PHYSICS 357S - Problem Set #2 - January 2016

Distributed **3rd February** due to be handed in by **22nd February** before 17:00. **Please** have a look at the problem set when it comes out. Decide whether it is going to cause you trouble or not…. And ask questions well before the due date. The problem sets are supposed to give you an opportunity to ask questions. There are SIX questions. *As usual, keep an eye out for typos! I am not a very good typist.*

The first two questions are just standard bookwork. BUT make sure you understand them, i..e. explicitly do the relativistic kinematics yourself!

1) In class we discussed how some of the kinetic energy of a *beam* particle colliding with a *target* particle can be transformed into the masses of new particles in the final state. Assume that a beam particle A of total energy E collides with a target particle B *(remember the LAB is defined as the frame where the target is at rest.)* New particles $C_1, C_2 \ldots$ are produced in the final state. We write this according to the notation:

$$A + B \to C_1 + C_2 + \ldots + C_n$$

a) Show that the minimum energy E for A is

$$E = \frac{M^2 - m_A^2 - m_B^2}{2m_B} c^2 \text{, where } M \equiv m_1 + m_2 + \ldots + m_n$$

This minimum energy is known as the *Threshold Energy* for producing the final state $C_1 + C_2 + \ldots + C_n$.

b) Imagine an experiment to produce a particle called the ψ. This particle is a bound state made of a c-quark and its anti-particle (the $\bar{c}$ -quark ... we will discuss all these concepts later). The ψ has a mass of $3.1\,\mathrm{GeV/c^2}$. Our experiment consists of firing a beam of positrons at a target containing stationary electrons. What energy does the positron beam have to have? *(Assume that the final state consists of a single ψ.)* Say we made a machine which collided electron and positron beams head on, with equal and opposite momentum. What would have to be the momentum of each beam?

c) Calculate the minimum π^- momentum in the laboratory necessary for the reaction

$$\pi^- p \to \Delta^{++} \pi^- \pi^-$$

to occur. This experiment consists of firing a *pion* beam in to a liquid hydrogen target. ($m_p = 0.981\,\mathrm{GeV/c^2}$, $m_{\pi^-} = 0.140\,\mathrm{GeV/c^2}$, $m_{\Delta^{++}} = 1.323\,\mathrm{GeV/c^2}$)

2) The new particles produced in these experiments are often unstable, and rapidly decay. Consider a particle A at rest *(i.e. consider the particle in its rest-frame, or CM {centre-of-mass, or centre-of-momentum} frame)* decaying according to the scheme:

$$A \to BC$$

Show that the energy of B is:

$$E_B = \frac{m_A^2 + m_B^2 - m_C^2}{2m_A} c^2$$

and also show that the outgoing momenta are given by

$$|\vec{p}_b| = |\vec{p}_c| = \frac{\sqrt{\lambda\left(m_A^2, m_B^2, m_C^2\right)}}{2m_A} c$$

where

$$\lambda(x, y, z) \equiv x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$$

Use these results to find the energy in the CM frame of each decay product in the following reactions.

a) $K^+ \to \mu^+ \nu_\mu$

b) $\eta^0 \to \gamma\gamma$

c) $K^0 \to \pi^- \pi^+$

d) $\Delta^+ \to p\pi^0$

c) $\Omega^- \to \Lambda K^+$

The μ has a mass of $106\, MeV/c^2$

The photon γ is the quantum of light, and is massless.

ν_μ is massless (well, it's so small that you can assume that it is zero). The η^0 is in some way a heavy version of the π^0

The Δ particle is in some way like a heavy proton. It has the same *quark structure*. In fact it can be considered as an *excited state* of the proton. The mass is $1232\, MeV/c^2$.

The K^+ is distinguished from the π^+ not only by being heavier; but it also carries one unit of a quantum number called *strangeness.* The K^+ and π^+ are mesons; we will learn that this means they are made up of a quark and an anti-quark. A state made of three quarks is called a baryon, The proton is a familiar *baryon.* The Ω^- is also a baryon; but it is a *strange baryon.* It carries three units of this strangeness quantum number. It has a mass of $1672\, MeV/c^2$.

3) In relativistic collisions the kinetic energy of the colliding particles can be transformed into the mass of *new particles.* So neither the number of particles, nor the total mass, is necessarily conserved in a relativistic collision. Very short lived particles can be produced, and we detect their presence by looking for a peak in the invariant mass of the products of the decay.

Say we suspected, or predicted, that there was a particle which decayed to an oppositely charged pair of *K* mesons. We could search for this particle by firing pions into a liquid hydrogen target.

$$\pi^+ p \to K^+ K^- \pi^+ p$$

We would then reconstruct the invariant masses of the oppositely charged *K* mesons, and build up a histogram of the invariant mass distribution over many events. If the invariant mass showed an enhancement at some mass, we could conclude that we had discovered the particle. Consider one event, and assume that the K^+ and the K^-, in the final state, are produced so that they have an angle of 6.3° between them , and that they have momenta 10.0 *GeV/c*, and 5.0 *GeV/c* respectively, in the LAB. What is the *invariant mass* of the K^+K^- system?

In fact there is a particle known as the ϕ meson, which consists of a s-quark and an anti-s quark bound together by the colour force. The ϕ has a mass of $1020\ GeV/c^2$. Do you think this one event would persuade you of the existence of the ϕ? Explain why or why not.

Take the mass of the K meson to be 493 MeV/c^2*, independent of its electric charge.*

4) The discovery of the heavy strangeness = -3 baryon, the Ω^-, was extremely important. It confirmed a theory originated by Gell-Mann and Ne'eman which they called *The Eightfold Way*. Essentially this theory was that baryons are made of three quarks, although it was not originally realized that this was what it meant. We now call this theory "$SU(3)$ of flavour". If you want to jump ahead a bit you can read section 3.3.2 of the text book (you don't have to). The Ω^- is discussed in section 5.1 and 6.5.2. The bubble chamber event of the first observation of the Ω^- will appear in future problem set.

The Ω^- was discovered in the reaction,

$$K^- p \to \Omega^- K^+ K^0 .$$

We will learn that the K^0 is a superposition of two eigenstates K_S^0 and K_L^0, which have different lifetimes. Assume that in this case the K^0 decays as a K_S^0 with a mean lifetime of 0.8954×10^{-10} s and has a mass of $497.7\ MeV/c^2$.

(a) Which force (weak, EM, or strong) do you think causes the K_S^0 to decay?

(b) What is the mean lifetime and mean decay distance of a $5\ GeV/c\ \ K_S^0$ in the LAB frame?

(c) In terms of the masses of the various particles, what is the threshold *kinetic energy* for the reaction to occur, if the proton is at rest?

(d) Suppose the K^0 travels at $0.8c$. It then decays in flight into two neutral pions. Find the maximum angle in the LAB frame that the pions can make with the K^0 line of flight. Express your answer in terms of the π and K masses.

5) The Large Hadron Collider is a proton collider designed to produce the Higgs boson; a particle thought to have a mass of about 200 GeV/c^2. Now that the Higgs has been discovered at the LHC, there are still be uncertainty about its properties, due to the fact that the colliding quarks inside the protons do not have a well defined momentum. The International Linear Collider is designed to study the couplings of the Higgs in detail, as the colliding electrons and positrons have well defined momenta. To go to even higher energies than the LHC, a Muon Collider has been proposed. This would be a synchrotron storage ring colliding μ^+ and μ^- head on.

a) How would you produce the muons necessary to inject into the storage ring? Have look at this link *http://www.physics.usyd.edu.au/hienergy/index.php/WANF*

b) With a site which is a square of 50 km on the side, what is the maximum beam energy of a muon collider, if the machine is built in a circular tunnel full of bending magnets with a field of 5 Tesla?

c) Assume that we want to produce the same CM energy by colliding muons from an accelerator with stationary protons in a liquid hydrogen target. What would have to be the beam energy of such the muons? What would be CM energy be if these muons collided with the atomic electrons in the liquid hydrogen?

d) In a synchrotron with the same beam momentum as the LHC, calculate the relative energy losses if the beams are protons, muons, or electrons. Look at section 4.2.2.2 of the textbook.

(6) The relationship between the "proper" force $\overline{f}$ and the change in energy and momentum is given in relativistic kinematics by,

$$\frac{d\overline{p}}{dt} = \overline{f}\ ;\ \ \frac{dE}{dt} = \overline{f}.\overline{v}\ .$$

The velocity in the observer's frame is $\overline{v}$, and the proper force is just the 4-force in the instantaneous rest frame. So,

$$\frac{d\overline{p}}{dt} = \frac{E}{c^2}\frac{d\overline{v}}{dt} + \frac{\overline{v}}{c^2}\frac{dE}{dt},$$

Derive this and then show that

$$m\frac{d\overline{v}}{dt} = \left(\overline{f} - \frac{\overline{v}\left(\overline{v}.\overline{f}\right)}{c^2}\right)\gamma^{-1} \qquad (1)$$

Where I have kept c explicitly. Keep in mind that $\overline{p} = m\frac{d\overline{x}}{dt}\gamma = m\overline{v}\gamma$ and $E = \gamma mc^2$. For a constant force, equation (1) can be integrated to give you the final velocity.

The International Linear Collider will consist of two linear accelerators (one accelerating electron, and the other accelerating positrons) colliding head-on. Assume that one of them is 15 km long and electrons are subject to a constant force of *eE*, in the LAB frame, and are accelerated to a momentum of 500 GeV/c in the LAB frame.

a) What is the final velocity if the electrons if they start from rest?

b) Find the field strength *E. I mean the strength of the electric field in Volts/metre in the direction of the particles, which accelerates them.*

c) How far would the electrons have to travel in this field in order to reach the same velocity, if Newtonian mechanics applied?

d) Estimate the length of the accelerator in the electrons' rest frame.

Possibly Useful Physical Constants:

Avogadro No:	$6\times10^{23}\,mole^{-1}$
pi	$\pi = 3.1416$
speed of light:	$c = 3.0\times10^{8}\,m/s$
Plank's constant:	$\hbar = 6.6\times10^{-22}\,MeV\cdot s$
	$\hbar c = 197\,MeV.fm$
	$(\hbar c)^2 = 0.4\,GeV^2\cdot mb$
	$1\ eV = 1.6\times10^{-19}\,Joules$
	$1\ eV/c^2 = 1.8\times10^{-36}\,kg$
	$1\,fm = 10^{-15}\,m$
	$1\ mb = 10^{-27}\,cm^2$
1 year	1 year $\approx \pi\times10^{7}s$
electron charge:	$e = 1.602\times10^{-19}\,C$
electron magnetic moment:	$\mu_e = 9.3\times10^{-24}\,Joules\cdot Tesla^{-1}$
fine structure constant:	$\alpha = e^2/(\hbar c) = 1/137.0360$
strong coupling constant:	$\alpha_s(M_Z) = 0.116\pm0.005$
Fermi coupling constant:	$G_F = 1.166\times10^{-5}\ GeV^{-2}$
Cabibbo angle:	$\sin\theta_C = 0.22$
Weak mixing angle:	$\sin^2\theta_W(M_Z) = 0.2319\pm0.0005$
Branching Ratios	$BR(Z\to e^+e^-) = 3.21\pm0.07\%$
	$BR(Z\to hadrons) = 71\pm1\%$

Particle Properties

Boson	Mass $\left(GeV/c^2\right)$
γ	$< 3\times10^{-36}$
$gluon$	~ 0
$W^{\pm}$	80.22
Z^0	91.187
H^0	~ 125

Lepton	Mass $\left(MeV/c^2\right)$
ν_e	$< 10^{-5}$
e	0.510999
ν_μ	< 0.27
μ	105.658
ν_τ	< 10
τ	1777

Hadron	Quark Content	Mass $\left(MeV/c^2\right)$	$\mathbf{I}\left(\mathbf{J}^{\mathbf{PC}}\right)$
π^+,π^0,π^-	$u\bar{d},\left(u\bar{u}-d\bar{d}\right)/\sqrt{2},d\bar{u}$	139.57,134.97, 139.57	$1\left(0^{-+}\right)$
K^+,K^-	$u\bar{s},s\bar{u}$	493.65	$\frac{1}{2}\left(0^-\right)$
$K^0,\bar{K}^0$	$d\bar{s},s\bar{d}$	497.67	$\frac{1}{2}\left(0^-\right)$
ρ^+,ρ^0,ρ^-	$u\bar{d},\left(u\bar{u}+d\bar{d}\right)/\sqrt{2},\bar{u}d$	775.7	$1\left(1^{--}\right)$
p , n	uud , udd	938.27, 939.57	$\frac{1}{2}\left(\frac{1}{2}^+\right)$
$\Delta^-,\Delta^0,\Delta^+,\Delta^{++}$	ddd,udd,uud,uuu	1232	$\frac{3}{2}\left(\frac{3}{2}^+\right)$
Λ^0	uds	1115.6	$0\left(\frac{1}{2}^+\right)$
$\bar{D}^0,D^0$	$u\bar{c},c\bar{u}$	1863	$\frac{1}{2}\left(0^-\right)$
D^-,D^+	$d\bar{c},c\bar{d}$	1869	$\frac{1}{2}\left(0^-\right)$
D_S^+,D_S^-	$c\bar{s},\bar{c}s$	1968	$0\left(0^-\right)$
B^+,B^-	$u\bar{b},\bar{u}b$	5279	$\frac{1}{2}\left(0^-\right)$
Λ_c^+	udc	2285	$0\left(\frac{1}{2}^+\right)$
$\Sigma^+,\Sigma^0,\Sigma^-$	uus,uds,dds	1189	$1\left(\frac{1}{2}^+\right)$
Ξ^0,Ξ^-	uss,dss	1315	$\frac{1}{2}\left(\frac{1}{2}^+\right)$
Ω^-	sss	1672	$0\left(\frac{3}{2}^+\right)$
Λ_b	udb	5624	$0\left(\frac{1}{2}^+\right)$