PHYSICS 357S - Problem Set #4 - March 2016

*There are **7** questions and **10** pages in this problem set.*

Distributed Thursday **10th March**. Due to be handed in by **23rd March** before 17:00. You can give it to me at class, give it to Laurelle, or put it in my mail box in room 804. Please be careful handing work in. Try to give it to one of us personally. Lost work cannot be given credit. **Please** have a look at the problem set when it comes out. Decide whether it is going to cause you trouble or not…. And ask questions well before the due date. The problem sets are supposed to give you an opportunity to ask questions.

There are SEVEN questions

There is a lot of text, but that is just to enlarge on some things in the lectures, and to try to help. I think the questions are actually quite simple.

As usual, keep an eye out for typos! I am not a very good typist.

(1) Classify the following experimentally observed process into strong, electromagnetic and weak interactions by considering the particles involved and the appropriate selections rules (i.e. quantum numbers conserved). Draw quark flow diagrams for all the decays.

$$\pi^- + p \to \pi^- + \pi^+ + n$$

$$\gamma + p \to \pi^0 + p$$

$$\overline{\nu}_\mu + p \to \mu^+ + n$$

$$\pi^0 \to e^+ + e^- + e^+ + e^-$$

$$p + \overline{p} \to \pi^+ + \pi^- + \pi^0$$

$$\tau^- \to \pi^- + \nu_\tau$$

$$D^- \to K^+ + \pi^- + \pi^-$$

$$\pi^- \to \pi^0 + e^- + \overline{\nu}_e$$

$$\Lambda + p \to K^- + p + p$$

(2) This question is about the shell model of the nucleus. I didn't explicitly talk about nuclear spin and parity. However, it is much the same as the argument for magnetic moments. Actually it IS the same. You just have to count the filled levels, and figure out what is the unpaired nucleon.

(i) the spin of all even-even nuclei is zero

(ii) the spin of odd-A nuclei are given by the unpaired nucleon.

We also haven't discussed parity at great length, but you must know roughly what that is from the quantum mechanics of the potential well. The nuclear parity is the product of all the single nucleon wave functions $\prod_A (-1)^l$. It follows that

(iii) the parity of the ground state of all even-even nuclei is even.

(iv) the parity of the ground state of all odd-A nuclei is that of the wavefunction of the unpaired nucleon.

Below is a table of the spins parities, and magnetic moments of some odd-A nuclei, and one which is even-A (it is actually odd-odd). Determine the type, neutron or proton, of the unpaired nucleon, or nucleons in the case of the even-A, and use the level ordering, in my notes on the shell model, to decide its configuration. Compare your predictions with the measurements. Be careful. In some of these nuclei, it is not true that *l* and *s* are parallel. The agreement is not perfect…right? Do you have any comments? Especially about the relative orientation of the spins in the even-A?

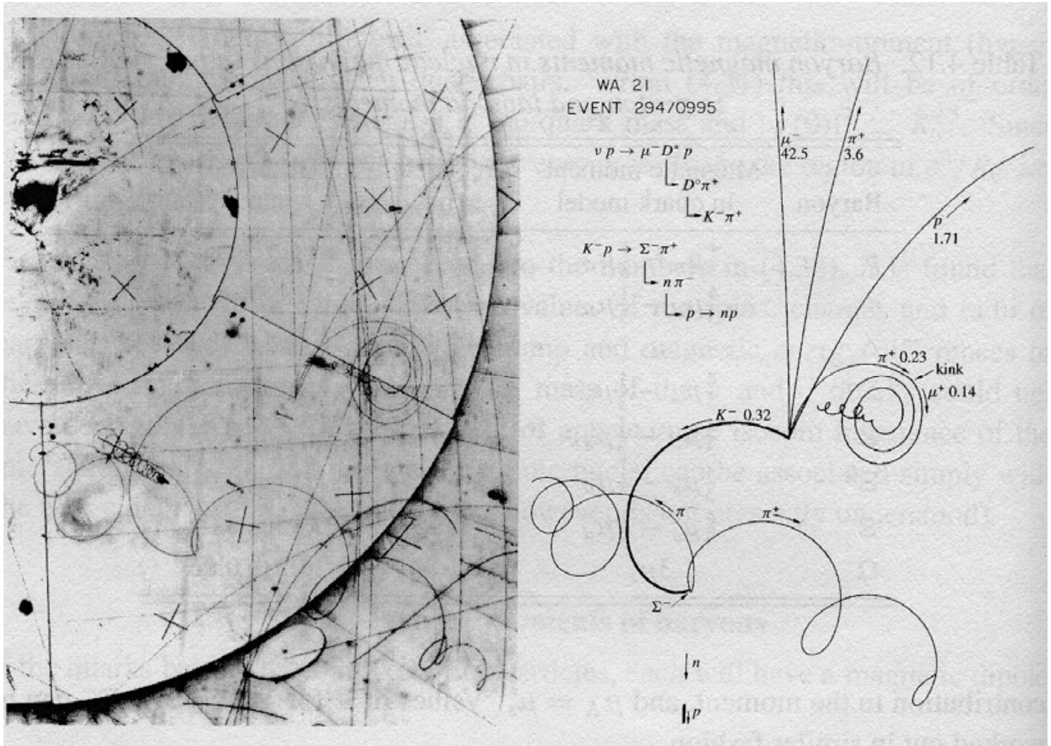
But, first, using the notation in the lecture about the shell model, show

$$g_j = g_l.\frac{j(j+1)+l(l+1)-s(s+1)}{2j(j+1)} + g_s.\frac{j(j+1)-l(l+1)+s(s+1)}{2j(j+1)},$$

if $\overline{\mu}_j = g_j \overline{J}$, $\overline{\mu}_s = g_s \overline{S}$ and $\overline{\mu}_l = g_l \overline{L}$. It's in the notes, so it's easy.

${}^{43}_{20}Ca\left(\frac{7^-}{2}, -1.32\mu_N\right)$, ${}^{93}_{41}Nb\left(\frac{9^+}{2}, 6.17\mu_N\right)$, ${}^{137}_{56}Ba\left(\frac{3^+}{2}, 0.931\mu_N\right)$, ${}^{197}_{79}Au\left(\frac{3^+}{2}, 0.145\mu_N\right)$,
${}^{26}_{13}Al\left(5^+,\ unknown\right)$

(3)



Heavy quark mass	Triplet–singlet difference
$M_c \simeq 1.86$ GeV	$M_{D^*} - M_D = 0.14$ GeV
$M_b \simeq 5.28$ GeV	$M_{B^*} - M_B = 0.046$ GeV

The above bubble chamber picture shows all aspects of a neutrino interaction. The neutrino interacts with a sea quark in a proton. The decay chains involved are beside the tracing of the event above. The different track momenta (in GeV/c) are also shown.

a) What do you think the flavor of the sea quark involved in this interaction is?

b) What is the lepton flavour of the incoming neutrino?

c) Write down what is the force producing each interaction and decay vertex in the picture.

d) Draw a quark line diagram for each interaction and decay in the picture.

e) There are two π^+ in this picture, only one of them comes from the decay of the D^0. From the properties of the D^0 and the momenta on the picture, calculate the angle between the K^- and each of the π^+, each time assuming that the K^- and the π^+ are the decay products of the D^0. Which π^+ is the correct one?

f) Why does the table below the bubble chamber picture allow you to identify the correct pion without any calculation?

(4) (a) An electron of energy 20 GeV is deflected through an angle of 5 degrees in an elastic collision with a stationary proton. What is the value of the 4-momentum transfer, q^2? If you didn't know the size of a proton (more accurately, the distribution of the proton's electric charge in space), what would be the limit that you could put on the size of a proton. *Neglect the mass of the electron compared to 20 GeV, and take the mass of the proton to be 0.938 GeV/c**2).*

(b) If the charge density distribution in a proton is $\rho(\bar{r}) = \rho_0 \exp(-mr)$ and is normalized to unity, show that the form factor is

$$F(q) = \frac{m^4}{\left(m^2 + q^2\right)^2}$$

(5) The simplest model of the distribution of electric charge in a nucleus is that the charge density as a function of radius is given by

$$\rho(r) = \rho_0 \quad ; \quad r \le a$$

$$\rho(r) = 0 \quad ; \quad r > a$$

where $r = |\boldsymbol{r}|$

(a) Sketch the form of this, as a function of *r*.

(b) Show that the form factor corresponding to this spatial distribution is

$$F(q^2) = \frac{3\left\{\sin\left(qa/\hbar\right) - \left(qa/\hbar\right)\cos\left(qa/\hbar\right)\right\}}{\left(qa/\hbar\right)^3}$$

You should start by writing down the general expression for the form factor. Then use the fact that, in a situation of spherical symmetry,

$$\iiint x(\bar{r}) \exp\left(i\bar{p}\cdot\bar{r}/n\right) r^2 \sin\theta \, d\theta \, d\phi \, dr = \frac{4\pi n}{p} \int x(r) r \sin\left(\frac{pr}{n}\right) dr$$

You have to *normalize* the form factor. This means dividing it by the integral of the charge density over the total volume of the nucleus. This is trivial since the nucleus is a sphere.

(c) Determine the Root Mean Square charge radius, in terms of *a*, for this charge density. *Again remember to normalize the result by dividing through by the integral over the charge density over the volume of the nucleus.*

(d) Now let's look at what the form factor does in numerical terms. An electron of momentum 500 MeV/c is scattered through an angle of 11° by an argon nucleus. Assume that there is no recoil and calculate the *momentum transfer* and also the *reduced de Broglie wavelength* of the electron. Further, calculate the *Mott differential cross section*, which corresponds to the case where the argon nucleus could be considered to be point-like. Finally, calculate by how much the *differential cross section* changes if the argon nucleus is considered to be represented by the spatial distribution in part **(a)** of this question. You should take the value of $r = 1.2 \times A^{1/3}\ fm$. *Argon has an Atomic Mass Number of 40, and an Atomic Number of 18. If you need more background on this question, you can find it in the book "Nuclear & Particle Physics" by W.S.C. Williams around page 44. You can find it in the library. However, I think I have given everything you need.*

(6) (a) Calculate the binding energy B, the binding energy per nucleon B/A and the neutron and proton separation energies *(these are defined on page 51 of the text book)* for ^{114}Cd given that the mass excesses *(the difference between the mass in atomic mass units and the mass number A)* are $p = 7.29\,MeV$, $n = 8.07\,MeV$, $^{114}Cd = -90.01\,MeV$, $^{113}Cd = -89.04\,MeV$, $^{113}Ag = -87.04\,MeV$.

(b) Compare your results from **(a)** to the semi-empirical mass formula, using the constants in the table at the end.

(6) (a) Consider nuclei with $Z = N = A/2$, neglect the pairing term, and show that the semi-empirical mass formula gives the binding energy per nucleon as

$$B/A = a - bA^{-\frac{1}{3}} - (d/4)A^{\frac{2}{3}}$$

Show that this reaches a maximum for iron at $Z = A/2 = 26$. So, iron is the most stable nucleus.

(b) In my rapid discussion of β-decay *(the overheads are on the web, and it is well-covered in the text book)* I showed that the bottom of the valley of stability is given by $Z = \beta/2\gamma$, where β and γ are combinations of the coefficients in the semi-empirical mass formula. Use this to calculate Z for $A=100$ and $A=200$. Compare your results to the attached plot of the valley of stability. Any comments about the result?

(7) (a) I'm afraid I was unconvincing when I discussed the Coulomb term in the semi-empirical mass formula. So, we'd better check it. Actually, this is a simple problem in electrostatics. Show that the potential energy due to electrostatic forces of a uniformly charged sphere of total charge Q and radius R is

$$\frac{3Q^2}{20\pi\varepsilon_0 R}$$

Consider the energy of a thin spherical shell of charge (which has a radius r and a charge density $\rho = 3Ze/4\pi R^3$ *) due to the charge enclosed in the volume* $4\pi r^3/3$.

The Coulomb term in the semi-empirical (liquid drop) mass formula is

$$\frac{a_c Z^2}{A^{1/3}}.$$

(b) Using the result of part **(a)** calculate the value of a_c in MeV/c^2. Assume as usual that the nuclear radius is given by $R = 1.24 \times A^{1/3}\ fm$, and use the fact that

$$\frac{e^2}{4\pi\varepsilon_0} = \frac{197.3}{137.04} MeV \bullet fm$$

Using the values of a_V, a_S and a_A given in the Table 4.1 below, and the fact that the binding energy of ${}^{181}_{73}Ta$ is 1454 MeV, check your value for a_c.

Now, interestingly enough, the nucleus ${}^{235}_{92}U$ can undergo spontaneous fission, one of the many fission channels is

$${}^{235}_{92}U \rightarrow {}^{87}_{36}Br + {}^{145}_{57}La + 3\,neutrons$$

Estimate the energy released in this process.

4.6 The β-stability valley. Filled squares denote the stable nuclei and long-lived nuclei occurring in nature. Neighbouring nuclei are unstable. Those for which data on masses and mean lives are known fill the area bounded by the lines. For the most part these unstable nuclei have been made artificially. (Data taken from *Chart of the Nuclides* (1977), Schenectady: General Electric Company.)

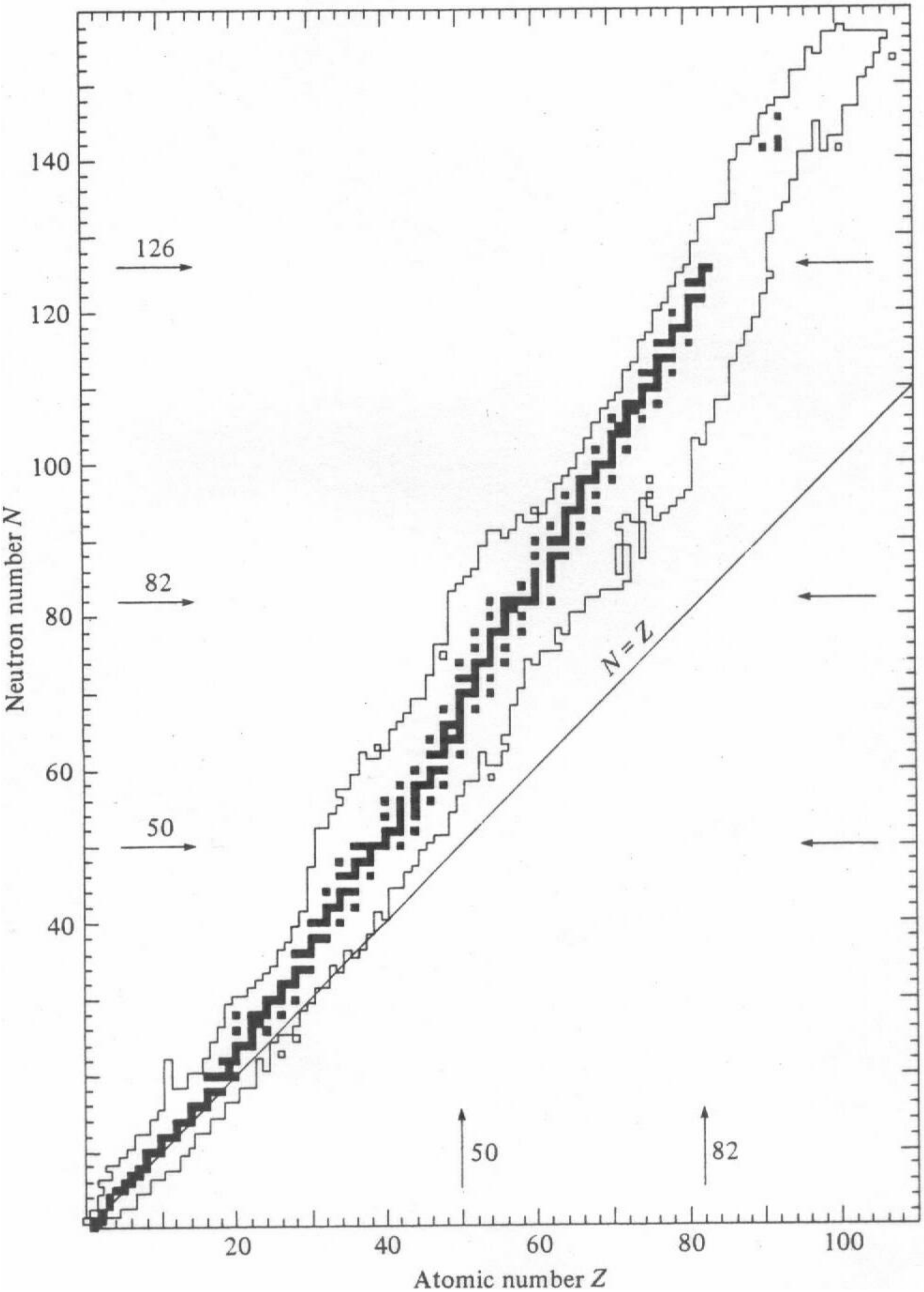


Table 4.1 The nuclear semi-empirical mass formula summarized.

$M(Z,A)c^2$	Nuclear rest mass energy
$= ZM_pc^2 + (A-Z)M_nc^2 - B(Z,A)$	Rest of mass of constituents less the binding energy

where

$B(Z,A) =$	$+a_V A$	Volume binding term
	$-a_S A^{2/3}$	Surface energy term
	$-a_C Z^2/A^{1/3}$	Coulomb term
	$-a_A (A-2Z)^2/A$	Asymmetry term
	$\left\{\begin{array}{ll} -a_P/A^{1/2} & \text{oo nuclei} \\ +0 & \text{eo and oe nuclei} \\ +a_P/A^{1/2} & \text{ee nuclei} \end{array}\right\}$	Pairing term

M_pc^2 = rest mass energy of the proton = 938.280 MeV.
M_nc^2 = rest mass energy of the neutron = 939.573 MeV.
A favoured set of values for the coefficients:

$$a_V = 15.56 \text{ MeV},$$
$$a_S = 17.23 \text{ MeV},$$
$$a_C = 0.697 \text{ MeV},$$
$$a_A = 23.285 \text{ MeV},$$
$$a_P = 12.0 \text{ MeV}.$$

To obtain the atomic rest mass energy, change M_P, the proton mass, to M_H, the mass of hydrogen atom, thus:

$\mathcal{M}(Z,A)c^2$ = atomic rest mass energy

$$\approx ZM_Hc^2 + (A-Z)M_nc^2 - B(Z,A).$$

(Note: $\approx$ because this formula neglects some atomic electron binding energy.)
M_Hc^2 = rest mass energy of the hydrogen atom = 938.791 MeV.

Figure 4.5 shows how the various contributions (except the pairing change with A throughout the periodic table. What is surprising is th formula is good from $A \simeq 20$ to the end of the periodic table with a pre better than $1\frac{1}{2}$% on the binding energy. This is shown in the case of the nuclei in Fig. 4.6.

Possibly Useful Physical Constants:

Avogadro No:	$6\times10^{23}\,mole^{-1}$
pi	$\pi = 3.1416$
speed of light:	$c = 3.0\times10^{8}\ m/s$
Plank's constant:	$\hbar = 6.6\times10^{-22}\,MeV\cdot s$
	$\hbar c = 197 MeV.fm$
	$(\hbar c)^2 = 0.4\,GeV^2\cdot mb$
	$1\ eV = 1.6\times10^{-19}\,Joules$
	$1\ eV/c^2 = 1.8\times10^{-36}\,kg$
	$1\ fm = 10^{-15}\,m$
	$1\ mb = 10^{-27}\,cm^2$
1 year	1 year $\approx \pi\times10^{7}s$
electron charge:	$e = 1.602\times10^{-19}\,C$
electron magnetic moment:	$\mu_e = 9.3\times10^{-24}\,Joules\cdot Tesla^{-1}$
fine structure constant:	$\alpha = e^2/(\hbar c) = 1/137.0360$
strong coupling constant:	$\alpha_s(M_Z) = 0.116\pm0.005$
Fermi coupling constant:	$G_F = 1.166\times10^{-5}\ GeV^{-2}$
Cabibbo angle:	$\sin\theta_C = 0.22$
Weak mixing angle:	$\sin^2\theta_W(M_Z) = 0.2319\pm0.0005$
Branching Ratios	$BR(Z\to e^+e^-) = 3.21\pm0.07\%$
	$BR(Z\to hadrons) = 71\pm1\%$

Particle Properties

Boson	Mass $\left(GeV/c^2\right)$
γ	$< 3\times10^{-36}$
$gluon$	~ 0
$W^{\pm}$	80.22
Z^0	91.187
H^0	> 116

Lepton	Mass $\left(MeV/c^2\right)$
ν_e	$< 10^{-5}$
e	0.510999
ν_μ	< 0.27
μ	105.658
ν_τ	< 10
τ	1777

Hadron	Quark Content	Mass $\left(MeV/c^2\right)$	$\mathbf{I}\left(\mathbf{J^{PC}}\right)$
π^+, π^0, π^-	$u\bar{d}, \left(u\bar{u} - d\bar{d}\right)/\sqrt{2}, d\bar{u}$	139.57,134.97, 139.57	$1\left(0^{-+}\right)$
K^+, K^-	$u\bar{s}, s\bar{u}$	493.65	$\frac{1}{2}\left(0^-\right)$
$K^0, \bar{K}^0$	$d\bar{s}, s\bar{d}$	497.67	$\frac{1}{2}\left(0^-\right)$
ρ^+, ρ^0, ρ^-	$u\bar{d}, \left(u\bar{u} + d\bar{d}\right)/\sqrt{2}, \bar{u}d$	775.7	$1\left(1^{--}\right)$
p, n	uud, udd	938.27, 939.57	$\frac{1}{2}\left(\frac{1}{2}^+\right)$
$\Delta^-, \Delta^0, \Delta^+, \Delta^{++}$	ddd, udd, uud, uuu	1232	$\frac{3}{2}\left(\frac{3}{2}^+\right)$
Λ^0	uds	1115.6	$0\left(\frac{1}{2}^+\right)$
$\bar{D}^0, D^0$	$u\bar{c}, c\bar{u}$	1863	$\frac{1}{2}\left(0^-\right)$
D^-, D^+	$d\bar{c}, c\bar{d}$	1869	$\frac{1}{2}\left(0^-\right)$
D_S^+, D_S^-	$c\bar{s}, \bar{c}s$	1968	$0\left(0^-\right)$
B^+, B^-	$u\bar{b}, \bar{u}b$	5279	$\frac{1}{2}\left(0^-\right)$
Λ_c^+	udc	2285	$0\left(\frac{1}{2}^+\right)$
$\Sigma^+, \Sigma^0, \Sigma^-$	uus, uds, dds	1189	$1\left(\frac{1}{2}^+\right)$
Ξ^0, Ξ^-	uss, dss	1315	$\frac{1}{2}\left(\frac{1}{2}^+\right)$
Ω^-	sss	1672	$0\left(\frac{3}{2}^+\right)$
Λ_b	udb	5624	$0\left(\frac{1}{2}^+\right)$