PHYSICS 357S - Problem Set #5 - March 2015

Distributed **12^{th} March**. Due to be handed in by **1st April.**

This problem set counts for 10% of the grade. It has 7 questions and 9 pages. If you don't understand a question ask me about it. If you think there is a bug (error, typo, etc) in a question..tell me. You might be right!

Please check the Web page and your email for any important messages as we approach the end of term. If you have any question in the period leading up to the exam, please feel free to just pass by my office and talk to me. Any day of the week is fine. It's usually best to email or phone in advance.

(1) This question is about the shell model of the nucleus. I didn't explicitly talk about nuclear spin and parity. However, it is much the same as the argument for magnetic moments. Actually in IS the same. You just have to count the filled levels, and figure out what is the unpaired nucleon.

(i) the spin of all even-even nuclei is zero

(ii) the spin of odd-A nuclei are given by the unpaired nucleon.

We also haven't discussed parity at great length, but you must know roughly what that is from the quantum mechanics of the potential well. The nuclear parity is the product of all the single nucleon wave functions $\prod_A (-1)^l$. It follows that

(iii) the parity of the ground state of all even-even nuclei is even.

(iv) the parity of the ground state of all odd-A nuclei is that of the wavefunction of the unpaired nucleon.

(b) From the equation $\bar{J} = \bar{L} + \bar{S}$ for a single particle in an eigenstate of operators $\hat{J}, \hat{L}, \hat{S}$, show that,

$$\left(\bar{L} \bullet \bar{S}\right) = \frac{1}{2}\left(\bar{J}^2 - \bar{L}^2 - \bar{S}^2\right)$$

Use this to show that the energy separation of two energy levels of a nucleon in a nucleus with spin-orbit coupling, is proportional to $l + \frac{1}{2}$.

(c) In the vector picture of quantum mechanical angular momentum, we can *define* the angle between the total angular momentum and the orbital angular momentum by,

$$\left\langle \frac{\bar{J} \bullet \bar{L}}{\sqrt{\bar{J}^2}\sqrt{\bar{L}^2}} \right\rangle .$$

For the case of $l = 1$, $j = \frac{3}{2}$, what is the angle between *l* and *j*?

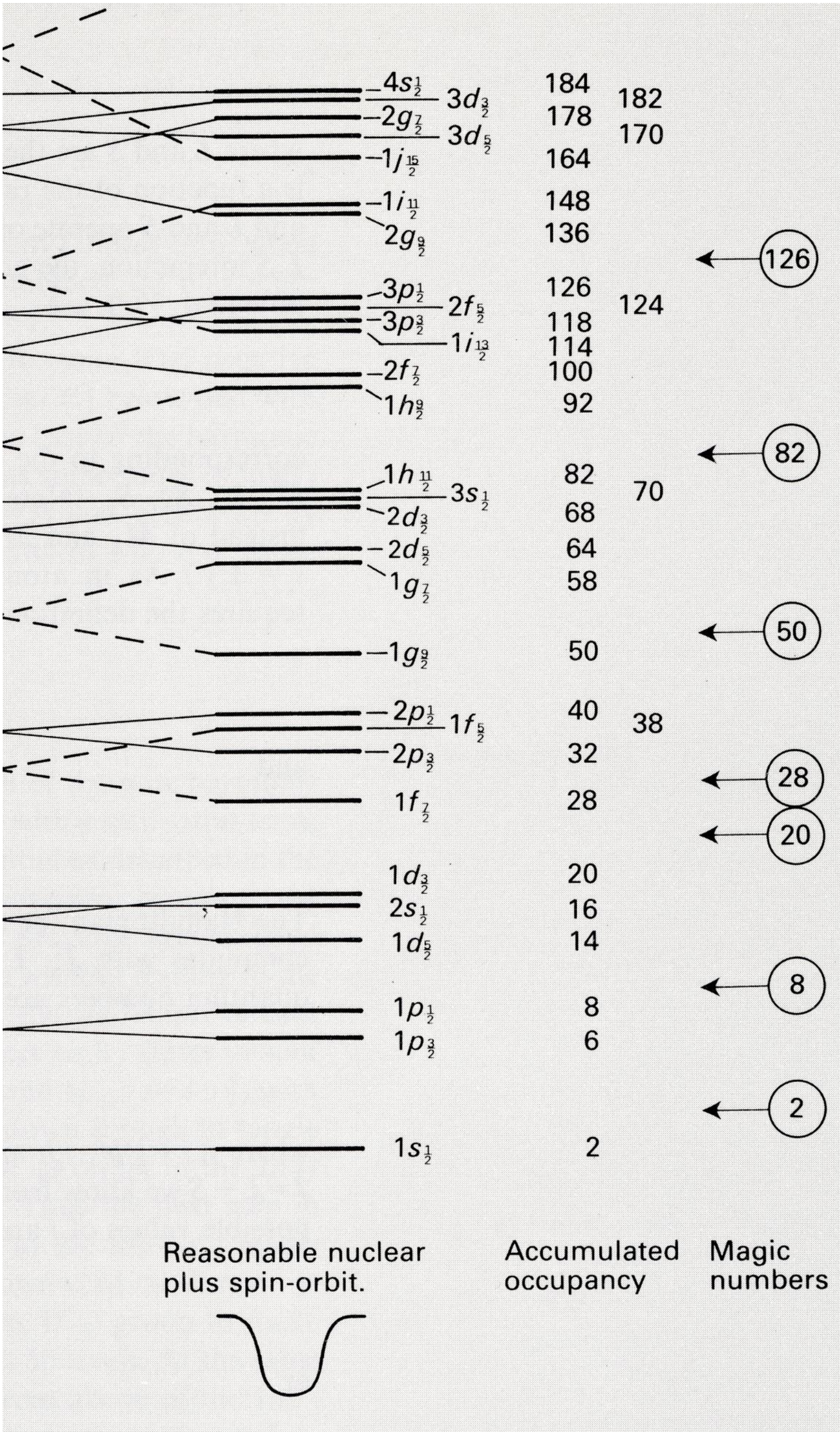
(d) The contribution of the total angular moment, the orbital angular momentum, and the nucleon spin of an unpaired nucleon in an A-odd nucleus are given by $\bar{\mu}_j = g_j \bar{J}$, $\bar{\mu}_l = g_l \bar{L}$, $\bar{\mu}_S = g_S \bar{S}$, in units of nuclear magnetons. Starting from the fact that $\bar{\mu}_j = g_j \bar{J} = g_l \bar{L} + g_S \bar{S}$, show that

$$g_j = g_l \left(\frac{\left[j(j+1) + l(l+1) - s(s+1) \right]}{2j(j+1)} \right) + g_S \left(\frac{\left[j(j+1) - l(l+1) + s(s+1) \right]}{2j(j+1)} \right)$$

Below is a table of the odd-A nuclei up to ${}^{19}_{9}F$. Determine the type, neutron or proton, of the odd nucleon and use the level ordering, in my notes on the shell mode to decide its configuration. Fill out the blank columns in the table. Be careful. In some of these nuclei, it is not true that *l* and *s* are parallel. The agreement is not perfect…right?

Nucleus	Odd nucleon type and configuration	Nuclear spin-parity	Magnetic dipole moment nuclear magnetons Calculated	Measured
$^{3}_{1}H$				2.9788
$^{3}_{2}He$				−2.1276
$^{7}_{3}Li$				3.2564
$^{9}_{4}Be$				−1.1776
$^{11}_{5}B$				2.6885
$^{11}_{6}C$				−1.0300
$^{13}_{6}C$				0.7024
$^{13}_{7}N$				0.3221
$^{15}_{7}N$				−0.2831
$^{15}_{8}O$				0.7189
$^{17}_{8}O$				−1.8937
$^{17}_{9}F$				4.7224
$^{19}_{9}F$				2.6288

NB: I have reproduced the shell structure diagram in the notes on the next page.


4s1/2
3d3/2
2g7/2
3d5/2
1j15/2
1i11/2
2g9/2
3p1/2
2f5/2
3p3/2
1i13/2
2f7/2
1h9/2
1h11/2
3s1/2
2d3/2
2d5/2
1g7/2
1g9/2
2p1/2
1f5/2
2p3/2
1f7/2
1d3/2
2s1/2
1d5/2
1p1/2
1p3/2
1s1/2
184
182
178
170
164
148
136
126
124
118
114
100
92
82
70
68
64
58
50
40
38
32
28
20
16
14
8
6
2
126
82
50
28
20
8
2
Reasonable nuclear plus spin-orbit.
Accumulated occupancy
Magic numbers

(2) One can perform deep inelastic scattering experiments using electrons, muons or neutrinos. Since electrons and muons will effectively interact via the electromagnetic interaction, they probe the electric charge distribution within the nucleon. Since neutrinos have no electric charge they will probe the distribution of matter independently of electric charge; in fact they interact via the weak interaction. The reason that neutrinos are interesting is that they can only exist with their spins parallel ("right-handed") or antiparallel ("left-handed") to their momentum, and they can be used to probe the existence of "right-handed" or "left-handed" objects within the nucleon. Since the weak interaction does not conserve parity (we will discuss what this means) right-handed and left-handed objects have different interactions.

i) Why does it not make sense to think of electrons or muons as intrinsically right or left handed?

ii) Why did experiments at SLAC use electrons to probe nucleon structure, while Fermilab used muons? *Think about how what particles are accelerated at SLAC and at Fermilab.*

iii) In deep inelastic scattering, the invariant mass of the final state hadronic system, W, is an indication of how inelastic the scattering is, and how "deeply" one is probing the nucleon structure. What are the maximum values of W that can be produced with electrons at SLAC and muons at Fermilab. *You need to make some assumption about the maximum muon beam energy ... Just assume that the muon energy is 1/3 of the proton energy of the Tevatron at Fermilab. see the textbook page 367 if you are interested in checking whether my guess makes any sense.*

iv) The total cross section for deep inelastic scattering of electrons from protons is calculated in the quark-parton model to be just the cross section for scattering off of three quarks with electric charges of either $2/3$ and $-1/3$. One averages over the quark charges, and assumes that the scattering is independent (see page 166 of Henley & Garcia). When the first deep inelastic experiments were performed the fact that quarks carried the *colour* charge was unknown. Show that the quark-parton result is unchanged by the fact that each quark can carry any of three different colour charges, as long as the three colour charges are present in the proton in equal proportions.

(3) (a) The Δ^{++} has $J^P = \frac{3}{2}^+$. It decays to p and a π^+ via the strong force. If the p has $J^P = \frac{1}{2}^+$, and the spin *0* π^+ is found to be in an *l=1* orbital state , what does that tell you about the parity of the pion?

(b) The major decay mode of the Ξ^0 is $\Xi^0 \to \Lambda\pi^0$. The Λ has the same spin and parity as the proton. Do you think this decay mode can be used to determine the spin and parity of the π^0? Why? Do you think that the Ξ^0 as a definite spin and parity? Why? *It might help to make a quark line diagram of the decay* $\Xi^0 \to \Lambda\pi^0$.

(4) Much of what was initially learned about subatomic symmetries was learned from the pattern of meson decays. (What is a meson, by the way?). There are many different kinds of mesons with different spins, parities, etc. These quantum numbers depend on the quark content, and relative angular momentum. There is a meson called the η which is neutral and is an "isosinglet"; that means it has no charged partners. The neutral pion is a member of an isotriplet; it has positive and negative charged partners. The η is useful for testing *C*- invariance. Which of the following decays are allowed and forbidden by *C*-invariance?

$$\eta \to \gamma\gamma$$
$$\eta \to \pi^0\gamma$$
$$\eta \to \pi^0\pi^0\pi^0$$
$$\eta \to \gamma\gamma\gamma$$
$$\eta \to \pi^+\pi^-\pi^0$$

The photon is $J^{PC} = 1^{--}$, and the η is $J^{PC} = 0^{-+}$

(5) A positive pion and a negative pion have orbital angular momentum *l* in their CM frame.

(a) Determine the *C* parity of this $\left(\pi^+\pi^-\right)$ system. *Think about the fact that the parity operation is equivalent to interchanging the two pions.*

(b) If *l=1*, can this system decay into two photons? Why?

(6) Show that the operator product $\overline{J}.\overline{p}$, is time reversal invariant. $\overline{J}$ is the angular momentum and $\overline{p}$ is the momentum of a particle. *You can do this by drawing a diagram like Figure 9.8 on page 259 of the text book*

(7) For two particles to interconvert or mix**,** $A \Leftrightarrow B$, it is necessary that they have the same mass (Why?), the same charge, and the same baryon and lepton numbers. In practice, this means that they have to be antiparticles of one another. In the standard model, with the usual three generations, show that *A* and *B* have to be neutral mesons, and identify all the possible quark contents. Figure out which of these particles have been observed so far, *(for example, look at the Particle Data Group website).* Why does a neutron not mix with an antineutron, in the same way that the K^0 and $\bar{K}^0$ to produce the K_1 and K_2? Why does one not see mixing in the *vector (i.e. spin 1, with quark spins aligned)* strange mesons K^{0*} and $\bar{K}^{0*}$? *For the neutron look at Figure 9.10 on page 263 of the text book. Can you have a transition like this between the neutron and the antineutron? For the* K^{0*} *about the force which causes it to decay and compare that to the force which causes the* K^0 decay*, again can you have a diagram which is the analogue of Figure 9.10?*

That's All, Folks!

Possibly Useful Physical Constants:

Avogadro No:	$6\times10^{23}\,mole^{-1}$
pi	$\pi = 3.1416$
speed of light:	$c = 3.0\times10^{8}\,m/s$
Plank's constant:	$\hbar = 6.6\times10^{-22}\,MeV\cdot s$
	$\hbar c = 197 MeV . fm$
	$(\hbar c)^2 = 0.4\,GeV^2\cdot mb$
	$1\ eV = 1.6\times10^{-19}\,Joules$
	$1\ eV/c^2 = 1.8\times10^{-36}\,kg$
	$1\ fm = 10^{-15}\,m$
	$1\ mb = 10^{-27}\,cm^2$
1 year	1 year $\approx \pi\times10^{7}\,s$
electron charge:	$e = 1.602\times10^{-19}\,C$
electron magnetic moment:	$\mu_e = 9.3\times10^{-24}\,Joules\cdot Tesla^{-1}$
fine structure constant:	$\alpha = e^2/(\hbar c) = 1/137.0360$
strong coupling constant:	$\alpha_s(M_Z) = 0.116\pm0.005$
Fermi coupling constant:	$G_F = 1.166\times10^{-5}\ GeV^{-2}$
Cabibbo angle:	$\sin\theta_C = 0.22$
Weak mixing angle:	$\sin^2\theta_W(M_Z) = 0.2319\pm0.0005$
Branching Ratios	$BR(Z\to e^+e^-) = 3.21\pm0.07\%$
	$BR(Z\to hadrons) = 71\pm1\%$

Particle Properties

Boson	Mass $\left(GeV/c^2\right)$
γ	$<3\times10^{-36}$
$gluon$	~ 0
$W^{\pm}$	80.22
Z^0	91.187
H^0	>116

Lepton	Mass $\left(MeV/c^2\right)$
ν_e	$<10^{-5}$
e	0.510999
ν_μ	<0.27
μ	105.658
ν_τ	<10
τ	1777

Hadron	Quark Content	Mass $\left(MeV/c^2\right)$	$\mathbf{I}\left(\mathbf{J}^{\mathbf{PC}}\right)$
π^+,π^0,π^-	$u\bar{d},\left(u\bar{u}-d\bar{d}\right)/\sqrt{2},d\bar{u}$	139.57,134.97, 139.57	$1\left(0^{-+}\right)$
K^+,K^-	$u\bar{s},s\bar{u}$	493.65	$\frac{1}{2}\left(0^-\right)$
$K^0,\bar{K}^0$	$d\bar{s},s\bar{d}$	497.67	$\frac{1}{2}\left(0^-\right)$
ρ^+,ρ^0,ρ^-	$u\bar{d},\left(u\bar{u}+d\bar{d}\right)/\sqrt{2},\bar{u}d$	775.7	$1\left(1^{--}\right)$
p,n	uud,udd	938.27, 939.57	$\frac{1}{2}\left(\frac{1}{2}^+\right)$
$\Delta^-,\Delta^0,\Delta^+,\Delta^{++}$	ddd,udd,uud,uuu	1232	$\frac{3}{2}\left(\frac{3}{2}^+\right)$
Λ^0	uds	1115.6	$0\left(\frac{1}{2}^+\right)$
$\bar{D}^0,D^0$	$u\bar{c},c\bar{u}$	1863	$\frac{1}{2}\left(0^-\right)$
D^-,D^+	$d\bar{c},c\bar{d}$	1869	$\frac{1}{2}\left(0^-\right)$
D_S^+,D_S^-	$c\bar{s},\bar{c}s$	1968	$0\left(0^-\right)$
B^+,B^-	$u\bar{b},\bar{u}b$	5279	$\frac{1}{2}\left(0^-\right)$
Λ_c^+	udc	2285	$0\left(\frac{1}{2}^+\right)$
$\Sigma^+,\Sigma^0,\Sigma^-$	uus,uds,dds	1189	$1\left(\frac{1}{2}^+\right)$
Ξ^0,Ξ^-	uss,dss	1315	$\frac{1}{2}\left(\frac{1}{2}^+\right)$
Ω^-	sss	1672	$0\left(\frac{3}{2}^+\right)$
Λ_b	udb	5624	$0\left(\frac{1}{2}^+\right)$