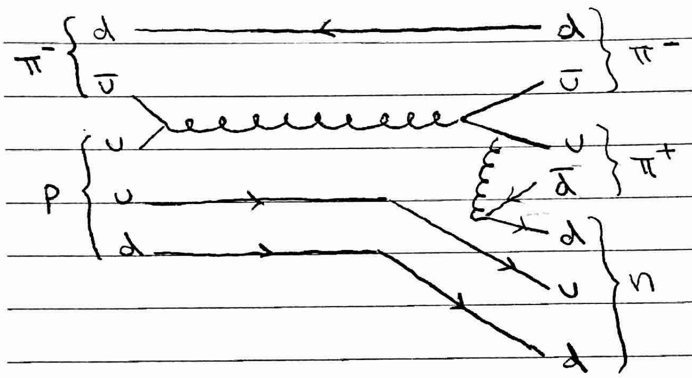
PHY 357 Problem Set #4

Question #1:

$\pi^- + p \rightarrow \pi^- + \pi^+ + n \Rightarrow (d\bar{u}) + (uud) \rightarrow (d\bar{u}) + (u\bar{d}) + (udd)$

$\Rightarrow$ Strong Interaction (all quarks, quark flavour conserved)

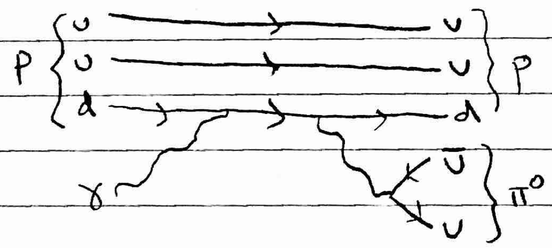
One possible Feynman diagram is:



$\gamma + p \rightarrow \pi^0 + p \Rightarrow \gamma + (uud) \rightarrow (u\bar{u}/d\bar{d}) + (uud)$

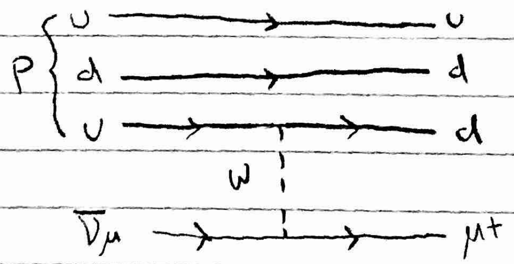
$\Rightarrow$ Electromagnetic interaction (γ is involved)

One possible Feynman diagram is:



$\bar{\nu}_\mu + p \rightarrow \mu^+ + n \Rightarrow \bar{\nu}_\mu + (uud) \rightarrow \mu^+ + (udd)$

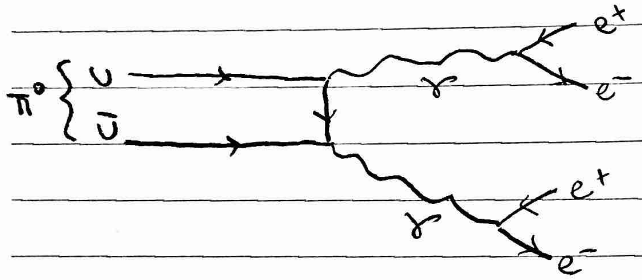
$\Rightarrow$ Weak interaction (neutrino is involved)



Question #1 continued...

$\pi^0 \rightarrow e^+ + e^- + e^+ + e^- \Rightarrow (u\bar{u}/d\bar{d}) \rightarrow e^+ + e^- + e^+ + e^-$

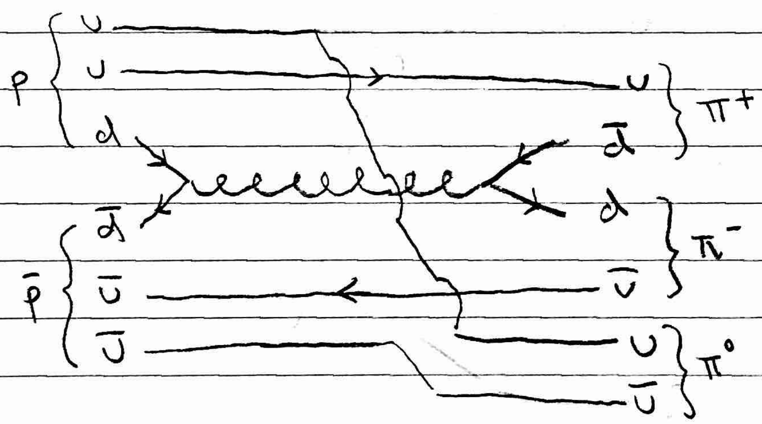
$\Rightarrow$ electromagnetic (electrons couple to γ from $u\bar{u}/d\bar{d}$ annihilation)



$p + \bar{p} \longrightarrow \pi^+ + \pi^- + \pi^0 \Rightarrow (uud) + (\bar{u}\bar{u}\bar{d}) \longrightarrow (u\bar{d}) + (\bar{u}d) + (u\bar{u}/d\bar{d})$

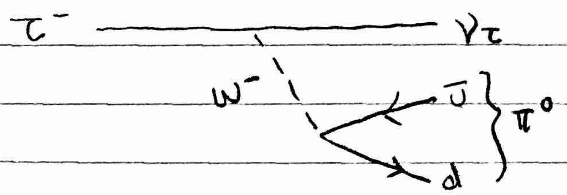
$\Rightarrow$ strong interaction (all quarks, flavour conserved)

One possible Feynman diagram is



$\tau^- \longrightarrow \pi^- + \nu_\tau \Rightarrow \tau^- \longrightarrow (\bar{u}d) + \nu_\tau$

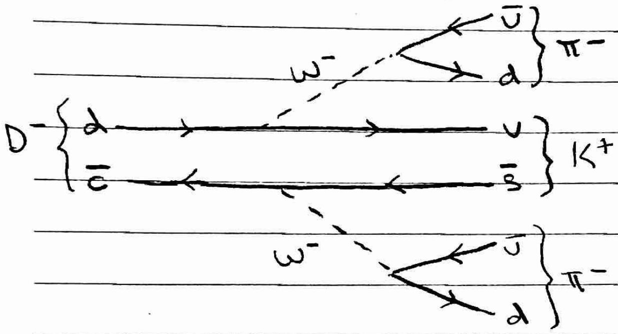
$\Rightarrow$ Weak interaction (neutrino is involved)



Question #1 continued . . .

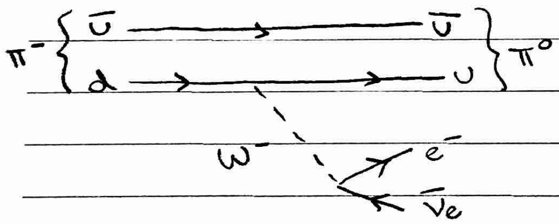
$D^- \rightarrow K^+ + \pi^- + \pi^- \Rightarrow (d\bar{c}) \longrightarrow (u\bar{s}) + (d\bar{u}) + (d\bar{u})$

$\Rightarrow$ weak interaction (quark flavour not conserved)



$\pi^- \longrightarrow \pi^0 + e^- + \bar{\nu}_e \Rightarrow (d\bar{u}) \longrightarrow (u\bar{u}/d\bar{d}) + e^- + \bar{\nu}_e$

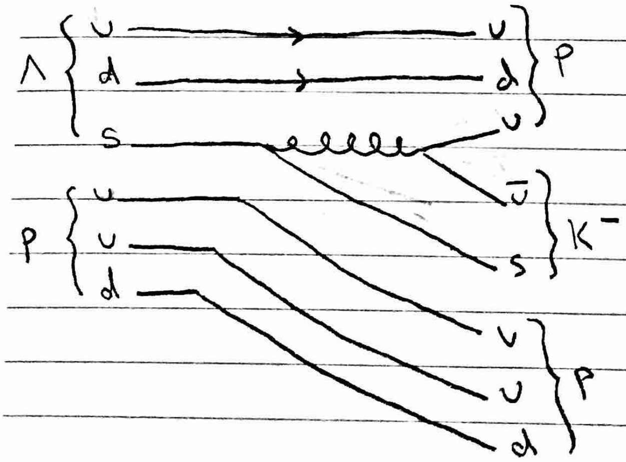
$\Rightarrow$ weak interaction (involves neutrino)



$\Lambda + p \longrightarrow K^- + p + p \Rightarrow (uds) + (uud) \longrightarrow (s\bar{u}) + (uud) + (uud)$

$\Rightarrow$ strong interaction (quarks, flavour conserved)

One possible diagram is:



Question #2:

Contribution of total angular momentum $\vec{\mu}_j = g_j \vec{J}$
Contribution of orbital angular momentum $\vec{\mu}_\ell = g_\ell \vec{L}$
Contribution from unpaired spin $\vec{\mu}_s = g_s \vec{S}$

Then, since $\mu_j = g_j J = g_\ell L + g_s S$ we can write $g_j J \cdot J = g_\ell L \cdot J + g_s S \cdot J$

$$g_j = \frac{1}{J^2}(g_\ell L \cdot J + g_s S \cdot J)$$

Now we need expressions for $L \cdot J$ and $S \cdot J$. Start with $J = L + S$:

(1) $S = J - L$
$$S^2 = J^2 + L^2 - 2L \cdot J$$
$$\Rightarrow L \cdot J = \frac{1}{2}(J^2 + L^2 - S^2)$$

(2) $L = J - S$
$$L^2 = J^2 + S^2 - 2S \cdot J$$
$$\Rightarrow S \cdot J = \frac{1}{2}(J^2 - L^2 + S^2)$$

$$\therefore g_j = \frac{1}{J^2}\left[g_\ell \frac{1}{2}(J^2 + L^2 - S^2) + g_s \frac{1}{2}(J^2 - L^2 + S^2)\right]$$
$$= g_\ell\left[\frac{j(j+1) + \ell(\ell+1) - s(s+1)}{2j(j+1)}\right] + g_s\left[\frac{j(j+1) - \ell(\ell+1) + s(s+1)}{2j(j+1)}\right]$$

Now for a detailed example for the shell model:

Nucleus ${}^{43}_{20}\text{Ca}\left(\frac{7}{2}^-, -1.32\mu\right)$

Unpaired nucleon: neutron
Configuration: $1s_{1/2}(2)$, $1p_{3/2}(4)$, $1p_{1/2}(2)$, $1d_{5/2}(6)$, $2s_{1/2}(2)$, $1d_{3/2}(4)$
$1f_{7/2}(3)$

Spin: $7/2$ since the unpaired nucleon is in $1f_{7/2}$ shell
Parity: $(-1)^\ell = (-1)^3 = -1$ (note: in spectroscopic notation $f = 3$)

The prediction matches the measured values given above. Note that I used the Saxon-Woods/Woods-Saxon potential.

Question #2 continued . . .

For the rest of the nuclei:

Nucleus	Unpaired	Energy level	Spin	Parity
$^{93}_{41}Nb(\frac{9}{2}^+, 6.17\mu_N)$	proton	$1g9/2$	$9/2$	+1
$^{137}_{56}Ba(3^+/2, 0.931\mu_N)$	neutron	$3s1/2$	$1/2$	+1
$^{197}_{79}Au(3^+/2, 0.145\mu_N)$	proton	$2d3/2$	$3/2$	+1
$^{26}_{13}Al(5^+, \text{unknown})$	both	$2d5/2$	0 -5	+1

note on the odd-odd nucleus:

For the spin, we have an unpaired proton and an unpaired neutron in $1d5/2$. Odd-odd nuclei have integer spins which lie in the range $|j_p - j_n|$ to $|j_p + j_n|$ so for $^{26}_{13}Al$ j can be $(5/2 - 5/2)$ to $(5/2 + 5/2) = 0, 1, 2, 3, 4, 5$.

In this case, experimentally it is found to be 5.

For the parity, $l = 2$ so $(-1)^2(-1)^2 = +1$

For the Woods-Saxon potential, we can see that the shell model correctly predicted all spins and parities, except for $^{137}_{56}Ba$, where experimentally it should be $\frac{3}{2}^+$ but the shell model gave $\frac{1}{2}^+$.

For the magnetic moment:

$^{34}_{20}Ca$ has an unpaired neutron, so $g_l = 0$ (for protons, $g_l = 1$)

$g_s = -3.826$ (for protons, $g_s = 5.586$)

From the energy level scheme, we see that $j = 7/2$, $l = 3$ $\therefore j = l + \frac{1}{2}$.

Then, $\langle\mu\rangle = [g_l(j + \frac{1}{2}) + \frac{1}{2}g_s]\mu_N$

$= [(0)(7/2 + \frac{1}{2}) + \frac{1}{2}(-3.826)]\mu_N$

$= -1.913\,\mu_N$ compared to $-1.32\,\mu_N$, not the greatest prediction ..

The rest of the calculated μ are:

$^{93}_{41}Nb$ $6.8\mu_N$

$^{137}_{56}Ba$ $-1.9\mu_N$

$^{197}_{79}Au$ $0.12\mu_N$

$^{26}_{13}Al$ $2.9\mu_N$

Question #3:

a) $\nu + p \rightarrow \mu^- + D^* + p$

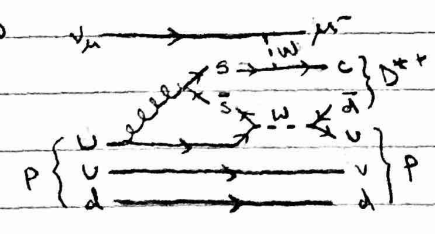
The neutrino interacts with a sea quark in the proton to produce a μ^-, D^*, and proton. Before the interaction, we have a total charge of +1. This tells us that the D^* must have positive charge, and $\therefore$ has quark content of $c\bar{d}$. Since there is a neutrino to muon involved, we know that the interaction must be weak via W-boson exchange. Quark flavour options include $s\bar{s}$, $c\bar{c}$, $d\bar{d}$, with $s\bar{s}$ being most likely. The creation of heavy quark pairs like $c\bar{c}$ is suppressed relative to $s\bar{s}$ because of their high invariant mass. The $c \rightarrow d$ or $d \rightarrow c$ transition is Cabbibo-suppressed (ie, due to quark mixing). This leaves us with a $s\bar{s}$ pair being most likely involved in the interaction.

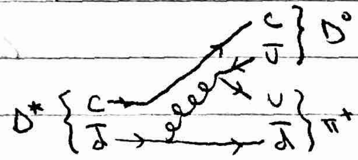
b) The lepton flavour of the incoming neutrino must be muon since there is a muon produced in the interaction.

c) The force involved in each interaction:

(1) $\nu_\mu p \rightarrow \mu^- D^+ p$ $\Rightarrow$ weak
(2) $D^+ \rightarrow D^0 \pi^+$ $\Rightarrow$ strong
(3) $D^0 \rightarrow K^- \pi^+$ $\Rightarrow$ weak
(4) $K^- p \rightarrow \Sigma^- \pi^+$ $\Rightarrow$ strong
(5) $\Sigma^- \rightarrow n \pi^-$ $\Rightarrow$ weak

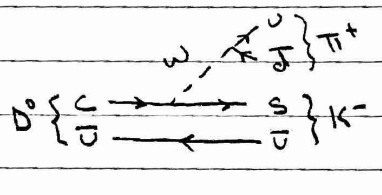
d) Feynman diagrams for each interaction:

(1)


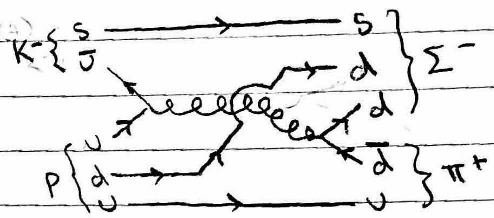
(2)


Question #3d) continued:

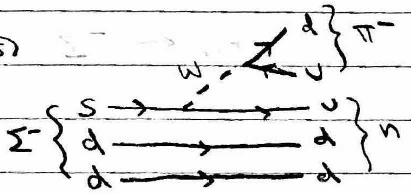
(3)



(4)



(5)



e) $D^0 \rightarrow K^- \pi^+$ where there are two π^+, one with $\vec{p} = 3.6$ GeV, the other with $\vec{p} = 0.23$ GeV.

Angle between K^- ($\vec{p} = 0.32$ GeV) and each π^+:

$$p_{D^0}^2 = (p_{K^-} + p_{\pi^+})^2$$

$$m_{D^0}^2 = m_{K^-}^2 + m_{\pi^+}^2 + 2(E_{K^-}E_{\pi^+} - \vec{p}_{K^-}\vec{p}_{\pi^+})$$

$$m_{D^0}^2 = m_{K^-}^2 + m_{\pi^+}^2 + 2(E_{K^-}E_{\pi^+} - |\vec{p}_{K^-}||\vec{p}_{\pi^+}|\cos\theta)$$

$$\Rightarrow \cos\theta = \frac{m_{D^0}^2 - m_{K^-}^2 - m_{\pi^+}^2 - 2E_{K^-}E_{\pi^+}}{-2|\vec{p}_{K^-}||\vec{p}_{\pi^+}|}$$

$$\theta = \cos^{-1}\left[\frac{m_{K^-}^2 + m_{\pi^+}^2 + 2E_{K^-}E_{\pi^+} - m_{D^0}^2}{2|\vec{p}_{K^-}||\vec{p}_{\pi^+}|}\right]$$

$\Rightarrow$ For $\vec{p}_{\pi^+} = 3.6$ GeV:

$$E_{\pi^+} = \sqrt{(3.6\,\text{GeV})^2 + (0.13957\,\text{MeV})^2}$$

$$= 3.6027\,\text{GeV}$$

$$\cong 3.6\,\text{GeV} \text{ so } E_{\pi^+} \sim \vec{p}_{\pi^+}$$

$$E_{K^+} = \sqrt{(0.32\,\text{GeV})^2 + (0.49365\,\text{GeV})^2}$$

$$\cong 0.588\,\text{GeV}$$

$$\theta = \cos^{-1}\left[\frac{m_{K^-}^2 + 2E_{K^-}|\vec{p}_{\pi^+}| - m_{D^0}^2}{2|\vec{p}_{K^-}||\vec{p}_{\pi^+}|}\right]$$

(in GeV!)

$$\theta = \cos^{-1}\left[\frac{(0.49365)^2 + 2(0.588)(3.6) - (1.863)^2}{2(0.32)(3.6)}\right]$$

$$= 1.11869\text{ rad}$$

$$\cong 64°$$

Question #3 continued:

(in GeV!)

For $\vec{p}_{\pi^+} = 0.23$ GeV, $E_{\pi^+} = 0.269$ GeV.

Then $\theta = \cos^{-1}\left[\dfrac{(0.49365)^2 + (0.13957)^2 + 2(0.269)(0.588) - (1.863)^2}{2(0.23)(0.32)}\right]$

$= \cos^{-1}(-19.64)$

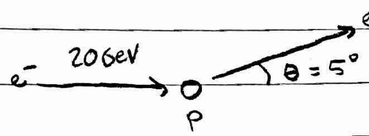
$=$ not defined.

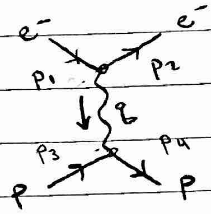
$\therefore$ the correct π^+ is the one with $\vec{p} = 3.6$ GeV. We could have guessed this without the calculation by simply looking at the table provided in the question which has $M_{D^*} - M_D = 0.14$ GeV, which is quite close to the pion mass of 0.13957 GeV. Thus we could estimate that the lower energy pion comes from the D^* decay, and the higher energy pion comes from the D^0 by process of elimination.

f) See part e) explanation :)

Question #4:

a) Elastic collision





Since this is an elastic collision, we can quickly calculate q^2 using the top vertice: $q^2 = (p_1 - p_2)^2$

$$= (E_1 - E_2)^2 - (\vec{p}_1 - \vec{p}_2)^2$$
$$= m_1^2 + m_2^2 - 2E_1E_2 + 2\vec{p}_1\vec{p}_2\cos\theta$$
$$= -2E_1E_2(1-\cos\theta) \quad \Rightarrow \text{assume } m \ll E$$
$$= -2(20\,\text{GeV})^2(1-\cos 5°) \quad \Rightarrow \text{assume } E_1 \sim E_2$$
$$\cong -3.044\ \text{GeV}$$

Alternatively, without assuming $E_1 \sim E_2$,

For top vertex $q^2 = -2E_1E_2(1-\cos\theta)$ as shown above

For bottom vertex
$$p_4 = p_3 + q$$
$$p_4^2 = (p_3+q)^2$$
$$m_p^2 = m_p^2 + q^2 + 2p_3q$$
$$q^2 = -2p_3q$$
$$q^2 = -2\begin{bmatrix} m_p \\ \vec{0} \end{bmatrix} \cdot \begin{bmatrix} E_1 - E_2 \\ \vec{p}_1 - \vec{p}_2 \end{bmatrix}$$
$$q^2 = -2m_p(E_1 - E_2)$$

Then $q^2 = -2E_1E_2(1-\cos\theta) \Rightarrow E_2 = {}^{-q^2}/_{2E_1(1-\cos\theta)}$

$q^2 = -2m_p(E_1 - E_2) \quad \Rightarrow E_2 = {}^{+q^2}/_{2m_p} + E_1$

$\therefore$ since $E_2 = E_2$, $-q^2/2E_1(1-\cos\theta) = +q^2/2m_p + E_1$

$$\Rightarrow q^2 = -E_1\left(\frac{1}{2m_p} + \frac{1}{2E_1(1-\cos\theta)}\right)^{-1}$$
$$\cong -2.81\ \text{GeV}$$

To estimate the size of the proton use $\Delta x \sim \hbar c/|q| = 197\ \text{MeV}\cdot\text{fm}/\sqrt{2.81\ \text{GeV}^2}$

$$= 0.117\ \text{fm}$$

According to Wikipedia, the proton radius is $\sim 0.84 - 0.87$ fm, so Δx is an underestimate.

Question #4 continued...

b) Charge density of proton given as $\rho(\vec{r}) = \rho_0 e^{-mr}$

The form factor is writen as $F(q^2) = \frac{1}{Ze} \int \rho(\vec{r})\, e^{i\vec{q}\cdot\vec{r}/\hbar}\, dV$

In this case, we can write $dV = r^2 \sin\theta\, dr\, d\theta\, d\phi$

$= r^2 d\cos\theta\, dr\, d\phi$ for spherically-symmetric distribution.

$$\therefore F(q^2) = \int_0^{2\pi} d\phi \underbrace{\int_{-1}^{+1}}_{\cos\theta} \underbrace{\int_0^{\infty}}_{r} \left(\frac{1}{Ze}\right) \rho(\vec{r})\, e^{i\vec{q}\cdot\vec{r}/\hbar}\, r^2\, d\cos\theta\, dr$$

$$= \frac{2\pi}{Ze} \int_{-1}^{+1} \int_0^{\infty} \rho_0 e^{-mr}\, e^{iqr\cos\theta/\hbar}\, r^2\, d\cos\theta\, dr$$

note: the normalization condition $\frac{4\pi}{Ze} \int_0^{\infty} r^2 \rho_0 e^{-mr} = 1$

$$\frac{4\pi}{Ze}\left(2/m^3\right)\rho_0 = 1$$

$$\Rightarrow \rho_0 = Ze\, m^3/8\pi$$

$$\therefore F(q^2) = \frac{2\pi}{Ze}\left[\frac{Ze\, m^3}{8\pi}\right] \int_0^{\infty} e^{-mr} \left[e^{iqr/\hbar} - e^{-iqr/\hbar}\right]\left(\hbar/iqr\right) r^2\, dr$$

$$= \frac{m^3}{4} \int_0^{\infty} e^{-mr} \sin(qr/\hbar)\,(2i)\,(\hbar/iqr)\, r^2\, dr \quad \Rightarrow \sin x = \frac{1}{2i}\left(e^{ix} - e^{-ix}\right)$$

$$= \frac{m^3}{2} \int_0^{\infty} e^{-mr} \sin(qr/\hbar)\, r^2\, dr\, (\hbar/q)$$

$$= \frac{m^3}{2}\left[2(q/\hbar)m / \left((q/\hbar)^2 + m^2\right)^2\right](\hbar/q)$$

$$= m^4 / \left((q/\hbar)^2 + m^2\right)^2$$

$$= m^4 / \left(q^2 + m^2\right)^2 \quad \text{setting } \hbar = 1$$

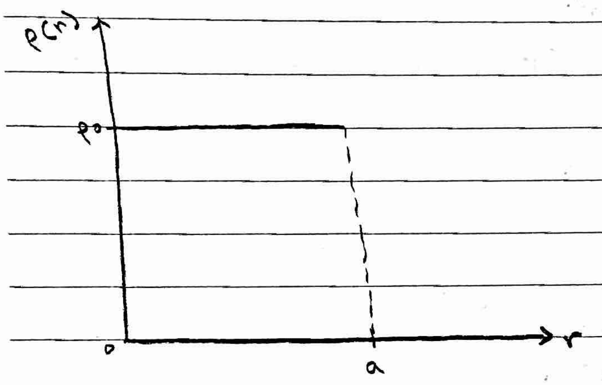
note: integrals done using Wolfram Alpha

Question #5:

a) Simplest model for charge-density of nucleus

$\rho(r) = \rho_0 \quad r \leq a$

$\rho(r) = 0 \quad r > a$



b) The form factor corresponding to this charge distribution is:

$$F(q^2) = \frac{1}{Ze} \int \rho(r) e^{iq\cdot r/\hbar} \, dV$$

$$= \frac{1}{Ze} \iiint \rho(r) e^{iq\cdot r/\hbar} r^2 \sin\theta \, d\theta \, d\phi \, dr$$

$$= \frac{1}{Ze} \left(\frac{4\pi\hbar}{q}\right) \int \rho(r) r \sin(qr/\hbar) \, dr$$ as given in question (and shown in #4b))

$$= \frac{4\pi\hbar}{Zeq} \int_0^a \rho_0 r \sin(qr/\hbar) \, dr + \frac{4\pi\hbar}{Zeq} \int_a^\infty (0) r \sin(qr/\hbar) \, dr$$

$$= \frac{4\pi\hbar}{Zeq} \int_0^a \rho_0 r \sin(qr/\hbar) \, dr + \text{const}$$

The normalization condition: $\frac{4\pi}{Ze} \int_0^\infty r^2 \rho(r) \, dr = 1$

$$\frac{4\pi}{Ze}\left[\int_0^a r^2 \rho_0 \, dr + \int_a^\infty r^2 (0) \, dr\right] = 1$$

$$\frac{4\pi}{Ze} \rho_0 \, r^3/3 \Big|_0^a + \frac{4\pi}{Ze} \text{const} = 1$$ const is arbitrary, set to 0.

$$4\pi/Ze \; \rho_0 (a^3/3) = 1$$

$$\Rightarrow \rho_0 = 3/4 \left(Ze/\pi a^3\right)$$

$$\therefore F(q^2) = \frac{4\pi\hbar}{Zeq} \left[\frac{3}{4} \frac{Ze}{\pi a^3}\right] \int_0^a r \sin(qr/\hbar) \, dr$$

$$= 3\hbar/qa^3 \left[(\hbar/q)^2 \left(\sin(q/\hbar \, a) - a(q/\hbar) \cos(q/\hbar \, a)\right)\right]$$

$$= 3\hbar/qa^3 \, (\hbar/q)^2 \left[\sin(qa/\hbar) - (qa/\hbar) \cos(qa/\hbar)\right]$$

$$= \frac{3\left[\sin(qa/\hbar) - (qa/\hbar)\cos(qa/\hbar)\right]}{(qa/\hbar)^3}$$

Question #5 continued...

c) Root mean square definition: $\langle r^2 \rangle = \frac{4\pi}{Ze} \int r^2 \rho(r) r^2 dr$

Then, after normalizing, $\langle r^2 \rangle = \frac{4\pi}{Ze}\left[\frac{3}{4}\frac{Ze}{\pi a^3}\right]\int_0^a r^2 r^2 dr$

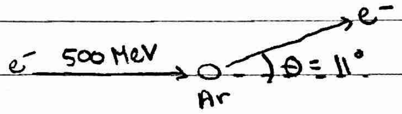
$$= \frac{4\pi}{Ze}\left[\frac{3}{4}\frac{Ze}{\pi a^3}\right]\int_0^a r^4 dr$$

$$= \frac{3}{a^3}\left[r^5/5\right]_0^a$$

$$= 3/a^3 \left(a^5/5\right)$$

$$= (3/5)\, a^2$$

d) Electron scattered off of argon nucleus



Momentum transfer $q^2 = (p_1 - p_2)^2 = (p_3 - p_4)^2$ is calculated in the same way as question #4a). ∴ we can write $q^2 = E_1\left(\frac{1}{2m_{Ar}} - \frac{1}{2E_1(1-\cos\theta)}\right)^{-1}$

$$\cong -9189 \text{ MeV}^2$$

$$\cong -9.189 \times 10^{-3} \text{ GeV}^2$$

The reduced deBroglie wavelength $\hbar c/pc = 197 \text{ MeV fm} / 500 \text{ MeV}$

$$\cong 0.394 \text{ fm}$$

The Mott differential cross section $\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = 4(Z\alpha)^2 \frac{E^2}{q^4}\left(1 - \beta^2 \sin^2 \theta/2\right)$

$$= 4\left(18 \cdot \tfrac{1}{137}\right)^2 \left[(500 \text{MeV})^2/(-9189 \text{MeV}^2)^2\right]\left(1 - \sin^2(11/2)\right)$$

$$\cong 7.86 \text{ fm}^2 \text{ assuming } \beta \sim 1$$

$$\cong 78.6 \text{ mb}$$

The change in $\frac{d\sigma}{d\Omega}$ is found using the form factor of part a), where $\frac{d\sigma}{d\Omega} = |F(q^2)|^2 \left(\frac{d\sigma}{d\Omega}\right)_{Mott}$

$\Rightarrow qa/\hbar = \sqrt{9189 \text{ MeV}^2} \cdot (1.2 \times 40^{1/3} \text{ fm}) / 197 \text{ fm} \cdot \text{MeV}$

$$\cong 1.99696$$

$\therefore |F(q^2)|^2 = \left[3\left[\sin(1.99696) - (1.99696 \cos(1.99696))\right] / (1.99696^3)\right]^2$

$$\cong 0.428$$

The differential cross section is ∴ 0.428x the Mott cross section.

Question #6a:

a) For ^{114}Cd:

For binding energy $B = a_v A - a_s A^{2/3} - a_c Z^2/A^{1/3} - a_A(A-2Z)^2/A + a_p/A^{1/2}$

$= (15.56\text{ MeV})(114) - (17.23\text{ MeV})(114)^{2/3}$ ↳ ^{114}Cd is ee (Z=48, N=66)

$-(0.697\text{ MeV})/(114)^{1/3}$

$-(23.285\text{ MeV})(114-2(48))^2/(114) + (12.0\text{ MeV})/(114)^{1/2}$

$\cong 972.47\text{ MeV}$

Alternatively, use $B = -M(Z,A) + ZM_p + (A-Z)M_n$

$= -(113.903u)(931.5\text{ MeV}/u) + (48)(938.27\text{ MeV}) + (66)(939.57\text{ MeV})$

$\cong 947.94\text{ MeV}$

For binding energy per nucleon $B/A = (947.94\text{ MeV}/114)$ OR $(972.47\text{ MeV}/114)$

$\cong 8.32\text{ MeV/nucleon}$ $\cong 8.53\text{ MeV/nucleon}$

(*)

b) The two methods were calculated in a), where the values are relatively close.

(*) I forgot about the neutron and proton energy separations! They are:

(1) $E_n = B(Z,A) - B(Z,A-1)$

$= M(Z,A-1) + M_n - M(Z,A)$

$\cong 8.09\text{ MeV}$

(2) $E_p = B(Z,A) - B(Z-1,A-1)$

$= M(Z-1,A-1) + M_p + m_e - M(Z,A)$

$\cong 7.3\text{ MeV}$

Question #6:

a) Nucleus with $Z = N = A/2$

Semi-empirical mass formula $M(Z,A)c^2 = ZM_pc^2 + (A-Z)M_nc^2 - B(Z,A)$

where $B(Z,A) = a_vA - a_sA^{2/3} - a_cZ^2/A^{1/3} - a_a(A-2Z)^2/A +$ ~~pairing term~~ (neglect)

$= a_vA - a_sA^{2/3} - a_c(A/2)^2/A^{1/3} - a_a(A-2(A/2))^2/A$

$= a_vA - a_sA^{2/3} - \frac{1}{4}a_cA^{5/3} - a_a(0)/A$

$\therefore B/A = a_v - a_sA^{-1/3} - \frac{a_c}{4}A^{2/3}$ as expected.

To find the maximum: $\frac{d}{dA}(B/A) = \frac{d}{dA}(a_v) - \frac{d}{dA}(a_sA^{-1/3}) - \frac{d}{dA}(\frac{a_c}{4}A^{2/3})$

$0 = 0 + \frac{1}{3}a_sA^{-4/3} - \frac{a_c}{2\cancel{4}}(\frac{\cancel{2}}{3})A^{-1/3}$

$0 = a_s/3\,A^{-4/3} - a_c/6\,A^{-1/3}$

$\Rightarrow a_c/6\,A^{-1/3} = a_s/3\,A^{-4/3}$

$a_c/6 = a_s/3\,A^{-4/3}A^{1/3}$

$a_c/6 = a_s/3\,A^{-1}$

$\Rightarrow A = 2a_s/a_c$

$= 2(17.23\text{ MeV})/(0.697\text{ MeV})$ using #s from Prob Set

$\cong 49.44$

$\therefore Z = A/2 \cong 24.72 \sim 25$ ... close to 26

b) Valley of stability $Z = \beta/2\gamma$ where $\beta = a_a + M_n - M_p - m_e$

$\gamma = a_a/A + a_c/A^{1/3}$

Then for $A = 100$, $Z = \beta/2\gamma$

$= (a_a + M_n - M_p - m_e)/(a_a/A + a_c/A^{1/3})$

$= (93.14\text{ MeV} + 939.57\text{ MeV} - 938.27\text{ MeV} - 0.511\text{ MeV})/2(\frac{93.14\text{ MeV}}{100} + \frac{0.697\text{ MeV}}{100^{1/3}})$

$= 93.929\text{ MeV}/2(1.0815\text{ MeV})$

$\cong 43.425$

$\cong 43$

Similarly, for $A = 200$, $Z \cong 80$

According to the table on pg. 8 of the Prob Set, for $Z = 43$, $N = 57$ is on the

Question #6z continued...

valley of stability. For $Z=80$ and $N=120$, we also fall on the valley of stability.

note: $Z=43 \Rightarrow$ Tc has no stable isotopes, though the "most stable" are ^{98}Tc, ^{97}Tc, ^{99}Tc with half-lives on the order of 200 000 – 4 million years. The specific isotope we considered, ^{100}Tc, has a half-life of 15.8 s, and can decay via β-decay.

$Z=80 \Rightarrow$ Hg has 7 stable isotopes, and ^{200}Hg happens to be one of them, however even though decays have not been observed it is believed to undergo α-decay. In any case, it is definitely β-stable.

Question #7:

a) Potential energy of a uniformly charged sphere of total charge Q and radius R

For $r<R$ the amount of charge is $q(r) = Qr^3/R^3$

For $r>R$ the amount of charge is $q(r) = Q$

The electric field is given by $\vec{E}(r) = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$

For $r<R$ $\vec{E}(r) = \frac{1}{4\pi\epsilon_0} \frac{Qr}{R^3}$

For $r>R$ $\vec{E}(r) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$

The electrostatic potential is given by $V = \frac{\epsilon_0}{2} \int E^2 \, dV$

$= 2\pi\epsilon_0 \int E^2 r^2 \, dr$ (integrating over θ, ϕ)

Now we integrate from $\{0, R\}$ and $\{R, \infty\}$

$$V = \frac{1}{8\pi\epsilon_0}\left[\int_0^R \frac{Q^2}{R^6} r^4 \, dr + \int_R^\infty Q^2/r^2 \, dr\right]$$

$$= \frac{1}{8\pi\epsilon_0}\left[\frac{Q^2}{R^6}\left(\frac{1}{5}\right)R^5 + \frac{Q^2}{r}(-1)\Big|_R^\infty\right]$$

$$= \frac{1}{8\pi\epsilon_0}\left[Q^2/5R + Q^2/R\right]$$

$$= 3Q^2/20\pi\epsilon_0 R$$

b) The value of a_c can be found by letting $Q = Ze$ and using the Coulomb term of the semi-empirical mass formula:

$$a_c Z^2/A^{1/3} = 3(Ze)^2/20\pi\epsilon_0 R$$

$$\Rightarrow a_c = 3A^{1/3} e^2 (20\pi\epsilon_0 \cdot 1.24 \text{ fm } A^{1/3})^{-1}$$

$$= 3e^2 (20\pi\epsilon_0 1.24 \text{fm})^{-1}$$

$$= \frac{3}{5}\left(\frac{e^2}{4\pi\epsilon_0}\right)\left(\frac{1}{1.24\text{fm}}\right)$$

$$= \frac{3}{5}\left(\frac{197}{137} \text{ MeV fm}\right)\left(\frac{1}{1.24 \text{fm}}\right)$$

$$= 0.696 \text{ MeV}$$

note: binding energy of 3 neutrons is zero

binding energies of nuclei found using formula

Solving the BE formula in the case of ^{181}Ta gives $a_c \sim 0.693$ MeV - not bad!

Spontaneous fission, energy released is $E = BE(^{235}U) - BE(^{87}Br) - BE(^{145}La)$

$\cong 153$ MeV