Problem Set #3 Solutions

Physics 357S

Question #1

a) Baryon with $L=0$

Baryons are made of 3 quarks, where quarks have spin $s=\frac{1}{2}$.
The intrinsic spin S of the baryon is $\therefore$ given by $3/2$ or $\frac{1}{2}$, which is found by adding the spin vectors of the individual quarks.
The total angular momentum of the baryon is $\therefore$

- for $S=3/2$, $J = L \oplus S = 0 \oplus 3/2 = 3/2$
- for $S=\frac{1}{2}$, $J = L \oplus S = 0 \oplus \frac{1}{2} = \frac{1}{2}$

b) Baryon with $L=1$

The total angular momentum of the baryon is:

- for $S=3/2$, $J = L \oplus S = 1 \oplus 3/2 = 5/2, 3/2, \frac{1}{2}$
- for $S=\frac{1}{2}$, $J = L \oplus S = 1 \oplus \frac{1}{2} = 3/2, \frac{1}{2}$

c) Electron in hydrogen atom with $\ell=1$, $j=3/2$, $m_j=\frac{1}{2}$, $m_s=+\frac{1}{2}$

Intrinsic spin of electron is $s=\pm\frac{1}{2}$, where $m_s=\pm\frac{1}{2}$. Since $m_j=\frac{1}{2}$, and we are given that $m_s=+\frac{1}{2}$, $m_\ell=0$ (ie, $m_j=m_s+m_\ell \Rightarrow m_\ell=0$). Reading off from the Clebsch Gordan table, the probability is $(\sqrt{2/3})^2 = 2/3$

(note: see table $1\times\frac{1}{2}$, where $J=3/2$, $M=\frac{1}{2}$, $m_1=0$ and $m_2=\frac{1}{2}$.)

d) Two particles in state $|2,+1\rangle$ collide and form bound state where $L=0$.

Adding the spins: $s_1 \oplus s_2 = 2 \oplus 2 = \{4,3,2,1,0\}$
Note that $m_{s_1}=1$ and $m_{s_2}=1$ so that $m_s=2$; $\therefore$ this eliminates states with $S=1$ and $S=0$.
For a bound state with $L=0$, the possibilities are:

- $|4,2\rangle$ where $J=4$, $M=2$, $m_1=1$, $m_2=1$ so $P=4/7$
- $|3,2\rangle$ where $J=3$, $M=2$, $m_1=1$, $m_2=1$ so $P=0$
- $|2,2\rangle$ where $J=2$, $M=2$, $m_1=1$, $m_2=1$ so $P=3/7$

As expected, total probability is $4/7+0+3/7=1$.

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c) ① Particle with $s = 1$, $m_s = -1$ so the state is $|1,-1\rangle$

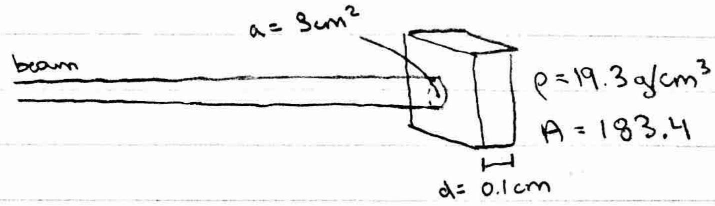
② Decays to (1) $s_1 = 1/2$, $m_{s_1} = +\frac{1}{2}$; state $|\frac{1}{2}, \frac{1}{2}\rangle$, $P = ?$ note: $L = 0$

(2) $s_2 = 3/2$, $m_{s_2} = ?$; state $|3/2, m_{s_2}\rangle$

Since $m_{s_1} = \frac{1}{2}$ and $m_s = -1$; $m_{s_2} = m_s - m_{s_1} = -3/2$.

Then, according to Clebsch-Gordan table, $P = 3/10$.

Question #2



a) Scattering centres intercepted by the beam.

Total # of scattering centres: $N = and$ where $a = 3cm^2$

n = atoms/volume

$d = 0.1cm$

Recall that $n = N_A \rho / A$ where N_A = Avogadro's Number = 6.022×10^{23} atoms/mole

$\rho = 19.3 \, g/cm^3$

$A = 183.4$

$$\therefore N = (a \cdot d)(N_A \rho / A)$$
$$= V N_A \rho / A$$
$$= (3cm^2 \cdot 0.1cm)(6.022 \times 10^{23})(19.3 \, g/cm^3)/183.4$$
$$\cong 1.96 \times 10^{22} \text{ scattering centres.}$$

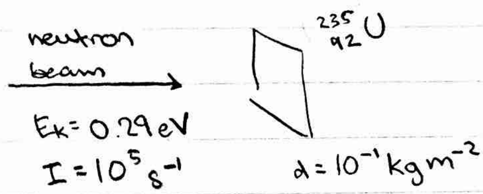
b) $\sigma_{TOT} = 0.1 mb$

$$\frac{N_{scat}}{N_{in}} = \frac{N \sigma_{TOT}}{a} = \frac{(1.96 \times 10^{22})(0.1 mb)}{(3cm^2)}$$

note: $1 mb = 10^{-27} cm^2$

$\cong 6.5 \times 10^{-7}$ is the is the scattered fraction of the incident beam.

Question #3



(i) elastic scattering $\sigma_e = 2\times10^{-30}\,m^2$

(ii) capture via γ-ray emission $\sigma_c = 7\times10^{-27}\,m^2$

(iii) capture via splitting of nucleus (fission) $\sigma_f = 2\times10^{-26}\,m^2$

a) Attenuation of the beam by the foil

Total # of Uranium atoms per unit area in foil:

$$n = d/A$$
$$= (10^{-1}\,kg\,m^{-2})N_A/(235\,kg)$$
$$\cong 2.56\times10^{23}\ \text{atoms}/m^2$$

The total cross section is given by:

$$\sigma_{TOT} = \sigma_e + \sigma_c + \sigma_f$$
$$= 2\times10^{-30}\,m^2 + 7\times10^{-27}\,m^2 + 2\times10^{-26}\,m^2$$
$$\cong 2.7\times10^{-26}\,m^2$$

Then, each neutron has a chance of being deflected of $N\sigma_{TOT} = (2.56\times10^{23}\,m^{-2})(2.7\times10^{-26}\,m^2)$
$= 6.9\times10^{-3}$

The intensity of the beam is $10^5\,s^{-1}$, so there are $IN\sigma_{TOT} = (10^5\,s^{-1})(6.9\times10^{-3})$
$= 691.25$ collisions/s

The ratio of the # of particles left after the foil over # of particles before is given by:

$$= (I - IN\sigma_{TOT})/I$$
$$= (10^5\,s^{-1} - 691.25\,s^{-1})/10^5\,s^{-1}$$
$$= 0.993$$

Then $1 - 0.993 \cong \underline{6.9\times10^{-3}}$ is the attenuation of the beam

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b) Number of fission reactions per second in the foil caused by the beam

Cross section for fission: $\sigma_f = 2\times10^{-26}\ m^2$

From a), number density of uranium atoms: $n = 2.56\times10^{23}\ m^{-2}$

Intensity of beam (# of neutrons per second): $I = 10^5\ s^{-1}$

$\therefore$ # of fissions per second $= nI\sigma_f$

$$= (2.56\times10^{23}\ m^{-2})(10^5\ s^{-1})(2\times10^{-26}\ m^2)$$

$$= 512\ s^{-1}$$

c) Flux of elastically scattered neutrons 10 m from the foil (assume isotropic distribution of scattered neutrons)

Cross section for elastic scattering: $\sigma_e = 2\times10^{-30}\ m^2$

$\therefore$ # of scattered neutrons per second $= nI\sigma_e$

$$= (2.56\times10^{23}\ m^{-2})(10^5\ s^{-1})(2\times10^{-30}\ m^2)$$

$$= 0.0512\ s^{-1}$$

Surface area of shell with $R = 10\,m$ around the foil: $A = 4\pi r^2$

$$= 4\pi(10m)^2$$

$$\cong 1256.6\ m^2$$

The flux is $\therefore$ $F =$ (# of scattered neutrons)$/A$

$$= (0.0512\ s^{-1})/(1256.6\ m^2)$$

$$\cong 4\times10^{-5}\ m^{-2}\ s^{-1}$$

Question #4

a) $\sigma = 10^{-43}\ cm^2$ (cross section for absorption of neutrinos)

Recall: Absorption length $L = 1/\sigma n$ where $n = $ # of targets/volume

Intensity $I = I_0 \exp[-x/L]$

$= I_0 \exp(-\sigma n x)$

First compute $n = N_A \rho / A$

$= (6.022\times10^{23})(999.97\ kg/m^3) / (18.01528\ g/mol)$

$\cong 3.34\times10^{28}\ m^{-3}$

Then, for 50% reduction of intensity in vacuum: $I/I_0 = \exp(-\sigma n x)$

$\ln(1/2) = -\sigma n x$

$x = -\ln(1/2)/(10^{-43} cm^2)(3.34\times10^{28} m^{-3})$

$x = 2.07\times10^{18}\ m$

b) Liquid scintillator with atomic ratio of $H/C = 1.10$

Density $\rho = 0.95\ g/cm^3$

Volume of tank $V = 10^3\ L = 10^3\ cm^3$

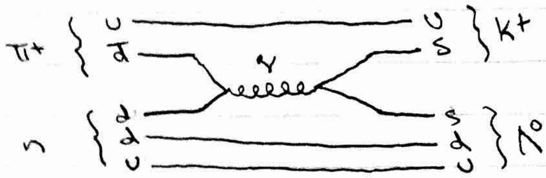
Flux of neutrinos $F = 10^{13}\ cm^{-2}\ s^{-1}$ with $\sigma = 10^{-43}\ cm^2$

Consider events of type $\bar{\nu}_e p \to e^+ n$ (ie: interactions with hydrogen)

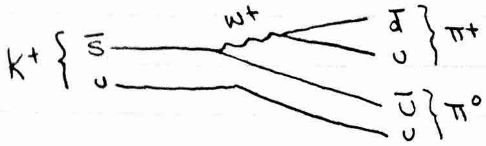
Captures/second $= F N \sigma$ where $N = $ capture sites $= Vn = V N_A \rho / A$

Question #5

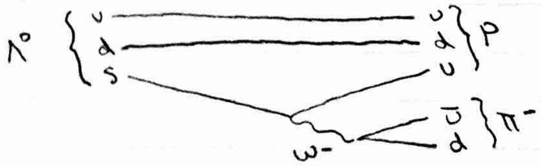
a) $\pi^+ n \rightarrow K^+ \Lambda^0$



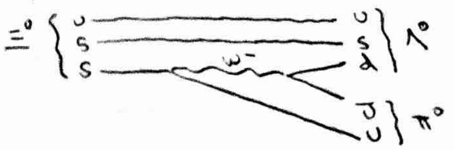
$K^+ \longrightarrow \pi^0 \pi^+$



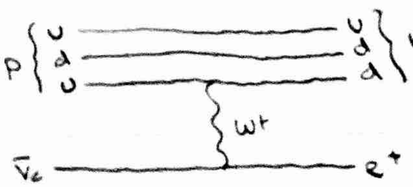
$\Lambda^0 \rightarrow p \pi^-$



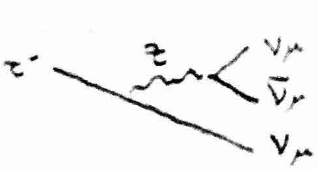
b) $\Xi^0 \rightarrow \Lambda^0 \pi^0$



$\bar{\nu}_e p \rightarrow e^+ n$



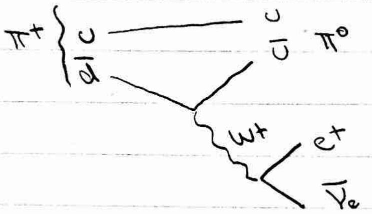
$\tau^- \rightarrow \nu_\mu \bar{\nu}_\mu \mu^-$

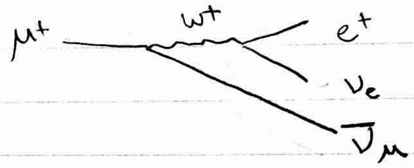


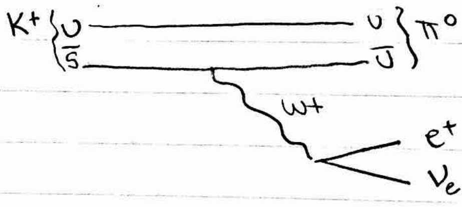
Question #6

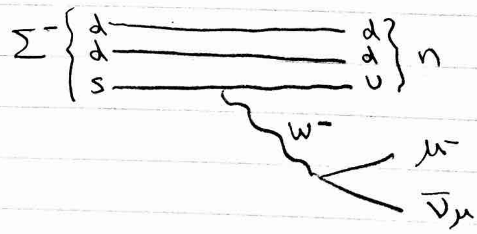
a) Δ^{++} (uuu) with spin 3/2. Since quarks each have a spin of ½, this tells us that all three up quarks are aligned. The Δ^{++} seems to violate Pauli's exclusion principle! This tells us that quarks must have an additional quantum number – in QCD it is called colour charge.

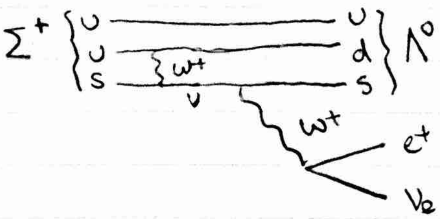
b) Lepton generations are electron (e), muon (μ), and tau (τ).

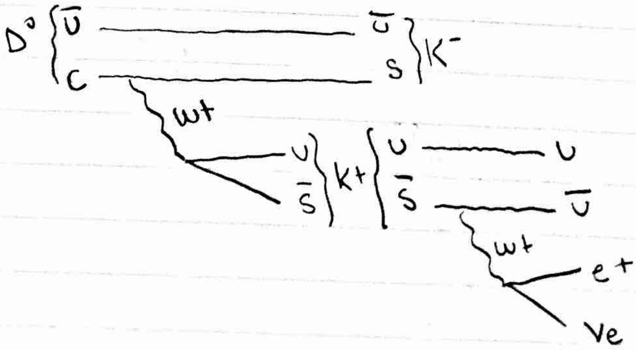












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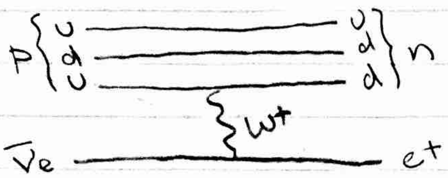

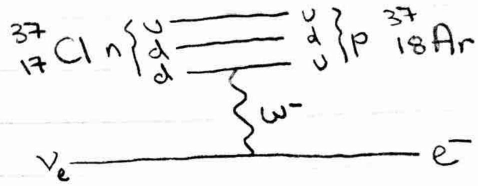

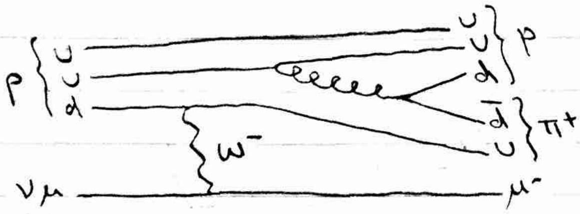

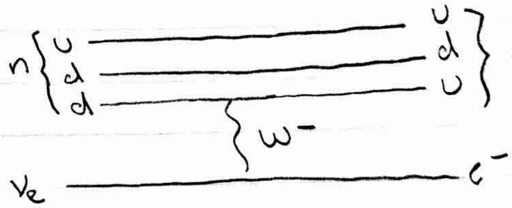

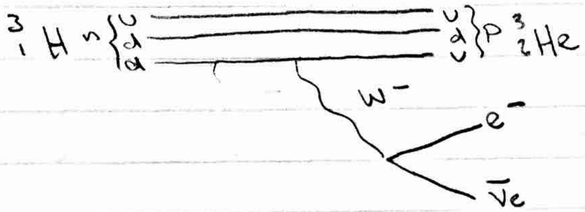

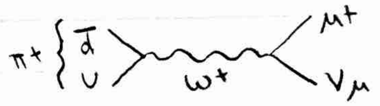

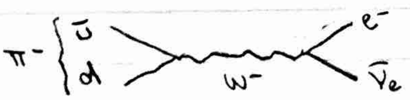

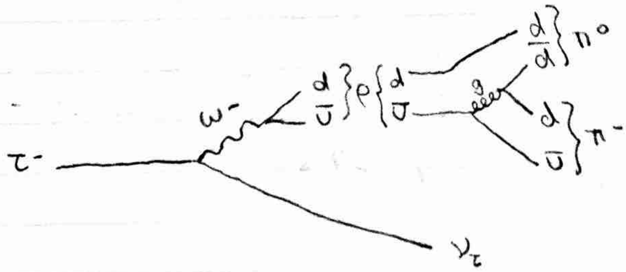

Question #7

Consider the International Linear Collider.

Luminosity is given by $L = \frac{N_1 N_2}{4\pi\sigma_x\sigma_y} f n_b$ where N_1, N_2 = # of particles in bunch

f = repitition frequency

n_b = # of bunches

σ_x, σ_y = rms beam size in $\{x,y\}$

a) The luminosity of the ILC:

$$L = \frac{N_1 N_2}{4\pi\sigma_x\sigma_y} f n_b = \frac{(2\times10^{10})^2(5\,Hz)(2625)}{4\pi(639\,nm)(5.7\,nm)}$$
$$\cong 1.15\times10^{38}\ m^{-2}\,s^{-1}$$

b) Cross section σ = 10 picobarns $= 10\,(10^{-40}\,m^2)$
$= 1\times10^{-39}\,m^2$

Then the # of scattering events per second is $L\sigma = (1.15\times10^{38}\,m^{-2}s^{-1})(1\times10^{-39}\,m^2)$
$= 0.115\,s^{-1}$

c) The average flux of electrons:

# of partices in beam/second · area $= \frac{N f n_b}{4\pi\sigma_x\sigma_y} \cong 5.7\times10^{27}\,m^{-2}s^{-1}$

d) Stationary target of LH where $\rho = 0.1\,g/cm^3$ and $d = 2\,m$

Then # of scattering events per second is $L\sigma = N_b f \sigma n_t$ where n_t = # atoms/cm²

note: $n_t = \rho d N_A / A$
$= (0.1\,g/cm^3)(2m)(6.022\times10^{23})/(1.0079\,u)$
$= 1.19\times10^{29}\,m^{-2}$

$\therefore L\sigma = (2\times10^{10})(5\,Hz)(1\times10^{-39}\,m^2)(1.19\times10^{29}\,m^{-2}) = 11.9\,s^{-1}$