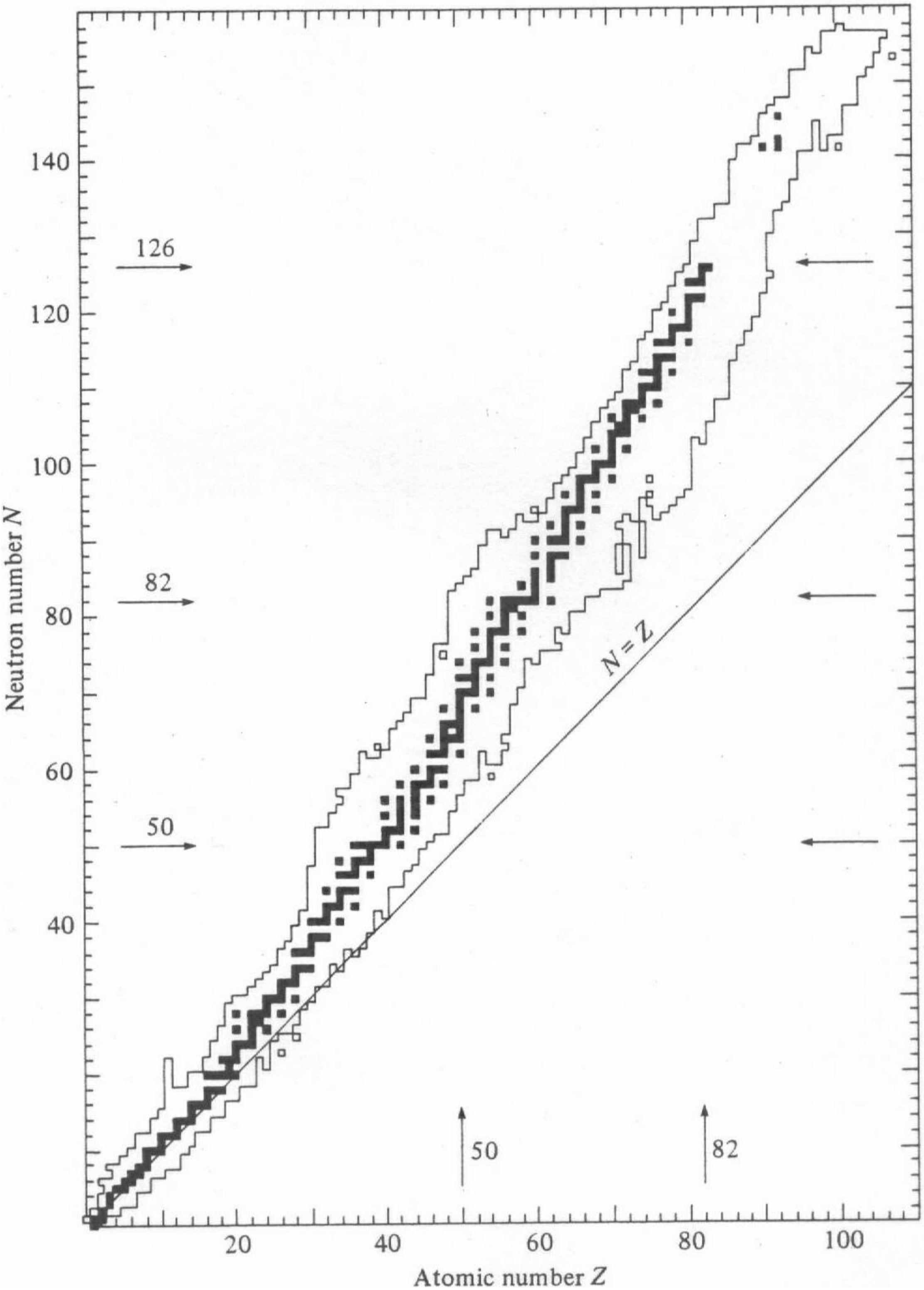


Table 4.1 The nuclear semi-empirical mass formula summarized.

$$M(Z,A)c^2 = ZM_pc^2 + (A-Z)M_nc^2 - B(Z,A)$$

Nuclear rest mass energy
Rest of mass of constituents
less the binding energy

where

$B(Z,A) =$	$+a_V A$	Volume binding term
	$-a_S A^{2/3}$	Surface energy term
	$-a_C Z^2/A^{1/3}$	Coulomb term
	$-a_A (A-2Z)^2/A$	Asymmetry term
	$\left\{\begin{array}{ll} -a_P/A^{1/2} & \text{oo nuclei} \\ +0 & \text{eo and oe nuclei} \\ +a_P/A^{1/2} & \text{ee nuclei} \end{array}\right\}$	Pairing term

M_pc^2 = rest mass energy of the proton = 938.280 MeV.
M_nc^2 = rest mass energy of the neutron = 939.573 MeV.
A favoured set of values for the coefficients:

$$a_V = 15.56 \text{ MeV},$$
$$a_S = 17.23 \text{ MeV},$$
$$a_C = 0.697 \text{ MeV},$$
$$a_A = 23.285 \text{ MeV},$$
$$a_P = 12.0 \text{ MeV}.$$

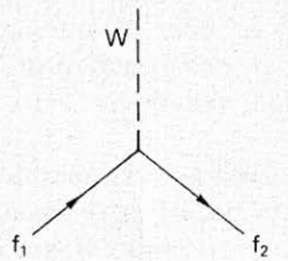
To obtain the atomic rest mass energy, change M_P, the proton mass, to M_H, the mass hydrogen atom, thus:

$$\mathcal{M}(Z,A)c^2 = \text{atomic rest mass energy} \approx ZM_Hc^2 + (A-Z)M_nc^2 - B(Z,A).$$

(Note: $\approx$ because this formula neglects some atomic electron binding energy.)
M_Hc^2 = rest mass energy of the hydrogen atom = 938.791 MeV.

Figure 4.5 shows how the various contributions (except the pairing change with A throughout the periodic table. What is surprising is th formula is good from $A \simeq 20$ to the end of the periodic table with a pre better than $1\frac{1}{2}\%$ on the binding energy. This is shown in the case of the nuclei in Fig. 4.6.

(a)



Rules for the Wf_1f_2 vertex

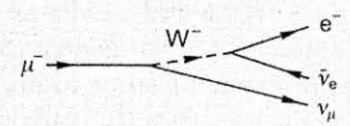
1. Electric charge conserved

and, from f_1 to f_2

2. Lepton (generation) number conserved
3. Quark number conserved
4. Quark flavour is not conserved.
5. Quark colour is conserved.

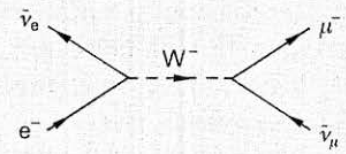
Examples follow in (b) to (f). Time from left to right

(b)



$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$

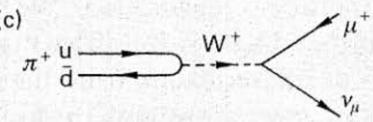
(μ^--decay)



$\bar{\nu}_e + e^- \to \bar{\nu}_\mu + \mu^-$

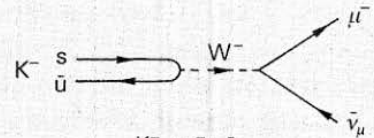
(Electron antineutrino scattering by electron)

(c)



$\pi^+ \to \mu^+ + \nu_\mu$

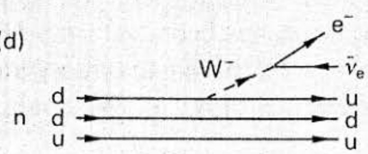
(The dominant decay mode of π^+)



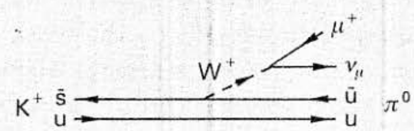
$K^- \to \mu^- + \bar{\nu}_\mu$

(A decay mode of the K^+)

(d)

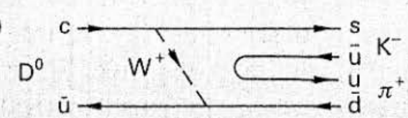


$n \to p + e^- + \bar{\nu}_e$



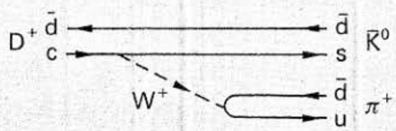
$K^+ \to \pi^0 + \mu^+ + \nu_\mu$

(e)



$D^0 \to K^- + \pi^+$

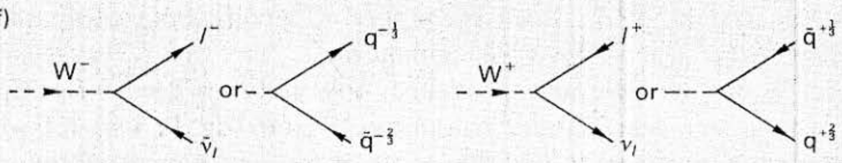
(A decay mode of the D^0)



$D^+ \to \bar{K}^0 + \pi^+$

(A decay mode of the D^+)

(f)



Lepton $l = e, \mu$ or τ. The $\pm\frac{2}{3}$, $\pm\frac{1}{3}$ in superscript refers to the charge of the quark. The $q\bar{q}$ final states will fragment into hadron jets.

Gluon exchange is omitted in the examples involving hadrons. The $u\bar{u}$ pair in $D^0 \to K^-\pi^+$ is created by gluon exchange with the other quarks.

Possibly Useful Physical Constants:

Avogadro No:	$6\times10^{23}\, mole^{-1}$
pi	$\pi = 3.1416$
speed of light:	$c = 3.0\times10^{8}\, m/s$
Plank's constant:	$\hbar = 6.6\times10^{-22}\, MeV\cdot s$
	$\hbar c = 197\, MeV.fm$
	$(\hbar c)^2 = 0.4\, GeV^2\cdot mb$
	$1\ eV = 1.6\times10^{-19}\, Joules$
	$1\ eV/c^2 = 1.8\times10^{-36}\, kg$
	$1\ fm = 10^{-15}\, m$
	$1\ mb = 10^{-27}\, cm^2$
1 year	$1\text{ year} \approx \pi\times10^{7}\, s$
electron charge:	$e = 1.602\times10^{-19}\, C$
electron magnetic moment:	$\mu_e = 9.3\times10^{-24}\, Joules\cdot Tesla^{-1}$
fine structure constant:	$\alpha = e^2/(\hbar c) = 1/137.0360$
strong coupling constant:	$\alpha_s(M_Z) = 0.116 \pm 0.005$
Fermi coupling constant:	$G_F = 1.166\times10^{-5}\ GeV^{-2}$
Cabibbo angle:	$\sin\theta_C = 0.22$
Weak mixing angle:	$\sin^2\theta_W(M_Z) = 0.2319 \pm 0.0005$
Branching Ratios	$BR(Z\to e^+e^-) = 3.21\pm0.07\%$
	$BR(Z\to hadrons) = 71\pm1\%$

Particle Properties

Boson	Mass $\left(GeV/c^2\right)$
γ	$<3\times10^{-36}$
$gluon$	~ 0
$W^{\pm}$	80.22
Z^0	91.187
H^0	>116

Lepton	Mass $\left(MeV/c^2\right)$
ν_e	$<10^{-5}$
e	0.510999
ν_μ	<0.27
μ	105.658
ν_τ	<10
τ	1777

Hadron	Quark Content	Mass $\left(MeV/c^2\right)$	$\mathbf{I}\left(\mathbf{J^{PC}}\right)$
π^+,π^0,π^-	$u\bar{d},\left(u\bar{u}-d\bar{d}\right)/\sqrt{2},d\bar{u}$	139.57,134.97, 139.57	$1\left(0^{-+}\right)$
K^+,K^-	$u\bar{s},s\bar{u}$	493.65	$\frac{1}{2}\left(0^-\right)$
$K^0,\bar{K}^0$	$d\bar{s},s\bar{d}$	497.67	$\frac{1}{2}\left(0^-\right)$
ρ^+,ρ^0,ρ^-	$u\bar{d},\left(u\bar{u}+d\bar{d}\right)/\sqrt{2},\bar{u}d$	775.7	$1\left(1^{--}\right)$
p,n	uud,udd	938.27, 939.57	$\frac{1}{2}\left(\frac{1}{2}^+\right)$
$\Delta^-,\Delta^0,\Delta^+,\Delta^{++}$	ddd,udd,uud,uuu	1232	$\frac{3}{2}\left(\frac{3}{2}^+\right)$
Λ^0	uds	1115.6	$0\left(\frac{1}{2}^+\right)$
$\bar{D}^0,D^0$	$u\bar{c},c\bar{u}$	1863	$\frac{1}{2}\left(0^-\right)$
D^-,D^+	$d\bar{c},c\bar{d}$	1869	$\frac{1}{2}\left(0^-\right)$
D_S^+,D_S^-	$c\bar{s},\bar{c}s$	1968	$0\left(0^-\right)$
B^+,B^-	$u\bar{b},\bar{u}b$	5279	$\frac{1}{2}\left(0^-\right)$
Λ_c^+	udc	2285	$0\left(\frac{1}{2}^+\right)$
$\Sigma^+,\Sigma^0,\Sigma^-$	uus,uds,dds	1189	$1\left(\frac{1}{2}^+\right)$
Ξ^0,Ξ^-	uss,dss	1315	$\frac{1}{2}\left(\frac{1}{2}^+\right)$
Ω^-	sss	1672	$0\left(\frac{3}{2}^+\right)$
Λ_b	udb	5624	$0\left(\frac{1}{2}^+\right)$