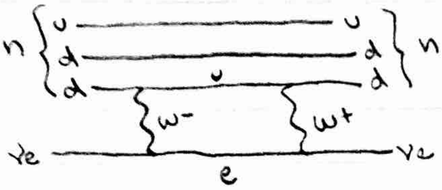
Question #1

a) Quark line diagrams for:

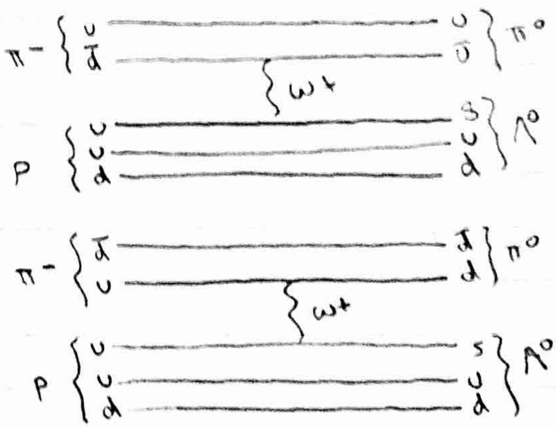
(1) $\nu_e e^- \rightarrow \nu_e e^-$

(3) $\nu_\mu n \rightarrow \mu^- p$

(2) $\nu_e n \rightarrow \nu_e n$

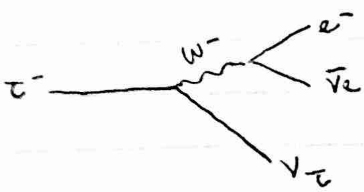


(4) $\pi^- p \rightarrow \Lambda \pi^0$

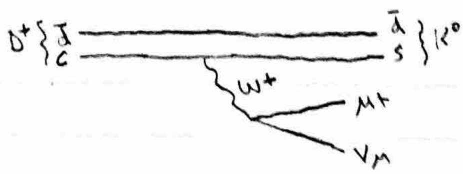


b) Quark line diagrams for:

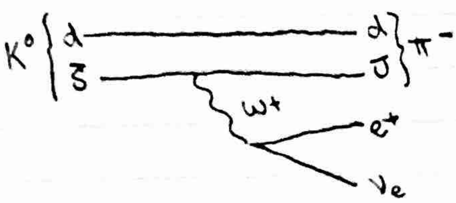
(1) $\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau$



(3) $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$



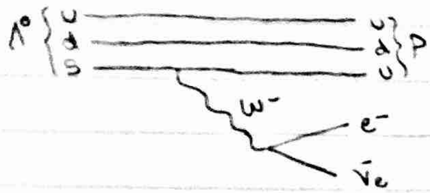
(2) $K_0 \rightarrow \pi^- e^+ \nu_e$



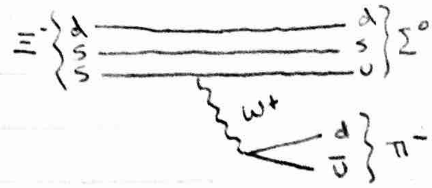
(4) $\tau^+ \rightarrow \pi^+ \nu_\tau$

Question #1 con't

(5) $\Lambda^0 \rightarrow p e^- \bar{\nu}_e$



(6) $\Xi^- \rightarrow \Sigma^0 \pi^-$



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Question #2

(1) $\pi^- + p \rightarrow \pi^- + \pi^+ + n$

$d\bar{u} + uud \rightarrow d\bar{u} + u\bar{d} + udd \rightarrow$ strong force (quark flavour/baryon # conserved)

(2) $\gamma + p \rightarrow \pi^0 + p$

$\gamma + uud \rightarrow u\bar{u}/d\bar{d} + uud \rightarrow$ EM force since $\gamma \rightarrow q\bar{q}$ is EM

(3) $\bar{\nu}_\mu + p \rightarrow \mu^+ + n$

$\bar{\nu}_\mu + uud \rightarrow \mu^+ + udd \rightarrow$ must be weak force b/c we have a neutrino

(4) $\pi^0 \rightarrow e^+ + e^- + e^+ + e^-$

$d\bar{d} \rightarrow e^+ + e^- + e^+ + e^- \rightarrow$ EM since $\pi^0 \rightarrow \gamma\gamma \rightarrow e^+e^-e^+e^-$

(5) $p + \bar{p} \rightarrow \pi^+ + \pi^- + \pi^0$

$uud + \bar{u}\bar{u}\bar{d} \rightarrow u\bar{d} + \bar{u}d + u\bar{u}/d\bar{d} \rightarrow$ strong force

(6) $\tau^- \rightarrow \pi^- + \nu_\tau$

$\tau^- \rightarrow \bar{u}d + \nu_\tau \rightarrow$ weak force

(7) $D^- \rightarrow K^+ + \pi^- + \pi^-$

$d\bar{c} \rightarrow u\bar{s} + \bar{u}d + \bar{u}d \rightarrow$ weak since strange/charm not conserved

(8) $\pi^- \rightarrow \pi^0 + e^- + \bar{\nu}_e$

$\bar{u}d \rightarrow u\bar{u}/d\bar{d} + e^- + \bar{\nu}_e \rightarrow$ weak

(9) $\Lambda + p \rightarrow K^- + p + p$

$uds + uud \rightarrow s\bar{u} + uud + uud \rightarrow$ strong

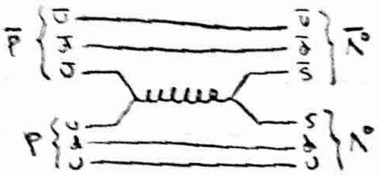
Question #3

a) $\bar{p} + p \rightarrow \bar{\Lambda} + \Lambda$

$\bar{u}\bar{u}\bar{d} + uud \rightarrow \bar{\Lambda} + uds \rightarrow$ strong force (see part b)

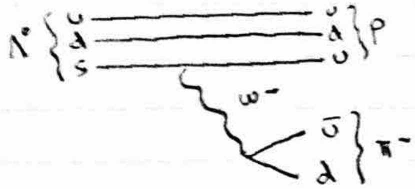
b)

	$\bar{p}$	p	Λ	$\bar{\Lambda}$
charge	-1	+1	0	0
baryon	-1	+1	+1	-1



The anti-lambda conserves charge and baryon number. Strangeness is also conserved.

c) The Λ^0 decays into a π^- and a proton.
The $\bar{\Lambda}^0$ decays into a π^+ and anti-proton.



∴ the weak force causes the neutral particle decays at the secondary vertices. We can infer this because of the displaced vertices – due to the longer lifetimes, the neutral particles had time to travel in the bubble chamber before decaying.

Also can have (from class notes):

→ still have an exchange of a weak vector boson ∴ weak force causes decay

note: this conclusion holds for $\bar{\Lambda}^0$ as well, Feynman diagrams are analogous.

d) See part b) for the quark line diagram.

e) See the Feynman diagrams in part c), but take the antiparticles.

Question #3 con't...

f) Interaction at B: $\bar{p} + p \rightarrow \pi^+ + \pi^- + \pi^+ + \pi^-$

$\bar{u}\bar{u}\bar{d} + uud \rightarrow u\bar{d} + \bar{u}d + u\bar{d} + \bar{u}d$

(charge)(baryon #): (-1)(-1) (+1)(+1) (+1)(0) (-1)(0) (+1)(0) (-1)(0)

Baryon # and charge are conserved. No surprising new quarks. ∴ strong interaction.

g) The rest mass of Λ can be determined based on the momenta of the charged particles, which can be determined from the curvature of their tracks.

(1) To get the momenta: $m\vec{a} = q\vec{v} \times \vec{B}$

$$\frac{mv^2}{R} = qvB$$

$$mv = qBR = \vec{p}_\perp$$

(2) To get the mass: $(p_\Lambda^\mu)^2 = (p_p^\mu + p_\pi^\mu)^2$

$$m_\Lambda^2 = (E_p + E_\pi)^2 - (\vec{p}_p + \vec{p}_\pi)^2$$

$$m_\Lambda^2 = (E_p + E_\pi)^2 - (\vec{p}_p^{\,2} + \vec{p}_\pi^{\,2} + 2\vec{p}_p\vec{p}_\pi)$$

angle between p and π^-

$$m_\Lambda^2 = (E_p + E_\pi)^2 - (\vec{p}_p^{\,2} + \vec{p}_\pi^{\,2} + 2|\vec{p}_p||\vec{p}_\pi|\cos\theta)$$

h) The distribution expected from observing many neutral decays, given that the mean lifetime of Λ is 2.6×10^{-10} s and the precision of a bubble chamber is ~MeV.

The width of a decay is given by $\Gamma = 1/\tau = 1/2.6\times10^{-10}\,\text{s}$

$= 2.53 \times 10^{-6}$ eV

∴ Since the width of Λ is much smaller than the resolution of the bubble chamber, we expect a mass peak with a width of ~ MeV.

Question #4

a) General expression for form factor in spherical coordinates: $F(q^2) = \frac{1}{Ze}\int \rho(\vec{r}) e^{i\vec{q}\cdot\vec{r}/\hbar} dV$

where $dV = r^2 \sin\theta \, dr \, d\theta \, d\phi$

$= r^2 \, d\cos\theta \, d\phi \, dr$ for spherically-symmetric distribution

Then,

$$F(q^2) = \int_0^{2\pi} d\phi \int_{-1}^{1} \int_0^{\infty} \left(\frac{1}{Ze}\right)\rho(\vec{r}) e^{i\vec{q}\cdot\vec{r}/\hbar} r^2 \, d\cos\theta \, dr$$

$$= \frac{2\pi}{Ze}\int \rho(r) e^{iqr\cos\theta/\hbar} r^2 \, d\cos\theta \, dr$$

$$= \frac{2\pi}{Ze}\int \rho(r)\left(\hbar/iqr\right)\left(e^{iqr/\hbar} - e^{-iqr/\hbar}\right) r^2 \, dr$$

$$= \frac{4\pi}{Ze}\int \rho(r)\left(\hbar/qr\right)\sin(qr/\hbar) r^2 \, dr$$

Expanding the sine gives:

$$\sin(qr/\hbar)\left(1/qr/\hbar\right) = \left(\frac{qr}{\hbar} - \frac{q^3r^3}{6\hbar^3} + \ldots\right)\left(\frac{\hbar}{qr}\right)$$

$$= 1 - \left(\frac{1}{6}\right)\left(\frac{qr}{\hbar}\right)^2 + \ldots$$

Then, we write the form factor as

$$F(q^2) = \frac{4\pi}{Ze}\int r^2 \rho(r)\left(1 - \left(\frac{1}{6}\right)\left(\frac{qr}{\hbar}\right)^2 + \ldots\right) dr$$

Imposing the normalization condition, we define the root-mean-square radius:

$$\frac{4\pi}{Ze}\int r^2 \rho(r) dr = 1 \text{ and } \langle r^2\rangle = \frac{4\pi}{Ze}\int r^2 \rho(r) r^2 dr$$

$\therefore$ the form factor can be written as,

$$F(q^2) = \frac{4\pi}{Ze}\int r^2\rho(r)dr - \frac{4\pi}{Ze}\int r^2 \rho(r) r^2 \left(\frac{1}{6}\right)\left(\frac{q}{\hbar}\right)^2 dr$$

$$= 1 - \langle r^2\rangle\left(1/6\right)\left(q/\hbar\right)^2$$

$$= 1 - \frac{q^2\langle r^2\rangle}{6\hbar^2} + O(q^4)$$

b) Now we have the charge distribution $\rho(r) = \rho_0 e^{-r/a}\left(\frac{1}{r}\right)$

As in part a), for spherically-symmetric charge distributions we have:

normalization

$$F(q^2) = \frac{4\pi}{Ze}\int r^2\rho(r)\sin(qr/\hbar)\left(\frac{\hbar}{qr}\right) dr$$

$$= \frac{4\pi}{Ze}\int r^2\rho(r)\sin(qr/\hbar)(\hbar/qr)\, dr \times \left[\frac{1}{Ze}\int d^3r \, \rho(r)\right]^{-1}$$

$$= \frac{4\pi}{Ze}\int r^2 \rho_0 e^{-r/a}\left(\frac{1}{r}\right)\sin(qr/\hbar)(\hbar/qr)\, dr \times \left[\frac{4\pi}{Ze}\int r^2 dr \, \rho_0 e^{-r/a}\left(\frac{1}{r}\right)\right]^{-1}$$

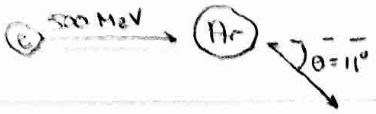
$$= \frac{4\pi}{Ze}\rho_0\left(\frac{\hbar}{q}\right)\int_0^{\infty} e^{-r/a}\sin(qr/\hbar)\, dr \times \left[\frac{4\pi}{Ze}\rho_0\int_0^{\infty} r e^{-r/a} dr\right]^{-1}$$

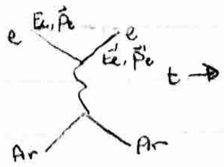
$$= \frac{\hbar}{q}\int_0^{\infty} e^{-r/a}\sin(qr/\hbar)\,dr \times \left[\int_0^{\infty} r e^{-r/a} dr\right]$$

$$= \frac{\hbar}{q}\left[\frac{q/\hbar}{(1/a)^2 + (q/\hbar)^2}\right] \times \left[a^2\right]^{-1}$$

$$= \left(1 + (qa/\hbar)^2\right)^{-1}$$

Question #4 con't...

c) 



(1) Momentum transfer: $q^2 = (E_e + E_e')^2 - (\vec{p}_e + \vec{p}_e')^2$ → (upper vertex)

$$= 2m_e^2 - 2E_eE_e' + 2p_ep_e'\cos\theta \quad (*)$$

$$p_{Ar}'^2 = (p_{Ar} + q)^2 \rightarrow \text{(lower vertex)}$$

$$m_{Ar}^2 = m_{Ar}^2 + q^2 + 2\vec{p}\cdot\vec{q}$$

$$\Rightarrow q^2 = m_{Ar}^2 - m_{Ar}^2 - 2\vec{p}\cdot\vec{q}$$

$$= -2\vec{p}\cdot\vec{q}$$

$$= -2(m_{Ar}, \vec{0})\cdot(E_e - E_e', \vec{p}_e - \vec{p}_e')$$

$$= -2m_{Ar}(E_e - E_e') \quad (*')$$

$(*)$ and $(*')$ give,

$$E_e' = {-q^2}/{2E_e(1-\cos\theta)} = {q^2}/{2m_{Ar}} + E_e$$

$$\Rightarrow q^2 = -E_e\left[\frac{1}{2m_{Ar}} + \frac{1}{2E_e(1-\cos\theta)}\right]^{-1}$$

$$= -500\text{ MeV}\left[\frac{1}{2\times40\times938} + \frac{1}{2\times500\times(1-\cos11^\circ)}\right]^{-1}$$

$$\simeq -9189\text{ MeV}^2$$

(2) Reduced deBroglie wavelength: $\frac{\hbar c}{pc} = \frac{197\text{ MeV fm}}{500\text{ MeV}}$

$$\simeq 0.394\text{ fm}$$

(3) Mott differential cross section: $\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = 4(Z\alpha)^2\frac{E^2}{q^4}\left(1-\beta^2\sin^2\theta/2\right)$

$$= 4\left(18\cdot\tfrac{1}{137}\right)^2(500\text{ MeV})^2/(9189\text{ MeV}^2)^2\left(1-\sin^2\tfrac{11^\circ}{2}\right) \quad (\text{assume } \beta\sim1)$$

$$\simeq 7.86\text{ fm}^2$$

$$\simeq 78.6\text{ mb}$$

(4) Change in $d\sigma/d\Omega$ is found using form factor of part a): $F(q^2) \simeq 1 - \frac{1}{6}\frac{q^2}{\hbar^2}\langle r^2\rangle$

$$= 1 - \tfrac{1}{6}(-9189\text{ MeV}^2)^2(1.2\times40^{1/3})^2(1/197^2)$$

$$\simeq 1.66$$

∴ the cross section increases by $\times1.66^2 = 2.76$

Question #5

a) $$B.E. = -a_1 A + a_2 A^{2/3} + a_3 \frac{Z^2}{A^{1/3}} + a_4 \frac{(N-Z)^2}{A} \pm a_5 A^{-3/4}$$

$$M(A,Z)c^2 = (A-Z)m_n c^2 + Z m_p c^2 - a_1 A + a_2 A^{2/3} + a_3 Z^2/A^{1/3} + a_4 \frac{(A-2Z)^2}{A} \pm a_5 A^{-3/4}$$

Differentiate $M(A,Z)$ for fixed A:

$$\frac{dM}{dZ} = -m_n + m_p + a_3 \frac{2Z}{A^{1/3}c^2} + a_4\left(\frac{1}{Ac^2}\right) 2(A-2Z)(-2)$$

$$0 = -m_n + m_p + 2a_3\left(\frac{1}{A^{1/3}c^2}\right)Z - a_4\left(\frac{4}{Ac^2}\right)(A-2Z)$$

$$m_n - m_p = 2a_3\left(\frac{1}{A^{1/3}}\right)Z - a_4\left(\frac{4}{A}\right)A + a_4\left(\frac{4}{A}\right)2Z$$

$$m_n - m_p + 4a_4 = \left[2a_3\left(\frac{1}{A^{1/3}}\right) + 8a_4\left(\frac{1}{A}\right)\right]Z$$

$$\Rightarrow Z = \frac{m_n - m_p + 4a_4}{2a_3/A^{1/3} + 8a_4/A}$$

For the plot of M vs Z, we can see that for constant A, M(Z) would be a parabola since we have Z^2 terms in the semi-empirical mass formula.

For $A = 16$; $Z = \dfrac{m_n - m_p + 4a_4}{\frac{2a_3}{A^{1/3}} + \frac{8a_4}{A}}$ note: $a_4 = 23.70$ MeV, $a_3 = 0.71$ MeV

$$= \frac{1.293\text{ MeV} + 4(23.70\text{ MeV})}{2(0.71\text{ MeV})/(16)^{1/3} + 8(23.70\text{ MeV})/16}$$

$\cong 8$ ∴ the isotope is ^{16}O, a stable isotope of Oxygen.

Similarly, for $A = 208$, $Z \cong 83$ ∴ the isotope is ^{208}Bi. This is off by 1 from Bi's "most stable" isotope of ^{209}Bi (also could be close to $^{208}_{82}Pb$).

b) Total binding energy using equation for B.E. given above. pairing term $\begin{cases} + & Z,N \text{ even} \\ 0 & A \text{ odd} \\ - & Z,N \text{ odd}\end{cases}$

For $^{8}_{4}Be$, $B.E = -a_1 A + a_2 A^{2/3} + a_3 Z^2/A^{1/3} + a_4 \frac{(N-Z)^2}{A} \pm a_5 A^{-3/4}$

$$= -(15.8)(8) + (18.3)(8^{2/3}) + (0.714)(4^2)(8^{-1/3}) + (23.2)\left(\tfrac{1}{8}(4-4)^2\right) + 12(8^{-3/4})$$

$\cong -44.96$ MeV

Similarly, for
$^{12}_{6}C$, $B.E \cong -80.59$ MeV
$^{56}_{26}Fe$, $B.E \cong -483.57$ MeV
$^{208}_{82}Pb$, $B.E \cong -1617.53$ MeV

note: used slightly different values for "a"s, but your values should be close.

Question #5 con't...

Binding energies using atomic masses of the nuclides are:

For ${}^{8}_{4}Be$, $BE = \Delta m = (Nm_n + Zm_p) - (m_{Be})$

$= (4 \cdot m_n + 4 \cdot m_p) - m_{Be}$

$= 8.06642u - 8.005305u$

$= 0.061115u$

$\cong 56.93\ MeV$

Similarly, ${}^{12}_{6}C$, $BE \cong 92.8\ MeV$

${}^{56}_{26}Fe$, $BE \cong 495.0\ MeV$

${}^{208}_{82}Pb$, $BE \cong 1645.2\ MeV$

Problem Set #4 Solutions — Physics 375S

Question #6

a) Electrostatic potential energy of a uniformly charged sphere,

For $r<R$, the amount of charge is $q(r) = Q\, r^3/R^3$

For $r>R$, the amount of charge is $q(r) = Q$

The electric field is given by: $\vec{E}(r) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$

For $r<R$, $\vec{E}(r) = \frac{1}{4\pi\epsilon_0}\frac{Qr}{R^3}$

For $r>R$, $\vec{E}(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$

Then the electrostatic potential is given by $V = \frac{\epsilon_0}{2}\int E^2\,dV$

$$= 2\pi\epsilon_0 \int E^2 r^2\,dr$$

Integrate from $\{0,R\}$ and $\{R,\infty\}$, $V = \frac{1}{8\pi\epsilon_0}\left[\int_0^R \frac{Q^2}{R^6} r^4\,dr + \int_R^\infty \frac{Q^2}{r^2}\,dr\right]$

$$= \frac{1}{8\pi\epsilon_0}\left[\frac{Q^2}{R^6}\left(\frac{1}{5}\right)R^5 + \frac{Q^2}{r}(-1)\Big|_R^\infty\right]$$

$$= \frac{1}{8\pi\epsilon_0}\left[\frac{Q^2}{5R} + \frac{Q^2}{R}\right]$$

$$= \frac{3Q^2}{20\pi\epsilon_0 R}$$

b) Let $Q = Ze$, then using the Coulomb term of the semi-empirical mass formula we have,

$$a_c Z^2/A^{1/3} = \frac{3(Ze)^2}{20\pi\epsilon_0 R}$$

$$\Rightarrow a_c = 3A^{1/3}e^2\left(20\pi\epsilon_0 \cdot 1.24\,\text{fm}\,A^{1/3}\right)^{-1}$$

$$= 3e^2\left(20\pi\epsilon_0\, 1.24\,\text{fm}\right)^{-1}$$

$$= \frac{3}{5}\left(\frac{e^2}{4\pi\epsilon_0}\right)\left(\frac{1}{1.24\,\text{fm}}\right)$$

$$= \frac{3}{5}\left(\frac{197}{137}\,\text{MeV fm}\right)\left(\frac{1}{1.24\,\text{fm}}\right)$$

$$= 0.696\ \text{MeV}$$

Solving the BE formula for a_c in the case of ^{181}Ta gives $a_c \sim 0.693$ MeV – not bad!

Spontaneous fission of $^{235}_{92}U \rightarrow {}^{87}_{36}Br + {}^{145}_{57}La + 3n$

The energy released can be estimated $E_{released} = BE(^{235}U) - BE(^{87}Br) - BE(^{145}La)$

$$\cong 153\ \text{MeV}$$

note: binding energy of 3 neutrons are zero, the binding energies of the rest of the nuclei are found using the formula from 5a).