# PHYSICS 357S - Problem Set #5 - March 2016

Distributed **22$^{nd}$ March**. Due to be handed in by **8$^{th}$ April.**
***8$^{th}$ April is the last day of classes. You can hand this in at the lecture on 6$^{th}$ April, or at my office on the 8$^{th}$.***
*This problem set counts for 10% of the grade. It has 6 questions and 7 pages. If you don't understand a question ask me about it. If you think there is a bug (error, typo, etc) in a question..tell me. You might be right!*
***Please check the Web page and your email for any important messages as we approach the end of term. If you have any question in the period leading up to the exam, please feel free to just pass by my office and talk to me. Any day of the week is fine. It's usually best to email or phone in advance.***

**(1)** The helicity of a particle with spin $\sigma$ is defined by $\lambda = \sigma.\hat{p}$ where $\hat{p}$ is a unit vector in the direction of the particle's momentum. By considering how the helicity expectation value behaves under the parity transformation show that the expectation value $\langle \lambda \rangle$ must be zero in a parity conserving interaction.

*Hint*: $\langle \lambda \rangle = \int \psi^*(r)(\sigma.\hat{p})\psi(r)dr$

**(2)** The charmed quark was not discovered until the early 1970's. It was discovered by looking at $e^+e^-$ annihilation to hadrons. When the machine energy is such that a pair of $c\bar{c}$ quarks can be produced, one sees a dramatic increase in the cross section due to the production of the $\psi$ particle, which is a $c\bar{c}$ bound state. The $\psi$ is a very massive particle, its mass is $3097\ MeV/c^2$. However, the mass of the charm quark is also large, $1650\ MeV/c^2$, so the system is non relativistic. Also, because of the mass of the $\psi$ is large, the strong coupling constant $\alpha_S$ is small. So the $\psi$ made of a $c\bar{c}$ pair behaves very much like *Positronium*, which is a bound state of an electron and a positron.

QED predicts that the decay rate for Positronium into three photons is given by

$$\omega = \frac{2\alpha^6(\pi^2 - 9)}{9\pi}\frac{m_e c^2}{\hbar}$$

The $\psi$ decays to three gluons, which materialize as hadrons. We can assume that this process can be treated in the same fashion as positronium. Use this assumption to estimate the value of the strong coupling constant $\alpha_S$. It will help to draw a "Feynman" diagram first. It looks a bit like the diagram I drew for $\pi^0 \to \gamma\gamma$, except there are three photons in the case of positronium, and three gluons in the case of the $\psi$. The full width of the $\psi$ is $63\ keV$, and 82% of all the decays go to hadrons.

The $\psi$ can also decay via the process $\psi \to \gamma + gluon + gluon$. In this case the two gluons will materialize as hadrons, while the photon will appear as a real photon. This is known as a *radiative decay*. In this case one of the *cc-gluon* couplings in the three gluon decay will become a *cc-photon* coupling. Modify the formula for the decay rate above and estimate the rate for this radiative decay of the $\psi$. Then estimate the *branching fraction* for this radiative decay mode.

**(3)** *The largest linear accelerator in the world was the SLAC (Stanford Linear Accelerator Center) electron LINAC. The accelerator produced electrons of* $40\,GeV/c$ *momentum, at the time of the discovery of quarks in deep inelastic scattering. It has now been converted into a coherent X-ray source; http://lcls.slac.stanford.edu/*

**a)** Show that if energy $\nu$ *(this is supposed to be the letter "nu")* ($\ll m_e c^2$) and 3-momentum $\vec{q}$ are transferred to a free stationary electron, then $\vec{q} \bullet \vec{q} = 2m_e \nu$. If $\nu > m_e c^2$ the problem is relativistic: in this case $q$ is the 4-momentum transfer. Show that $q^2 = -2m_e \nu$ .

**b)** For quasi-elastic scattering of a nucleon ( mass $m_N$ ) from a nucleus, show that the energy transfer to the nucleon is given by:

$$\nu = \frac{-q^2}{2m_N} .$$

How do you expect the energy transfer distribution to differ in **a)** (relativistic case) and **b)?**

**c)** In the quark-parton model the proton is modeled as a beam of non-interacting point particles. Show that in the high $q^2$ regime, the fractional momentum carried by a quark is:

$$x = \frac{-q^2}{2m_P \nu} .$$

Sketch how you might expect this *x*-distribution to look, and explain the shape of it in qualitative terms.

**d)** Show that the mass of the hadronic system produced in deep inelastic scattering is

$$W^2 = m_P{}^2 + q^2 + 2m_P \nu .$$

What is the maximum $W^2$ that the SLAC LINAC could produce? What is the minimum? Remember that $0 \le x \le 1$.

**e)** $q^2$ is a Lorentz scalar. What about $\nu$? A useful Lorentz scalar in deep inelastic scattering is $(p.q)/(p.k)$. Show that

$$y = \frac{\nu}{E_e} .$$

*p*,*k*,*q* are the 4-momentum of the proton, the incoming electron, and the 4-momentum transfer, $E_e$ is the energy of the incoming electron in the LAB frame. What is the range of *y*? It is then easy to see that $-q^2 = 2m_P E_e xy$ . What is the maximum value of $-q^2$, and what can you say about the struck parton, and the 4-momentum transfer to it.

**(4) (a)** The $\Delta^{++}$ has $J^P = \frac{3}{2}^+$. It decays to $p$ and a $\pi^+$ via the strong force. If the $p$ has $J^P = \frac{1}{2}^+$, and the spin *0* $\pi^+$ is found to be in an *l=1* orbital state . What does that tell you about the parity of the pion?

**(b)** The major decay mode of the $\Xi^0$ is $\Xi^0 \to \Lambda\pi^0$. The $\Lambda$ has the same spin and parity as the proton. Do you think this decay mode can be used to determine the spin and parity of the $\pi^0$? Why? Do you think that the $\Xi^0$ as a definite spin and parity? Why?

**(c)** Show that the operator product $\overline{J}.\overline{p}$, is time reversal invariant. $\overline{J}$ is the angular momentum and $\overline{p}$ is the momentum of a particle. *You can do this by considering the diagram below, which is Figure 9.8 on page 259 of the text book by Henley & Garcia.*

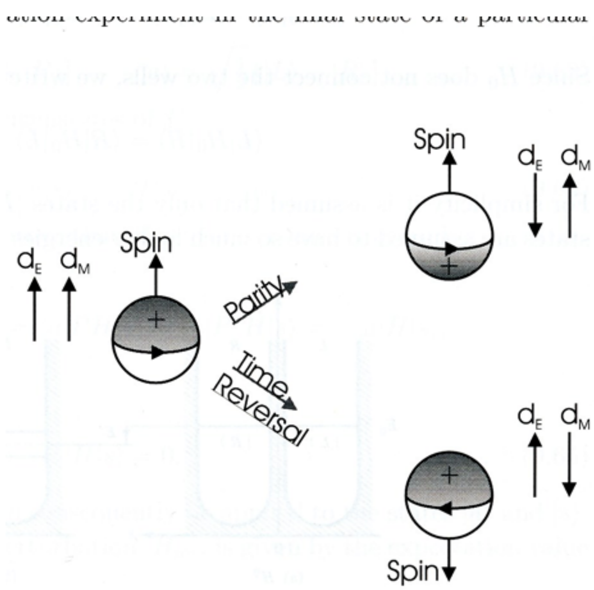


Figure 9.8: A spinning positively charged particle is represented here as a spherical object. Its mirror (located in the horizontal midplane) image and its time reversed image are shown. The magnetic $d_M$ and electric $d_E$ dipole moments are also shown.

**(5) (a)** Which of the following decays are forbidden by charge conjugation invariance?

$$\omega^0 \to \pi^0\gamma$$
$$\eta' \to \rho^0\gamma$$
$$\pi^0 \to \gamma\gamma\gamma$$
$$J/\psi \to p\overline{p}$$
$$\rho^0 \to \gamma\gamma$$

Note that the $\pi^0, \eta'$ have positive charge conjugation, while the $\omega, \rho, J/\psi$ are all negative.

**(b)** A positive pion and a negative pion have orbital angular momentum *l* in their CM frame.

**(i)** Determine the *C* parity of this $\left(\pi^+\pi^-\right)$ system.

**(ii)** If *l=1*, can this system decay into two photons? Why?

**(c)** What is the charge conjugate reaction corresponding to $K^- + p \rightarrow \overline{K}^0 + n$? Can a $K^- p$ system be an eigenstate of the charge conjugation operator?. Give a similar discussion of the reaction $\overline{p}p \rightarrow \pi^+\pi^-$.

**(6) (a)** Describe, using a sketch, Mme Wu's experiment to determine whether parity is conserved in weak interactions.

**(b)** Question **(4)** ask how $\vec{J} \cdot \vec{p}$ behaves under time reversal? Does Wu's experiment tell you anything about time reversal invariance?

**(c)** The electric dipole moment of a particle is $\vec{\sigma} \cdot \vec{E}$, where $\vec{\sigma}$ is the spin and $\vec{E}$ is an external electric field. $\vec{\sigma} \cdot \vec{E}$ changes sign under both the *T* and *P* operations. What generic kind of interaction would you study to search for *T* –invariance violations, as distinct from *P*? *Think about parity invariance of the three forces.*

***That's All, Folks!***

**Possibly Useful Physical Constants:**

| | |
|---|---|
| Avogadro No: | $6\times10^{23}\,mole^{-1}$ |
| pi | $\pi = 3.1416$ |
| speed of light: | $c = 3.0\times10^{8}\,m/s$ |
| Plank's constant: | $\hbar = 6.6\times10^{-22}\,MeV\cdot s$ |
| | $\hbar c = 197\,MeV.fm$ |
| | $(\hbar c)^2 = 0.4\,GeV^2\cdot mb$ |
| | $1\,eV = 1.6\times10^{-19}\,Joules$ |
| | $1\,eV/c^2 = 1.8\times10^{-36}\,kg$ |
| | $1\,fm = 10^{-15}\,m$ |
| | $1\,mb = 10^{-27}\,cm^2$ |
| 1 year | 1 year $\approx \pi\times10^{7}\,s$ |
| electron charge: | $e = 1.602\times10^{-19}\,C$ |
| electron magnetic moment: | $\mu_e = 9.3\times10^{-24}\,Joules\cdot Tesla^{-1}$ |
| fine structure constant: | $\alpha = e^2/(\hbar c) = 1/137.0360$ |
| strong coupling constant: | $\alpha_s(M_Z) = 0.116\pm0.005$ |
| Fermi coupling constant: | $G_F = 1.166\times10^{-5}\,GeV^{-2}$ |
| Cabibbo angle: | $\sin\theta_C = 0.22$ |
| Weak mixing angle: | $\sin^2\theta_W(M_Z) = 0.2319\pm0.0005$ |
| Branching Ratios | $BR(Z\to e^+e^-) = 3.21\pm0.07\%$ |
| | $BR(Z\to hadrons) = 71\pm1\%$ |

# Particle Properties

| **Boson** | **Mass** $\left(GeV/c^2\right)$ |
|---|---|
| $\gamma$ | $< 3\times10^{-36}$ |
| $gluon$ | $\sim 0$ |
| $W^{\pm}$ | 80.22 |
| $Z^0$ | 91.187 |
| $H^0$ | $> 116$ |

| **Lepton** | **Mass** $\left(MeV/c^2\right)$ |
|---|---|
| $\nu_e$ | $< 10^{-5}$ |
| $e$ | 0.510999 |
| $\nu_\mu$ | $< 0.27$ |
| $\mu$ | 105.658 |
| $\nu_\tau$ | $< 10$ |
| $\tau$ | 1777 |

| **Hadron** | **Quark Content** | **Mass** $\left(MeV/c^2\right)$ | $\mathbf{I}\left(\mathbf{J}^{\mathbf{PC}}\right)$ |
|---|---|---|---|
| $\pi^+,\pi^0,\pi^-$ | $u\bar{d},\left(u\bar{u}-d\bar{d}\right)/\sqrt{2},d\bar{u}$ | 139.57,134.97, 139.57 | $1\left(0^{-+}\right)$ |
| $K^+,K^-$ | $u\bar{s},s\bar{u}$ | 493.65 | $\frac{1}{2}\left(0^-\right)$ |
| $K^0,\bar{K}^0$ | $d\bar{s},s\bar{d}$ | 497.67 | $\frac{1}{2}\left(0^-\right)$ |
| $\rho^+,\rho^0,\rho^-$ | $u\bar{d},\left(u\bar{u}+d\bar{d}\right)/\sqrt{2},\bar{u}d$ | 775.7 | $1\left(1^{--}\right)$ |
| $p,n$ | $uud,udd$ | 938.27, 939.57 | $\frac{1}{2}\left(\frac{1}{2}^+\right)$ |
| $\Delta^-,\Delta^0,\Delta^+,\Delta^{++}$ | $ddd,udd,uud,uuu$ | 1232 | $\frac{3}{2}\left(\frac{3}{2}^+\right)$ |
| $\Lambda^0$ | $uds$ | 1115.6 | $0\left(\frac{1}{2}^+\right)$ |
| $\bar{D}^0,D^0$ | $u\bar{c},c\bar{u}$ | 1863 | $\frac{1}{2}\left(0^-\right)$ |
| $D^-,D^+$ | $d\bar{c},c\bar{d}$ | 1869 | $\frac{1}{2}\left(0^-\right)$ |
| $D_S^+,D_S^-$ | $c\bar{s},\bar{c}s$ | 1968 | $0\left(0^-\right)$ |
| $B^+,B^-$ | $u\bar{b},\bar{u}b$ | 5279 | $\frac{1}{2}\left(0^-\right)$ |
| $\Lambda_c^+$ | $udc$ | 2285 | $0\left(\frac{1}{2}^+\right)$ |
| $\Sigma^+,\Sigma^0,\Sigma^-$ | $uus,uds,dds$ | 1189 | $1\left(\frac{1}{2}^+\right)$ |
| $\Xi^0,\Xi^-$ | $uss,dss$ | 1315 | $\frac{1}{2}\left(\frac{1}{2}^+\right)$ |
| $\Omega^-$ | $sss$ | 1672 | $0\left(\frac{3}{2}^+\right)$ |
| $\Lambda_b$ | $udb$ | 5624 | $0\left(\frac{1}{2}^+\right)$ |