1)

~~a)~~ $\langle \lambda \rangle = \int \psi^*(\bar{r})(\bar{\sigma}\cdot\bar{p})\,\psi(\bar{r})\,dr$

under parity transformation

$$\langle \lambda \rangle \rightarrow \int \psi^*(-\bar{r})(-\bar{\sigma}\cdot\bar{p})\,\psi(-\bar{r})\,dr$$

if $\psi(-\bar{r})$ is an eigenstate of Parity

then $\psi(-\bar{r}) = \pm\psi(\bar{r})$

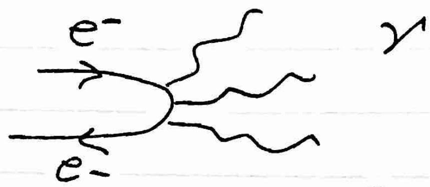
$\therefore \quad \psi^*(-\bar{r})\cdot\psi(-\bar{r}) = \psi^*(\bar{r})\,\psi(\bar{r})$

$\therefore \langle \lambda \rangle \rightarrow -\int \psi^*(\bar{r})\,\hat{\sigma}\cdot\bar{p}\,\psi(\bar{r})\,dr,$

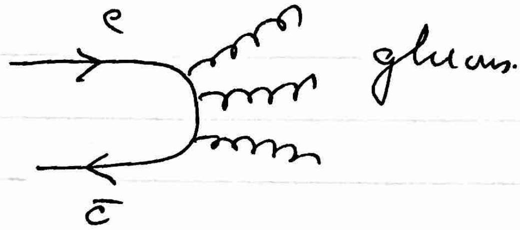
$\therefore \langle \lambda \rangle \rightarrow -\langle \lambda \rangle$

$\therefore \langle \lambda \rangle = 0$

2) Positronium



$J/\psi \to 3g$ (hadrons)



So we substitute

$$\alpha \to \alpha_s$$

$$m_e \to m_c \quad (1500\ MeV/c^2)$$

ω is decay rate (s^{-1})

$$\Gamma\ (keV) = \frac{\hbar}{\tau} = \omega$$

$$\omega = \frac{\Gamma}{\hbar}$$

$$\frac{\Gamma_{3g}}{\Gamma_{full}} = \text{Branching fraction.}$$

$$\Gamma_{3g} = \Gamma_{full} \times 0.82$$

$$= 0.82 \times 63\,keV \approx 52\ keV$$

$$\omega = \frac{\Gamma_{3g}}{\hbar} = \frac{2(\alpha_s)^6(\pi^2-9)}{9\pi}\frac{m_c c^2}{\hbar}$$

$$(\alpha_s)^6 = \frac{9\pi\,\Gamma_{3g}}{2(\pi^2-9)\,m_c c^2}$$

$$= \frac{9\pi\ 0.052\ \text{MeV}}{2(\pi^2-9)\ 1500\frac{\text{MeV}}{c^2}\cdot c^2}$$

$$\underline{\alpha_s = 0.287}$$

For $J/\psi \rightarrow \gamma gg$

$$\omega = \frac{\Gamma_{2g\gamma}}{\hbar} = \frac{2(\alpha)^2(\alpha_s)^4(\pi^2-9)}{9\pi}\frac{m_c c^2}{\hbar}$$

$$\therefore \frac{\Gamma_{\gamma 2g}}{\Gamma_{3g}} = \frac{\alpha^2(\alpha_s)^4}{(\alpha_s)^6} = 6.47\times10^{-4}$$

$$BR \quad \frac{\Gamma_{2g\gamma}}{\Gamma} = \frac{6.47\times10^{-4}\times\Gamma_{3g}}{\Gamma_{full}} = \frac{6.47\times10^{-4}\times5.2\times10^{-2}}{6.3\times10^{-2}}$$

$$= 5.34\times10^{-4}$$

3a) Non relativistic, energy transfered to stationary electron is just

$\nu \quad \frac{1}{2} m v^2$

$$2 m_e \nu = m_e^2 \cdot v^2$$

$$= |\bar{p}|^2$$

b) for relativistic case of scattering off of one nucleon inside a nucleus

nucleon inside nucleus absorbs 4-momentum q^2 from virtual γ

initial 4-momentum of NUCLEON p_N

$$(p_N + q)^2 = m_N^2$$

$$\underbrace{p_N^2}_{m_N^2} + q^2 + 2 p_N \cdot q = m_N^2$$

$$q^2 = -2 p_N \cdot q$$

$$= -2 (m_N, 0)(\nu, \bar{q})$$

$$= -2 m_N \nu$$

$$\nu = \frac{-q^2}{2 m_N}$$

c) this is similar to quasi elastic scattering in b)

proton has 4-momentum P

parton has 4-momentum xP

$$(xP + q)^2 = m_{parton}^2 \sim 0.$$

virtual γ

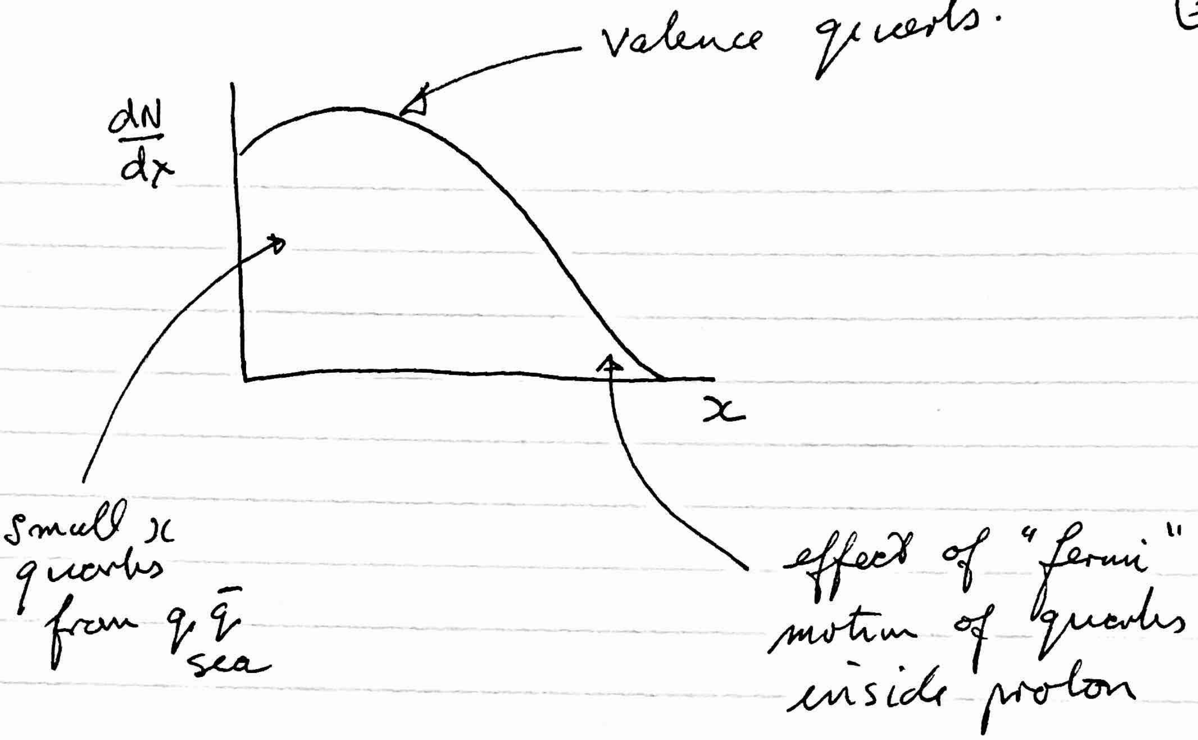
$$x^2P^2 + q^2 + 2xP\cdot q = 0$$

$$x^2P^2 = x^2 M_{proton}^2 \ll q^2$$

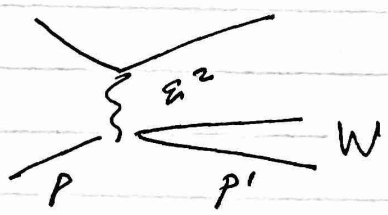
$$\Rightarrow q^2 + 2xP\cdot q = 0$$

$$x = \frac{-q^2}{2P\cdot q} \rightarrow 2(M_p, 0)(\nu, q^2)$$

$$x = \cancel{2M_p}\ \frac{-q^2}{2M_p\cdot\nu}$$



d)



$$q^2 = (p' - p)^2$$

$$= p'^2 + p^2 - 2p' \cdot p$$

$$= W^2 + m_p^2 - 2(\nu + m_p, \bar{p}')(m_p, 0)$$

$$= W^2 + m_p^2 - 2\nu m_p - 2 m_p^2$$

$$q^2 = W^2 - m_p^2 - 2\nu m_p.$$

$$W^2 = q^2 + m_p^2 + 2\nu m_p$$

d) when $x=1$; $q^2 = -2m\nu$

then. $W^2 = m_p^2 - 2m_p\nu + 2m_p\nu$

$$W^2 = m_p^2 \qquad \underline{min}$$

this makes sense — the smallest hadronic mass you can make is m_p

when $x=0$ $\quad q^2=0$

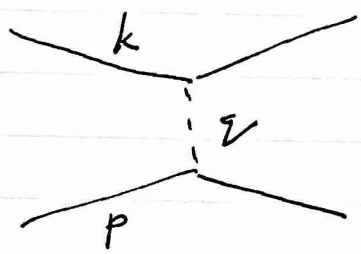
$$W^2 = m_p^2 + 2m_p\nu \qquad max$$

that also makes sense, if you transfer all the beam energy to the target that is the highest mass you can make.

$$W^2 = 1^2 + 2.1.40 \approx 80 \ \frac{GeV^2}{c^2}$$

e) Lorentz scalar in question is

$$y = \frac{P \cdot q}{P \cdot k}$$



in LAB $\quad y = \dfrac{(m_p, 0)(\nu, \bar{q})}{(m_p, 0)(E_e, \bar{p}_e)} = \dfrac{\nu}{E_e}$

obviously $0 < y < 1$

from definition of x & y

the maximum value of $|-q^2| = 2 m_p E_e$

At this point the struck quark absorbs all the energy of the beam

4) a) $\Delta^{++} \rightarrow p \, \pi^+$

$J^P = \frac{3}{2}^+ \rightarrow \frac{1}{2}^+ \; 0^?$

initial parity = +

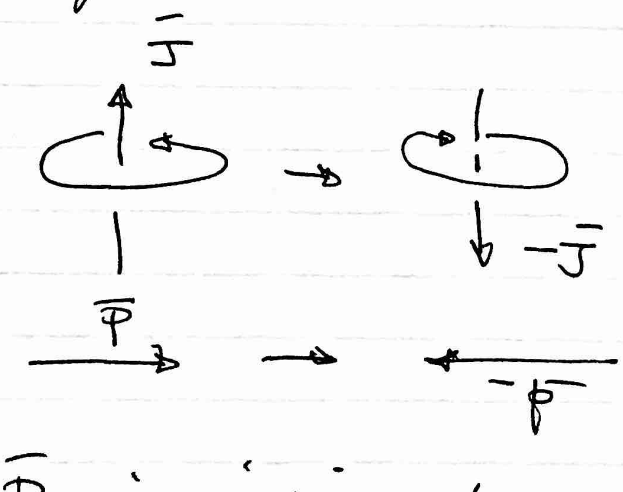
final equals $(+)(-1)^1(?)$

$\therefore \pi$ must have -ve PARITY

b) you can't measure parity in this decay mode. It is a week decay and violates parity conservation.

Yes it has definite spin + parity as it consists of quarks bound together by the colour force which conserves parity

c) under T



$\therefore \bar{J} \cdot \bar{p}$ is invariant

5a)

	C
ω^0 is	$-$
η' is	$+$
π^0 is	$+$
ρ^0 is	$-$
γ is	$-$

$$\underset{-}{\omega^0} \rightarrow \underset{+}{\pi^0}\ \underset{-}{\gamma} \qquad \checkmark$$

$$\underset{+}{\eta'} \rightarrow \underset{-}{\rho^0}\ \underset{-}{\gamma} \qquad \checkmark$$

$$\underset{+}{\pi^0} \rightarrow \underset{-\,-\,-}{\gamma\gamma\gamma} \qquad \times$$

$$\underset{-}{\rho^0} \rightarrow \underset{-\,-}{\gamma\gamma} \qquad \times$$

$$J/\psi \rightarrow p\bar{p}$$

$$1^{--} \qquad \tfrac{1}{2} \quad \tfrac{1}{2}$$

final state C is $(-1)^{L+S} = (-1)^{0+1}$ or $(-1)^{1+0}$

$= -$

$-$ $\qquad$ $-$

$\therefore$ so it is allowed

5) b) $\pi^+\pi^- \xrightarrow{C} \pi^-\pi^+$

or

$\pi^+ \xleftarrow{-r} \quad \xrightarrow{+r} \pi^- \longrightarrow \pi^- \xleftarrow{-r} \quad \xrightarrow{+r} \pi^+$

changes sign of radius vector ∴ same as parity

$$C|\pi^+\pi^-\rangle = (-1)^\ell |\pi^-\pi^+\rangle$$

$\ell = 1 \rightarrow C$ is $-$
$2\gamma \rightarrow C$ is $+$
} NOT POSSIBLE

$$K^- p \rightarrow \bar{K}^0 n$$

$$\downarrow C$$

$$K^+ \bar{p} \rightarrow K^0 \bar{n}$$

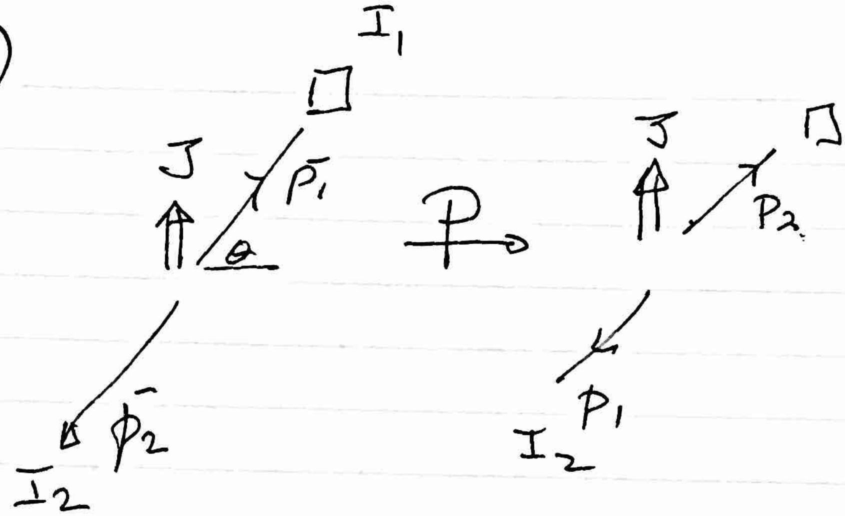
no it can't → basically because the q change to $\bar{q}$

$\bar{p} p \rightarrow \pi^+\pi^-$ is OK

$$\downarrow$$

$p\bar{p} \rightarrow \pi^-\pi^+$ ⟹ have same number of q & $\bar{q}$ in initial & final states

6) a)



$^{60}Co \to {}^{60}Ni\, e^- \bar{\nu}_e$

experiment is to count intensity of electrons going in upper hemisphere and in lower hemisphere, where hemisphere is defined by $\bar{J}$

Parity leaves $\bar{J}$ invariant
changes $\bar{p}_1 \to \bar{p}_2$

the effect of parity is achieved by reversing sign of $\bar{J}$. $\bar{J}$ of nuclei are aligned by magnetic field. Reverse magnetic field & look for change in I_2 & I_1 if they change — Parity not conserved symbollically $\frac{\bar{J}\cdot\bar{p}}{|\bar{J}|\cdot|p|} \xrightarrow{P} \frac{-\bar{J}\cdot\bar{p}}{|J|\cdot|p|}$

this is $\langle \cos\theta_e \rangle$

for Parity conservation

$$\langle \cos\theta_e \rangle = - \langle \cos\theta_e \rangle$$

$\therefore$ average value of $\cos\theta_e$ must be zero for parity invariance

Any asymmetry in ~~cos~~ $\cos\theta_e$ demonstrates parity nonconservation

b) $\bar{J}\cdot\bar{p}$ does not change sign under T

$\therefore$ you can learn nothing

c) You need an experiment which involves the strong interaction, which conserves P and C & could be sensitive to T

eg neutron electric dipole moment