PHYSICS 357S - Problem Set #3 - February 2016

Distributed **22nd Feb** and be handed in by **9th March** before 17:00. **Please** have a look at the problem set when it comes out. Decide whether it is going to cause you trouble or not…. And ask questions well before the due date. The problem sets are supposed to give you an opportunity to ask questions. There are 7 questions. *As usual, keep an eye out for typos! I am not a very good typist.*

(1) Baryons like the proton, or neutron, are bound states of three quarks. Since we can have all sorts of orbital and spin angular momentum combinations, the spin (by spin I mean the modulus of the maximum z-component of the angular momentum. An electron is spin $\frac{1}{2}$) The spin spectrum of baryons is much more complex than mesons, which consist of a quark and an anti-quark.

a) What are the possible total spins of a mesons in which the quarks have no relative angular momentum?

b) What are the possible total spins possible for mesons in which the quarks have one unit of orbital angular momentum?

c) A meson has total spin of $\frac{5}{2}$, is it possible for the constituent quarks to have a relative orbital angular momentum of 2?

d) Suppose you had two particles of spin 1 each in the states $|1,0\rangle$. They collide and form a composite system (bound state). What values could you get for the total angular momentum of this system? What is the probability of each value. You get some value with probability 1, so check that the sum of your probabilities is unity.

e) A particle has intrinsic spin $\frac{3}{2}$, and is in a $\left|\frac{3}{2},+\frac{1}{2}\right\rangle$ state. What are the possible spin states of its decay products, and what are their probabilities?

(2) (a) An electron beam of 10 GeV energy and a current of 10^{-6} amps is focused onto an area of $0.5\,cm^2$ What is the flux of electrons?

(b) The total number of protons stored in a synchrotron is such that if they are all extracted in one orbit period, the flux is much larger than a normal fixed target experiment could cope with. In order to avoid saturation of the experiments, the internal circulating beam is slowly extracted over a number of orbit periods, a little bit of the beam being extracted on each turn. Assume that the circulating beam of a 100 GeV accelerator contains 10^{13} protons. In a fixed target experiment these protons are focused onto an area of 2 cm^2 and extracted uniformly over a time of 0.5 s. What is the flux of protons onto the experiment? If the accelerator is 7 *km* in circumference, how many beam orbit periods does this extraction correspond to?

(c) A copper target of thickness 0.1 *cm* intercepts a particle beam of area 4 cm^2. Compute the number of scattering centers intercepted by the beam. Then assume a total interaction cross section of 10 *mb* and calculate what fraction of the beam is scattered.

(3) A cross section allows you to calculate the attenuation of a beam of particles passing through a material. It also allows you to calculate the *probability* that an individual particle will interact in that amount of material. As you probably know, we now know that the different flavours of neutrino can change into each other….. neutrino oscillations. The most famous result on neutrino oscillations comes from the SNO detector in Sudbury, Ont. Their results resolved the long standing "solar neutrino problem". Neutrinos produced by β-decay in the sun are electron type neutrinos. We can calculate the rate of electron neutrinos produced in the sun (amazing, I think), and too few reach the Earth… they oscillate into other flavours on the way.

The first observation of neutrino oscillations was by the Kamiokande experiment in Japan. Neutrinos are produced high in the atmosphere by cosmic ray collisions, and the decay of pions and kaons. Both electron and muon neutrinos are produced with a known ratio of production. The Kamiokande experiment looked for neutrinos coming from the other side of the Earth, i.e. they penetrate all the way through the Earth and appear as upwards going neutrinos in Japan. Kamiokande found that the ratio of electron to muon neutrinos was different for downward and upward going neutrinos. The downward going ones had just come from a few km up in the atmosphere, whereas the upward going ones had crossed the Earth's diameter, and there was time for the oscillation to manifest itself.

a) Calculate the probability of a neutrino crossing the Earth's Diameter without interacting. Assume that the cross section for interaction is $10^{-38} cm^2$, the diameter of the Earth is 12,756 km, it's mean density is $5500 kg.m^{-3}$ and that the "atomic wt" is 40. I chose 40 because it is half way between silicon and iron. This is just an approximate calculation, if you come up with better input numbers for the Earth's composition; good for you. But don't waste too much time doing research on the structure of the Earth.

b) Assume that the Kamiokande detector is cube of 1000 tons of water. Take the "atomic wt" of water to be 16, and its density to be $1 gm.cm^{-3}$. Calculate the probability of a neutrino interacting in this detector. Just assume the same cross section for interaction. Assume that Kamiokande is uniformly illuminated by this "beam" of neutrinos.

c) If the Kamiokande detector sees 10 upward going neutrino interactions per day, roughly estimate the flux of neutrinos incident on the other side of the Earth. Neglect oscillations, this is just a simple cross section calculation.

(4) If one had a particular stable nuclide, one might want to fire neutrons into it and change it into a new nuclide with more neutrons. This is how some of the *transuranic elements* were made. The bombarded nuclides can fly apart into lighter nuclides, or they might fuse. By studying these *nuclear reactions*, a great deal has been learned about the internal dynamics of nuclei. There are also various practical applications. Thermonuclear weapons get some (secret) amount of their yield from:

$$^{7}_{3}Li + n \rightarrow {}^{3}_{1}T + {}^{4}_{2}He + n \, .$$

The first US lithium based thermonuclear weapon test was Castle Bravo. The expected yield was 6 megatons. The designers had neglected this reaction, and the yield was 15 megatons, almost leading to disaster. The cross section for this is probably a secret, so I can't do a calculation on it. However, a much better application is in a fusion reactor, where most of the energy will come from the reaction

$$^{3}_{1}T + {}^{2}_{1}D \rightarrow {}^{4}_{2}He + n \, .$$

At an energy of $65\, keV$ this produces $17\, MeV$ neutrons. These are absorbed in the walls of the reactor (a Tokamak) according to

$$^{6}_{3}Li + n \rightarrow {}^{4}_{2}He + {}^{3}_{1}T \, ,$$

producing heat and more $^{3}_{1}T$. Let's consider the reaction $^{3}_{1}T + {}^{2}_{1}D \rightarrow {}^{4}_{2}He + n$. It has a cross section of $5.0\, barn$. In a cm^{3} of $^{2}_{1}D$,calculate what fraction of the $^{2}_{1}D$ is used up in a year, if the incident flux of $^{3}_{1}T$ is $10^{15}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$.

(5) a) At the end of the lecture on “particle Classification & Lepton Number”. I drew diagrams showing the flow of the quarks and force bosons for the interaction

$$K^- p \to \Lambda^0 \pi^0 .$$

Draw the same kind of diagrams for the interaction $K^- p \to \Xi^0 K^0 \pi^0$, followed by $K^0 \to \pi^0 \pi^0$, $\Xi^0 \to \Lambda \pi^0$ and $\pi^0 \to \gamma\gamma$. There is an $\overline{s}$ quark in the K^0, and two *s* quarks in the Ξ^0. You have to produce an $s\overline{s}$ pair from a *gluon*, just as in $K^- p \to \Lambda^0 \pi^0$. This is an example of the colour force *conserving strangeness, or quark flavour.* Also note that the weak interaction can change an s quark into a u quark or an $\overline{s}$ into a $\overline{u}$. This is an example of the weak force *not conserving quark flavour.* In each of these quark flow diagrams, explicitly state which force is acting in each decay.

b) At the end of the lecture on “Scattering and Feynman Diagrams”, I showed the quark flow diagrams for various weak processes. By analogy, draw the quark flow diagrams for the weak interaction processes,

$$\Xi^0 (uss) \to \Sigma^+ e^- \overline{\nu}_e$$
$$\nu_e n \to e^- p$$
$$\tau^- \to \nu_\mu \overline{\nu}_\mu \mu^-$$

(6) (a) The Δ^{++} has spin $\frac{3}{2}$. What does that tell you about the angular momentum state of the quarks in it? How does that make you conclude that the quarks must carry an additional quantum number, as well as spin, electric charge, and mass? Think about the spin-statistics of spin $\frac{1}{2}$. What is that quantum number? If you can't figure this out, it is a well know argument, you'll find it by Googling Δ^{++}

(b) Assign the lepton generation *(this is the same as lepton flavour)* subscript and distinguish antineutrinos from neutrinos in the reactions and decays and decays on the next page. Use the symbols $\nu_e, \overline{\nu}_e, \nu_\mu, \overline{\nu}_\mu, \nu_\tau, \overline{\nu}_\tau$. Draw the lowest order Feynman diagram for each process. Where there are mesons or baryons in both sides of the process, just show how the quarks flow through the interaction; but remember to label them with their flavours and charges. The chart at the end of the problem set will help you with this. It is from the book Nuclear and Particle Physics by W.S.C. Williams

$\pi^+ \to \pi^0 e^+ \nu$	$\nu p \to n e^+$
$\mu^+ \to e^+ \nu \nu$	$\nu\ {}^{37}_{17}Cl \to {}^{37}_{18}Ar\ e^-$
$\mu^- \to e^- \nu \nu$	$\nu p \to \mu^- p \pi^+$
$K^+ \to \pi^0 e^+ \nu$	$\nu n \to e^- p$
$\overline{K}^0 \to \pi^+ e^- \nu$	${}^3_1H \to {}^3_2He\, e^- \nu$
$\Sigma^- \to n \mu^- \nu$	$\pi^+ \to \mu^+ \nu$
$\Sigma^+ \to \Lambda^0 e^+ \nu$	$\pi^- \to e^- \nu$
$D^0 \to K^- \pi^0 e^+ \nu$	$\tau^- \to \pi^- \pi^0 \nu$

(7) I gave the formula for the luminosity of a colliding beam accelerator

$$\mathcal{L} = \frac{N_1 N_2}{4\pi\sigma_x\sigma_y} f n_b \,.$$

N_1 and N_2 are the number of particles per bunch in each of the colliding beams, f is the rotation frequency of bunches around the machine, n_b is the number of bunches around the ring, and σ_x and σ_y are the r.m.s. size of the beams in the x and y directions. The beams are supposed to have a Gaussian profile. The relevant parameters of the LHC are:

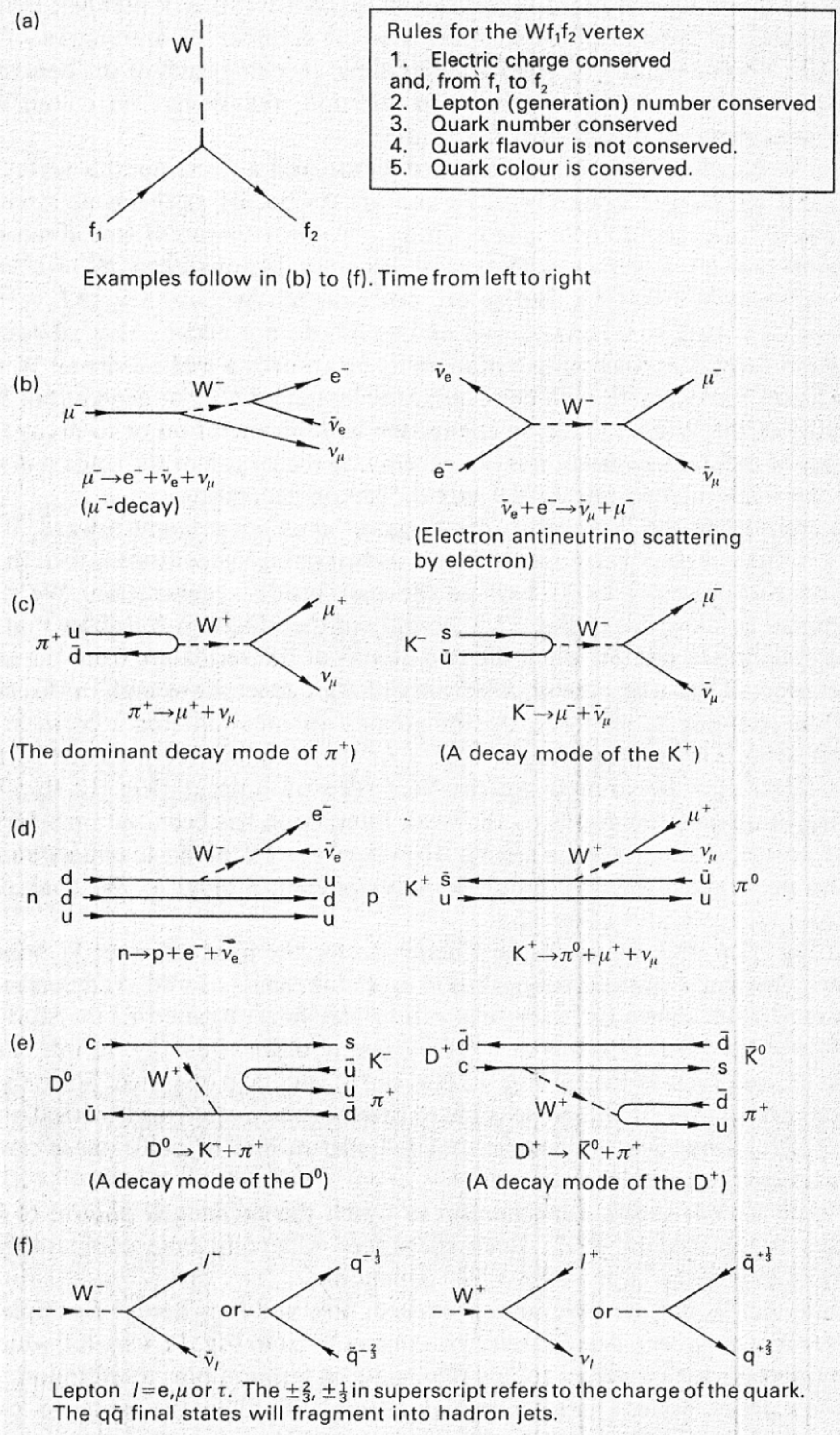
Beam Parameters for Large Hadron Collider	
Number of Particles per Bunch N	1.5×10^{11}
Number of bunches n_b	2808
R.M.S. beam size σ_x (microns)	16
R.M.S. beam size σ_y (microns)	16

(a) what is the luminosity?

(b) If the cross section for collisions is 10 picobarns, determine the total number of scattering events per second.

(c) Find the *average* flux of protons

(d) If a beam of protons is extracted from the machine and allowed to scatter from a stationary target of liquid hydrogen (density = 0.1 g.cm^{-3}) 2 m. long, find the number of scattering events per second and compare to the answer in (b). You have to assume that the scattering cross section in this case is the same as in the colliding beam case. This is unrealistic........ but there we are.



Gluon exchange is omitted in the examples involving hadrons. The $u\bar{u}$ pair in $D^0 \rightarrow K^- \pi^+$ is created by gluon exchange with the other quarks.

Possibly Useful Physical Constants:

Avogadro No:	$6\times10^{23}\,mole^{-1}$
pi	$\pi = 3.1416$
speed of light:	$c = 3.0\times10^{8}\,m/s$
Plank's constant:	$\hbar = 6.6\times10^{-22}\,MeV\cdot s$
	$\hbar c = 197\,MeV.fm$
	$(\hbar c)^2 = 0.4\,GeV^2\cdot mb$
	$1\,eV = 1.6\times10^{-19}\,Joules$
	$1\,eV/c^2 = 1.8\times10^{-36}\,kg$
	$1\,fm = 10^{-15}\,m$
	$1\,mb = 10^{-27}\,cm^2$
1 year	1 year $\approx \pi\times10^{7}s$
electron charge:	$e = 1.602\times10^{-19}\,C$
electron magnetic moment:	$\mu_e = 9.3\times10^{-24}\,Joules\cdot Tesla^{-1}$
fine structure constant:	$\alpha = e^2/(\hbar c) = 1/137.0360$
strong coupling constant:	$\alpha_s(M_Z) = 0.116\pm0.005$
Fermi coupling constant:	$G_F = 1.166\times10^{-5}\,GeV^{-2}$
Cabibbo angle:	$\sin\theta_C = 0.22$
Weak mixing angle:	$\sin^2\theta_W(M_Z) = 0.2319\pm0.0005$
Branching Ratios	$BR(Z\to e^+e^-) = 3.21\pm0.07\%$
	$BR(Z\to hadrons) = 71\pm1\%$

Particle Properties

Boson	Mass $\left(GeV/c^2\right)$
γ	$< 3\times10^{-36}$
gluon	~ 0
$W^{\pm}$	80.22
Z^0	91.187
H^0	125

Lepton	Mass $\left(MeV/c^2\right)$
ν_e	$<10^{-5}$
e	0.510999
ν_μ	< 0.27
μ	105.658
ν_τ	<10
τ	1777

Hadron	Quark Content	Mass $\left(MeV/c^2\right)$	$\mathbf{I}\left(\mathbf{J^{PC}}\right)$
π^+,π^0,π^-	$u\bar{d},\left(u\bar{u}-d\bar{d}\right)/\sqrt{2},d\bar{u}$	139.57,134.97, 139.57	$1\left(0^{-+}\right)$
K^+,K^-	$u\bar{s},s\bar{u}$	493.65	$\frac{1}{2}\left(0^-\right)$
$K^0,\bar{K}^0$	$d\bar{s},s\bar{d}$	497.67	$\frac{1}{2}\left(0^-\right)$
ρ^+,ρ^0,ρ^-	$u\bar{d},\left(u\bar{u}+d\bar{d}\right)/\sqrt{2},\bar{u}d$	775.7	$1\left(1^{--}\right)$
p,n	uud,udd	938.27, 939.57	$\frac{1}{2}\left(\frac{1}{2}^+\right)$
$\Delta^-,\Delta^0,\Delta^+,\Delta^{++}$	ddd,udd,uud,uuu	1232	$\frac{3}{2}\left(\frac{3}{2}^+\right)$
Λ^0	uds	1115.6	$0\left(\frac{1}{2}^+\right)$
$\bar{D}^0,D^0$	$u\bar{c},c\bar{u}$	1863	$\frac{1}{2}\left(0^-\right)$
D^-,D^+	$d\bar{c},c\bar{d}$	1869	$\frac{1}{2}\left(0^-\right)$
D_S^+,D_S^-	$c\bar{s},\bar{c}s$	1968	$0\left(0^-\right)$
B^+,B^-	$u\bar{b},\bar{u}b$	5279	$\frac{1}{2}\left(0^-\right)$
Λ_c^+	udc	2285	$0\left(\frac{1}{2}^+\right)$
$\Sigma^+,\Sigma^0,\Sigma^-$	uus,uds,dds	1189	$1\left(\frac{1}{2}^+\right)$
Ξ^0,Ξ^-	uss,dss	1315	$\frac{1}{2}\left(\frac{1}{2}^+\right)$
Ω^-	sss	1672	$0\left(\frac{3}{2}^+\right)$
Λ_b	udb	5624	$0\left(\frac{1}{2}^+\right)$