Question #1

a) Show that $(\vec{L}\cdot\vec{S}) = \frac{1}{2}(\vec{J}^2 - \vec{L}^2 - \vec{S}^2)$ from $\vec{J} = \vec{L} + \vec{S}$:

$$J\cdot J = (L+S)\cdot(L+S)$$
$$J^2 = L\cdot L + L\cdot S + S\cdot L + S\cdot S$$
$$J^2 = L^2 + S^2 + 2L\cdot S$$
$$L\cdot S = \frac{1}{2}(J^2 - L^2 - S^2)$$

Finding energy separation using $L\cdot S$:

$$L\cdot S = \frac{1}{2}(J^2 - L^2 - S^2) = \frac{1}{2}(j(j+1) - l(l+1) - s(s+1))$$
$$= \frac{1}{2}(j(j+1) - l(l+1) - 3/4)$$

The energy-level correction due to spin-orbit coupling is $E_{nl} = E_n - \mu_s \cdot B_L$

Since $\mu_s \cdot B_L \propto S\cdot L$, the energy level separations are $\propto$ to $L\cdot S$ derived above.

Then, for aligned we have $j = l + \frac{1}{2}$, for antialigned we have $j = l - \frac{1}{2}$, the difference in energy levels is proportional to:

$$\Delta E \propto |L\cdot S[\text{aligned}] - L\cdot S[\text{anti-aligned}]|$$
$$= |[(l+\tfrac{1}{2})(l+\tfrac{1}{2}+1) - l(l+1) - 3/4] - [(l-\tfrac{1}{2})(l-\tfrac{1}{2}+1) - l(l+1) - 3/4]|$$
$$= |(l+\tfrac{1}{2})(l+3/2) - l(l+1) - 3/4 - (l-\tfrac{1}{2})(l+\tfrac{1}{2}) + l(l+1) + 3/4|$$
$$= (l+\tfrac{1}{2})(l+3/4) - (l-\tfrac{1}{2})(l+\tfrac{1}{2})$$
$$= (l+\tfrac{1}{2})[(l+3/4) - (l-\tfrac{1}{2})]$$
$$= l + \frac{1}{2}$$

b) Angle between $\vec{J}$ and $\vec{L}$: $\theta = \langle \vec{J}\cdot\vec{L} / \sqrt{\vec{J}^2}\sqrt{\vec{L}^2} \rangle$

For $l = 1$, $j = 3/2$, the angle between l and j is:

$\rightarrow$ For $J\cdot L$, $J = L + S$

$$J - L = S$$
$$(J-L)\cdot(J-L) = S^2$$
$$J^2 + L^2 - 2J\cdot L = S^2$$
$$\Rightarrow J\cdot L = (J^2 + L^2 - S^2)\tfrac{1}{2}$$

$$\therefore \theta = \frac{\frac{1}{2}(j(j+1) + l(l+1) - 3/4)}{\sqrt{j(j+1)}\sqrt{l(l+1)}}$$
$$= \frac{\frac{1}{2}(2 + 15/4 - 3/4)}{(\sqrt{2})(\sqrt{15/4})}$$
$$\cong 0.91$$

Question #1 con't ...

d) Contribution of total angular momentum: $\bar{\mu}_j = g_j \bar{J}$

Contribution of orbital angular momentum: $\bar{\mu}_\ell = g_\ell \bar{L}$

Contribution of nucleon spin of unpaired nucleon in A-odd nucleus: $\bar{\mu}_s = g_s \bar{S}$

Thus, since $\bar{\mu}_j = g_j \bar{J} = g_\ell \bar{L} + g_s \bar{S}$, we can write:

$$g_j J = g_\ell L + g_s S$$
$$g_j J \cdot J = g_\ell L \cdot J + g_s S \cdot J$$
$$g_j = \frac{1}{J^2}\left(g_\ell L \cdot J + g_s S \cdot J\right)$$

Find expressions for $L \cdot J$ and $S \cdot J$, similarly as in part b), so that;

$$g_j = g_\ell\left[\frac{j(j+1) + \ell(\ell+1) - s(s+1)}{2j(j+1)}\right] + g_s\left[\frac{j(j+1) - \ell(\ell+1) + s(s+1)}{2j(j+1)}\right]$$

Nucleus	Odd nucleon type and configuration	Nuclear spin-parity	Magnetic moment (calculated)
${}^3_1H$	proton, $(1s_{1/2})^1$	$(1/2)^+$	2.793
${}^3_2He$	neutron, $(1s_{1/2})^1$	$(\frac{1}{2})^+$	-1.913
${}^7_3Li$	proton, $(1s_{1/2})^2(1p_{3/2})^1$	$(3/2)^-$	3.793
${}^9_4Be$	neutron, $(1s_{1/2})^2(1p_{3/2})^3$	$(3/2)^-$	-1.913
${}^{11}_5B$	proton, $(1s_{1/2})^2(1p_{3/2})^3$	$(3/2)^-$	3.793

etc...

Detailed example:

For ${}^3_1H$, we have one proton and two neutrons, $\therefore$ the proton is unpaired and is in the $(1s_{1/2})$ state, where $\ell = 0$. $\therefore$ its spin-parity is $(\frac{1}{2})^+$.

For the magnetic moment, there are two possibilities:

(1) $j = \ell + \frac{1}{2}$

$\Rightarrow \langle\mu\rangle = \left[g_\ell(j - \frac{1}{2}) + \frac{1}{2}g_s\right]\mu_N$

(2) $j = \ell - \frac{1}{2}$

$\Rightarrow \langle\mu\rangle = \left[g_\ell \frac{j(j+3/2)}{(j+1)} - \frac{1}{2}\left(\frac{1}{j+1}\right)g_s\right]\mu_N$

note: $g_\ell = 1$, $g_s = 5.586$ for protons

$g_\ell = 0$, $g_s = -3.826$ for neutrons

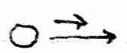
For ${}^3_1H$, $j = \frac{1}{2}$ and $\ell = 0$ so that $j = \ell + \frac{1}{2}$. The magnetic moment is calculated as:
$$\langle\mu\rangle = \left[g_\ell(j - \tfrac{1}{2}) + \tfrac{1}{2}g_s\right]\mu_N$$
$$= \left[(1)(\tfrac{1}{2} - \tfrac{1}{2}) + \tfrac{1}{2}(5.586)\right]\mu_N$$
$$= 2.793\mu_N$$

This is close to the measured value of 2.9788 (~6%).

Question #2

a) Electrons and muons do not have a meaningful right-handed or left-handed characterization. This is because a change in reference frame can flip the "handedness" of the particle.

ie,



right handed

Reference frame moving slower than particle sees particle moving to the right.

left handed

Reference frame moving faster than particle sees particle moving to left.

This is why it does not make sense to think of electrons or muons as right or left handed.

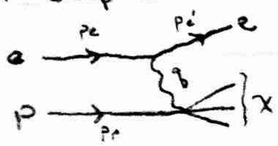
b) SLAC used electrons vs. Fermilab used muons, why?

→ SLAC is a linear accelerator

→ Fermilab is circular

Electrons are harder to accelerate along a curved path than heavier particles like muons - this is because the electron will radiate energy as it travels along a curved path (ie, bremsstrahlung radiation: for a given energy, bremsstrahlung losses increase with decreasing particle mass). Linear colliders avoid bremsstrahlung losses since the particles are not moving in a circle (ie, accelerating towards the centre). Muons are heavier and so bremsstrahlung losses are not as significant

c) For deep inelastic scattering with electrons at a linear collider:



→ $p_e^\mu = E(1,0,0,1)$ since $m_e \ll m_p$, $p_e'^\mu = E'(1, \sin\theta, 0, \cos\theta)$

→ $p_p^\mu = m_p(1,0,0,0)$ since p is stationary

→ The invariant mass of the final state hadronic system is given by: $W^2 = p_x^{\mu\,2} = (E_x^2 - |\vec{p}_x|^2) = (q + p_p)^2$

→ Then, $q^2 = (p_e - p_e')^2$

$$= (E_e - E_e')^2 - (\vec{p}_e - \vec{p}_e')^2$$

$$= 2EE'(1-\cos\theta)$$

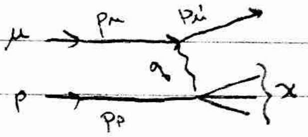
→ Finally, $W^2 = (q + p_p)^2$

$$= m_p^2 + q^2 + 2 p_p \cdot q$$

$$= m_p^2 + q^2 + 2m_p(E - E')$$

Question #2 con't . . .

For deep inelastic scattering with muons at a circular collider:



→ $p_\mu = (E_\mu, 0, 0, p_\mu)$, $p_\mu' = (E_\mu', |\vec{p}_\mu'|\sin\theta, 0, |\vec{p}_\mu'|\cos\theta)$

→ $p_p = (m_p, 0, 0, 0)$ → target is stationary ....?

→ Then, as before, $q^2 = (p_\mu - p_\mu')^2$

$$= -2EE' + 2\vec{p}\cdot\vec{p}' + 2m_\mu^2$$

$$= -2EE'(1-\cos\theta) \quad \text{for } EE' >> m_\mu$$

→ Similarly, $W^2 = (q + p_p)^2$

$$= m_p^2 + 2m(E - E') + q^2$$

Problem Set #5

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Question #3

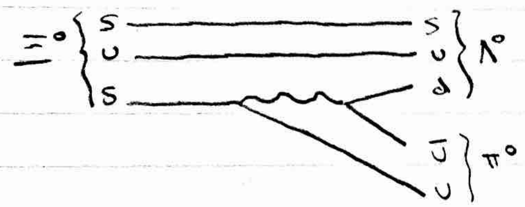
a) $\Delta^{++} \rightarrow p + \pi^+$ where Δ^{++} $J^P = \frac{3}{2}^+$

p $J^P = \frac{1}{2}^+$

π^+ $\ell = 1,\ s = 0$

Mesons carry parity $P = (-1)^{\ell+1}$, $\therefore$ the pion has a parity of $P = (-1)^{1+1} = +1$.

b) $\Xi^0 \rightarrow \Lambda + \pi^0$



This process involves a weak decay of the strange quark! We cannot use this process to determine the parity of the π^0 since the weak interaction violates parity.

Looking up the Ξ^0 in the PDG, we can see that its parity has not yet been measured, but is predicted to be +. Looking at its decay modes, we see that the weak $\Xi^0 \rightarrow \Lambda^0 \pi^0$ decay has a branching fraction of >99.5%, and in addition the production of Ξ^0 from non-strangeness initial states have a very small cross section — this makes it difficult to study the Ξ^0.

Question #4

Mesons contain one quark and one antiquark ex// $\pi^+ = u\bar{d}$

note: γ $J^{PC} = 1^{--}$

η $J^{PC} = 0^{-+}$

Then, $\eta \rightarrow \gamma\gamma \Rightarrow C(+) \rightarrow C(-)\,C(-)$ $\therefore$ allowed

$\eta \rightarrow \pi^0 \gamma \Rightarrow C(+) \rightarrow C(+)\,C(-)$ $\therefore$ forbidden

$\eta \rightarrow \pi^0\pi^0\pi^0 \Rightarrow C(+) \rightarrow C(+)\,C(+)\,C(+)$ $\therefore$ allowed

$\eta \rightarrow \gamma\gamma\gamma \Rightarrow C(+) \rightarrow C(-)\,C(-)\,C(-)$ $\therefore$ forbidden

$\eta \rightarrow \pi^+\pi^-\pi^0 \Rightarrow C(+) \rightarrow C(+)\,C(+)\,C(+)$ $\therefore$ allowed

MIQUELRIUS

Question #5

a) Parity of $\pi^+\pi^-$ system: $P = P(\pi_1) \cdot P(\pi_2) \cdot (-1)^l$ where l = relative orbital angular momentum

$= (-1)(-1)(-1)^l$

$= (-1)^l$

b) If $l = 1$ then $P = (-1)^{(1)} = -1$.

If the system decays into a two photon final state, then $P_{\gamma\gamma} = (-1)(-1) = +1$

Since EM interaction conserves C, this decay is not possible.

Question #6

Show that $\vec{J} \cdot \vec{p}$ is time-reversal invariant, where $\vec{J}$ = angular momentum

$\vec{p}$ = momentum

note: linear momentum reverses sign if the direction in time is reversed

angular momentum reverses sign under time reversal as well

$\therefore$ $\vec{J} \cdot \vec{p}$ is time reversal invariant since $\vec{J} \cdot \vec{p} \xrightarrow[t]{} (-\vec{J}) \cdot (-\vec{p}) = \vec{J} \cdot \vec{p}$.

$\Rightarrow$ You can check this by applying time-reversal to the expectation value of the momentum operator, where $\psi^T(x, t) = \psi^*(r, -t)$

Question #7

For two particles to mix, $A \Leftrightarrow B$, they must have the same mass because

With 3 quark generations, candidate particles for mixing are either baryons or mesons. We exclude baryons because baryons must have baryon number of ± 1, and baryon mixing would therefore violate conservation of baryon number.

We are now left with neutral mesons (neutral because, as stated in the question, for two particles to mix, they must have the same charge). For $q\bar{q}$, these are their own antiparticles and so are not interesting. What is left are $q_1\bar{q}_2$ neutral mesons.

The quark combinations are, for $-1/3$ charge: $d\bar{s} \Leftrightarrow s\bar{d}$, $d\bar{b} \Leftrightarrow b\bar{d}$, $s\bar{b} \Leftrightarrow b\bar{s}$

for $+2/3$ charge: $u\bar{c} \Leftrightarrow c\bar{u}$ (note: t does not form bound states)

The meson candidates are $K^0/\bar{K}^0$, $B^0/\bar{B}^0$, $B_s^0/\bar{B}_s^0$, $D^0/\bar{D}^0$. See the PDG for which have been observed so far.

The neutron does not mix with an antineutron due to conservation of baryon number.

For the vector mesons, check out their decay lifetimes. Would they have time to mix?