PHY 357 Problem Set #3 March 2016

Question #1

a) Mesons with $L=0$

Mesons are made of 2 quarks, each with spin $\pm\frac{1}{2}$. The possible spins are therefore $\frac{1}{2}+\frac{1}{2}=1$ (aligned) or $\frac{1}{2}-\frac{1}{2}=0$ (anti-aligned).

$\therefore$ for $S=1$, $J=L\oplus S=0\oplus 1=1$

$S=0$, $J=L\oplus S=0\oplus 0=0$

b) Mesons with $L=1$

As in a), the possible intrinsic spins are $\{1,0\}$

$\therefore$ for $S=1$, $J=L\oplus S=1\oplus 1=0,1,2$

$S=0$, $J=L\oplus S=1\oplus 0=1$ (note: j runs from $|l+s|$ to $|l-s|$)

c) Meson with total spin $5/2$.

The relative orbital momentum cannot be 2 since the intrinsic spin S of a meson is always a whole number (0 or 1). If L is also a whole number there is no way to combine orbital angular momentum and spin in order to get a fractional total spin.

d) Two particles in state $|s,m_s\rangle=|1,0\rangle$ combine to form bound state w/ $L=0$

Adding the spins $S_1\oplus S_2=1\oplus 1=0,1,2$

Since $m_{s1}=0$ and $m_{s2}=0$, $m_{sT}=0$. Since m_s runs from $-s$ to s, this works for all possible spin combinations.

For $L=0$, $l_T=0$ and $m_{lT}=0$, so the possibilities are:

- $|2,0\rangle$ where $j_T=2$, $m_T=0$ $\therefore P=2/3$
- $|1,0\rangle$ where $j_T=1$, $m_T=0$ $\therefore P=0$
- $|0,0\rangle$ where $j_T=1$, $m_T=0$ $\therefore P=1/3$

As expected, the total probability is $2/3+0+1/3=1$

Question #1 continued ...

e) Intrinsic spin $3/2$ in state $|3/2, 1/2\rangle$

We assume this particle decays into two particles which have a relative orbital angular momentum of 0 (ie// s of particle is $3/2$, in state with $j = 3/2$ $\therefore$ $\ell = 0$).

So before the decay we have: $j_T = 3/2$ $\quad m_{jT} = 1/2$

$\ell_T = 0$ $\quad m_{\ell T} = 0$

$s_T = 1/2$ $\quad m_{sT} = 1/2$

For the lowest possible states, $j_1 + j_2 = 1 + 1/2 = 3/2$ and $\ell_1 = \ell_2 = 0$

Then for particle #1 we have $j_1 = 1$ $\quad m_{j_1} = -1, 0, 1$

$\ell_1 = 0$ $\quad m_{\ell_1} = 0$

$s_1 = 1$ $\quad m_{s_1} = -1, 0, 1$

And for particle #2 we have $j_2 = 1/2$ $\quad m_{j_2} = -1/2, 1/2$

$\ell_2 = 0$ $\quad m_{\ell_2} = 0$

$s_1 = 1/2$ $\quad m_{s_2} = -1/2, 1/2$

The possibilities are $|1, 0\rangle$ and $|1/2, 1/2\rangle$

$|1, 1\rangle$ and $|1/2, -1/2\rangle$

Looking at the Clebsch-Gordon table, we have two particles, one with spin 1 and one with spin $1/2$. So look to the $1 \times 1/2$ table. Our total is $3/2, 1/2$, so look at that column. The probabilities are then:

- for $|1,1\rangle|1/2, -1/2\rangle$ $\quad P = 1/3$ $\quad \rightarrow$ the 1st # in $|a,b\rangle$ is from the "title" of
- for $|1,0\rangle|1/2, 1/2\rangle$ $\quad P = 2/3$ $\quad$ the table, $1 \times 1/2$, while the second # comes from the row.

$\therefore |3/2, 1/2\rangle = \sqrt{1/3}\, |1,1\rangle|1/2, -1/2\rangle + \sqrt{2/3}\, |1,0\rangle|1/2, 1/2\rangle.$

Question #2

a) Electron beam 10 GeV, 10^{-6} A, 0.5 cm^2

The flux can be found by using the current, 10^{-6}A, of the beam:

$\Rightarrow 10^{-6} A = 10^{-6}$ C/s where $1e = 1.6 \times 10^{-19}$C

$\therefore N_{electrons} = 10^{-6} C/s \,/\, 1.6 \times 10^{-19} C$

$= 6.2 \times 10^{12}$ electrons/s

The flux is $\therefore$ $\text{Flux} = \dfrac{\#\text{ of particles / unit time}}{\text{area}}$

$\Phi = (6.2 \times 10^{12} \text{ electrons/s}) / (0.5 \text{ cm}^2)$

$\cong 1.25 \times 10^{13}$ e/s-cm^2

b) Proton beam 10 GeV, 10^{13} protons

For fixed target, $A = 2\text{ cm}^2$

$t = 0.5$s

The flux is then : Flux = # of particles / time · area

$= 10^{13} \text{ protons} / (0.5\text{s})(2\text{cm}^2)$

$= 1 \times 10^{13}$ p/s·cm^2

For accelerator, $C = 7$ km (circumference)

The number of orbital periods corresponds to the # of times the protons travel around the circumference in 0.5s. Assume $v \sim c$.

Then 1 period is: $t_p = C/v = 7\text{km}/c \cong 2.33 \times 10^{-5}$ s

The total # of periods is $\therefore$ $t/t_p = 0.5\text{s} / 2.33 \times 10^{-5}\text{s}$

$\cong 2.14 \times 10^4$

Question #2 continued . . .

c) Copper target $t = 0.1\,cm$

Beam $A = 4\,cm^2$

N = number of scattering centres per unit volume.

Recall that $n = N_A \rho / A$

$= (6.02\times10^{23}\,mol^{-1})(3.96\,g/cm^3)/(63.546\,g/mol)$

$\cong 8.49\times10^{22}$ atoms/cm^3

The total number of scattering centres is then $N = nV$

$= (8.49\times10^{22}\,cm^{-3})(4\,cm^2)(0.1\,cm)$

$\cong 3.4\times10^{22}$ atoms

Assuming a cross section $\sigma = 10\,mb$, fraction scattered $= \sigma n \Delta x$

$= (10\times10^{-27}\,cm^2)(8.49\times10^{22}\,cm^{-3})(0.1\,cm)$

$\cong 8.49\times10^{-5}$

$\therefore$ % scattered = 0.00849%

Question #3

a) Probability of neutrino crossing earth without interacting

$\sigma = 10^{-38}\ cm^2$
$d = 12756\ km$
$\rho = 5500\ kg/m^3$
$A = 40u$

From slide 8 of lecture 10, number of particles interacting is given by $(N_0 - N) = N_0(1 - e^{-n\sigma x})$

Then $n = N_A \rho / A$
$= (6.022 \times 10^{23}\ mol^{-1})(5500\ kg/m^3)/(40 g/mol)$
$= (6.022 \times 10^{23})(5.5\ g/cm^3)/(40g)$
$= 8.28 \times 10^{22}\ cm^{-3}$

The # of scatters $(N_0 - N)$ so the number of particles that are not scattered is simply $N = N_0 e^{-n\sigma x}$. The % is $\therefore$ $e^{-n\sigma x} = \exp(-8.28 \times 10^{22}\ cm^{-3} \cdot 10^{-38}\ cm^2 \cdot 1.2756 \times 10^9\ cm)$
$\cong 100\%$

b) Kamiokande detector 1000 tons of water

$A = 16\ g/mol$
$\rho = 1\ g/cm^3$

Probability of interaction $(N_0 - N) = N_0(1 - e^{-n\sigma x})$

For water, $n = N_A \rho / A$
$= (6.022 \times 10^{23}\ mol^{-1})(1\ g/cm^{-3})/(16\ g/mol)$
$= 3.764 \times 10^{22}\ cm^{-3}$

Then $P \sim (1 - \exp(-n\sigma x))$
$= (1 - \exp(-3.764 \times 10^{22}\ cm^{-3})(10^{-38}\ cm^2)(968 cm))$
$\cong 3.64 \times 10^{-13}$

note: 1000 tons = 907185 kg
$\Rightarrow V = 907185\ kg / 1\ g/cm^3$
$= 9.072 \times 10^8\ cm^3$
$\Rightarrow d = \sqrt[3]{9.072 \times 10^8\ cm^3}$
$= 968.06\ cm$

Question #3 continued . . .

c) 10 upward neutrinos per day

Flux can be estimated by: Flux = # of particles per m^2 per s

Captures/s = Flux × capture sites × cross section

$= \Phi N \sigma$

$\therefore \phi = (\text{captures}/s) / N\sigma$

$= (10\ \text{day}^{-1}) / (Vn)\sigma$

$= (10\ \text{day}^{-1}) / (9.072 \times 10^8\ cm^3)(3.764 \times 10^{22}\ cm^{-3})(10^{-38}\ cm^2)$

$= (1.16 \times 10^{-4}\ s^{-1}) / (3.41 \times 10^{-7}\ cm^2)$

$\approx 339.7\ cm^{-2}\ s^{-1}$

$\approx 2.93 \times 10^{11}\ m^{-2}\ \text{day}^{-1}$

note:

For this question, I accepted most answers that were within the correct order of magnitude as some students used approximations

Question #4

$^3_1T + ^2_1D \rightarrow ^4_2He + n$

$\sigma = 5.0b$

$\phi = 10^{15}\ cm^{-2}\ s^{-1}$

$\Delta t = 1y = 3.15 \times 10^7 s$

Then $\frac{N}{N_0} = (1 - e^{-\sigma\phi\Delta t}) = 1 - \exp[(-5\times10^{-28} m^2)(10^{15})(10^4 m^2 s^{-1})(3.15\times10^7 s)]$

$\cong 0.1457$

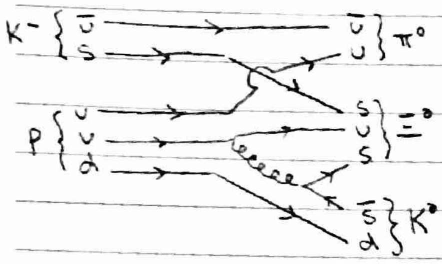
$= 14.6\%$

$\therefore$ the fraction of ^2_1D used up in 1 year is $N/N_0 \cong 14.6\%$

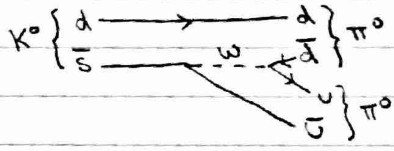
(note: can also use approx. $(1 - e^{-\sigma\phi\Delta t}) \sim \sigma\phi\Delta t$ which gives $\sim 15.75\%$)

Question #5

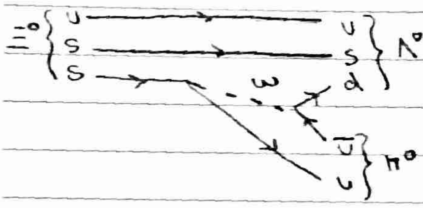
a) $K^- p \to \Xi^0 K^0 \pi^0$ (strong)



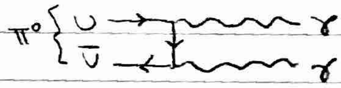
$K^0 \to \pi^0 \pi^0$ (weak)



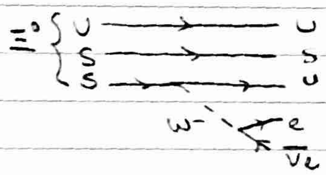
$\Xi^0 \to \Lambda \pi^0$ (weak)



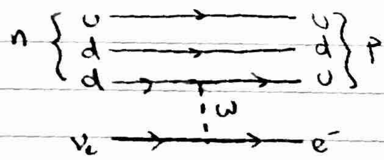
$\pi^0 \to \gamma\gamma$ (electromagnetic)



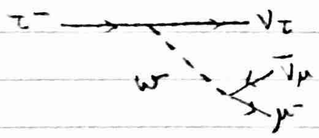
b) $\Xi^0 \to \Sigma^+ e \bar{\nu}_e$



$\nu_e n \to e p$



$\tau^- \to \nu_\tau \bar{\nu}_\mu \mu^-$

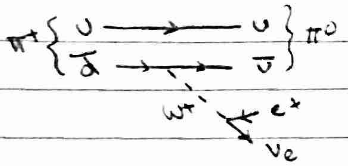


Question #6

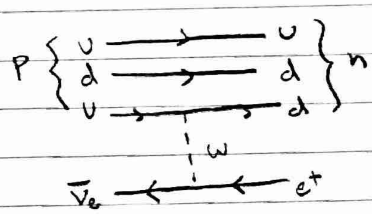
a) Δ^{++} spin $3/2$

Δ^{++} has quark content of uuu – three up quarks, each with spin $1/2$, same charge, and same flavour. But, if we remember the Pauli exclusion principle, having all three quarks aligned to give spin $3/2$ state seems to violate this. This issue is solved if we introduce a new quantum number, known as colour charge.

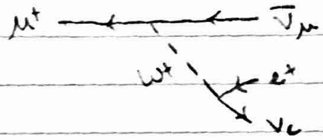
b) $\pi^+ \to \pi^0 e^+ \nu_e$



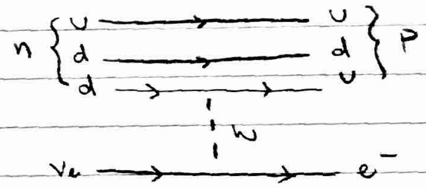
$\bar{\nu}_e p \to n e^+$



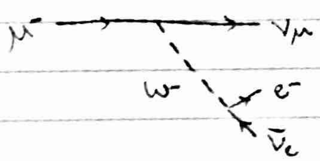
$\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$



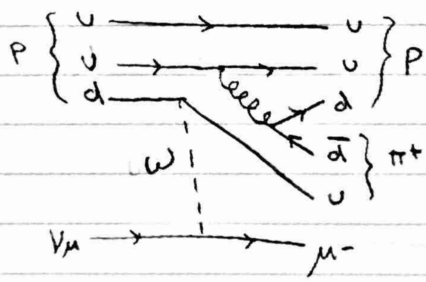
$\nu_e\, {}^{37}_{17}Cl \to {}^{37}_{18}Ar\, e^-$



$\mu^- \to e^- \bar{\nu}_e \nu_\mu$

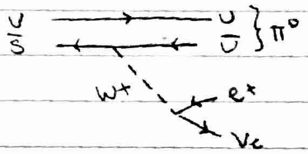


$\nu_\mu p \to \mu^- p \pi^+$

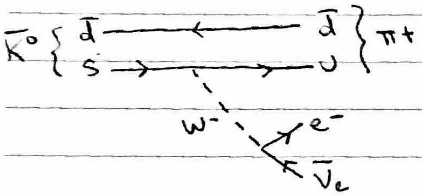


Question #6 continued ...

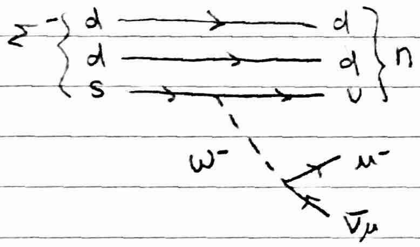
$K^+ \longrightarrow \pi^0 e^+ \nu_e$



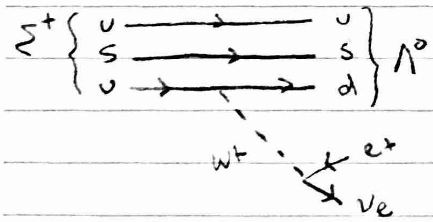
$\bar{K}^0 \longrightarrow \pi^+ e^- \bar{\nu}_e$



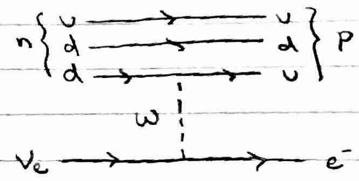
$\Sigma^- \longrightarrow n \mu^- \bar{\nu}_\mu$



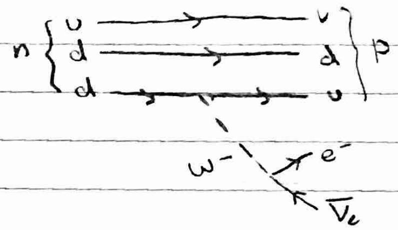
$\Sigma^+ \longrightarrow \Lambda^0 e^+ \nu_e$



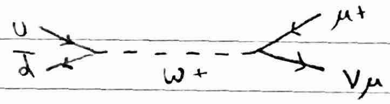
$\nu_e n \longrightarrow e^- p$



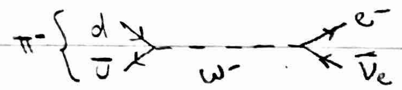
${}^3_1He \longrightarrow {}^3_2He \; e^- \bar{\nu}_e$



$\pi^+ \longrightarrow \mu^+ \nu_\mu$

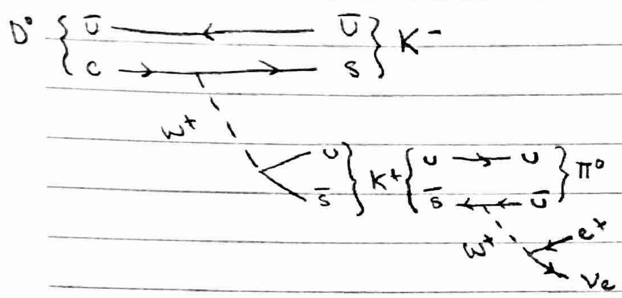


$\pi^- \longrightarrow e^- \bar{\nu}_e$

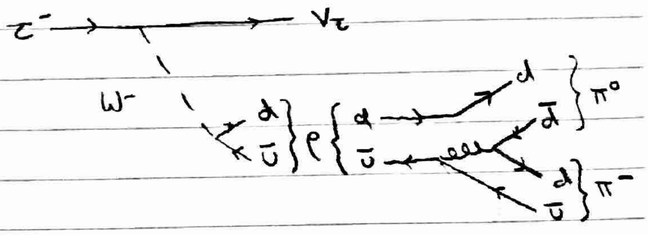


Question #6 continued . . .

$D^0 \rightarrow K^- \pi^0 e^+ \nu_e$



$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$



Question #7

a) Luminosity of Large Hadron Collider

$$\mathcal{L} = \frac{N_1 N_2}{4\pi\sigma_x\sigma_y} f n_b$$

$$= (1.5\times10^{11})^2 (11\ \text{kHz})(2808) / 4\pi(16\ \mu m)^2$$

$$\cong 2.16\times10^{38}\ m^{-2}s^{-1}$$

b) Cross section of $10\ pb = 10(10^{-40}\ m^2) = 1\times10^{-39}\ m^2$

Total # of scattering events/s $= \mathcal{L}\sigma = (2.16\times10^{38}\ m^{-2}s^{-1})(10^{-39}\ m^2)$

$= 0.216\ s^{-1}$

c) The average flux of protons

# of particles / second · area $= N f n_b / 4\pi\sigma_x\sigma_y \cong 1.44\times10^{27}\ m^{-2}s^{-1}$

d) Stationary LH target $\rho = 0.1\ g/cm^3$

$\ell = 2\ m$

# of scattering events/s $= \mathcal{L}\sigma = N f \sigma n_t$ where $n_t =$ # atoms/cm² for target

$= \rho d N_A / A$

$= (0.1\ g/cm^3)(2m) N_A / (1.00794u)$

$= 1.19\times10^{29}\ m^{-2}$

$\therefore\ \mathcal{L}\sigma = (1.5\times10^{11})(11\ \text{kHz})(10^{-39}\ m^2)(1.19\times10^{29}\ m^{-2})$

$= 196350\ s^{-1}$

$= 1.96\times10^5\ s^{-1}$