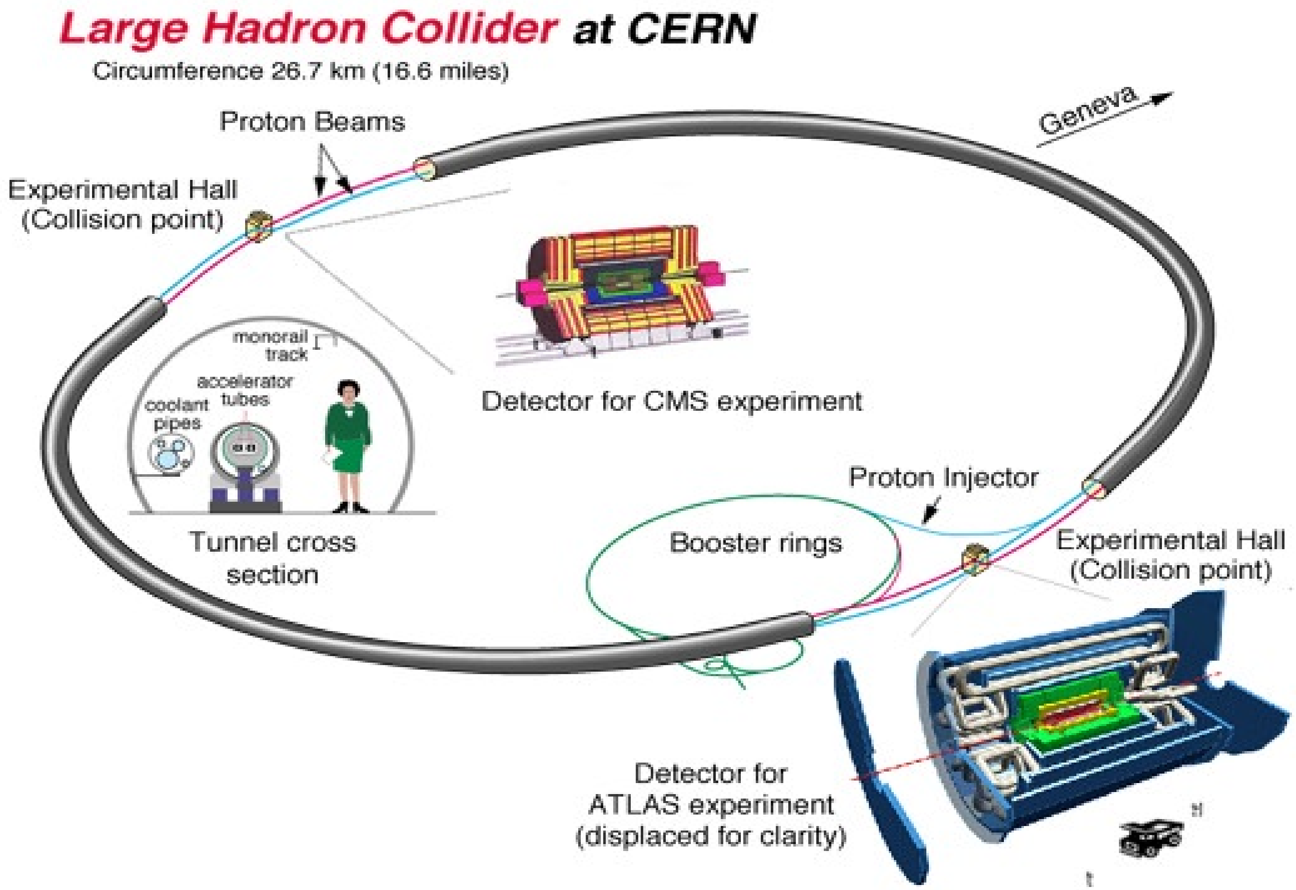
First some Introductory Stuff => On The Web

http://hep.physics.utoronto.ca/~orr/wwwroot/phy357/PHY357S.htm

PHY357 = What is the Universe Made Of?



SubAtomic Physics

Nuclear Physics

- Understand the Atomic Nucleus in terms of the interaction of Protons and Neutrons.

- An enormously important subject; both in order to understand matter on Earth and in Stars and the Universe, and for technological reasons

Particle Physics/High Energy Physics

- Understand Reality at its most Basic Level

- Experiment ↕ Constituents Interactions

- Theory — Why these Consituents and Interactions? Why this Space-Time

Micro Level ⟷ Macro Level → History of Cosmos

"All these things being considered, it seems probable to me that God in the beginning formed matter in solid, massy, hard, impenetrable, moveable particles of such sizes and figures, and with such other properties, and in such proportion to space, as most conduced to the end for which he formed them; and that these primitive particles being solids, are incomparably harder than any porous bodies compounded of them; even so very hard, as never to wear or break in pieces; no ordinary power being able to divide what God himself made in the first creation. While the particles continue entire, they may compose bodies of one and the same nature and texture in all ages: but should they wear away, or break in pieces, the nature of things depending on them would be changed. Water and earth, composed of old worn particles and fragments of particles, would not be of the same nature and texture now, with water and earth composed of entire particles in the beginning. And there, that nature may be lasting, the changes of corporal things are placed only in the various separations and new associations and motions of these permanent particles"

Isaac Newton - Optics

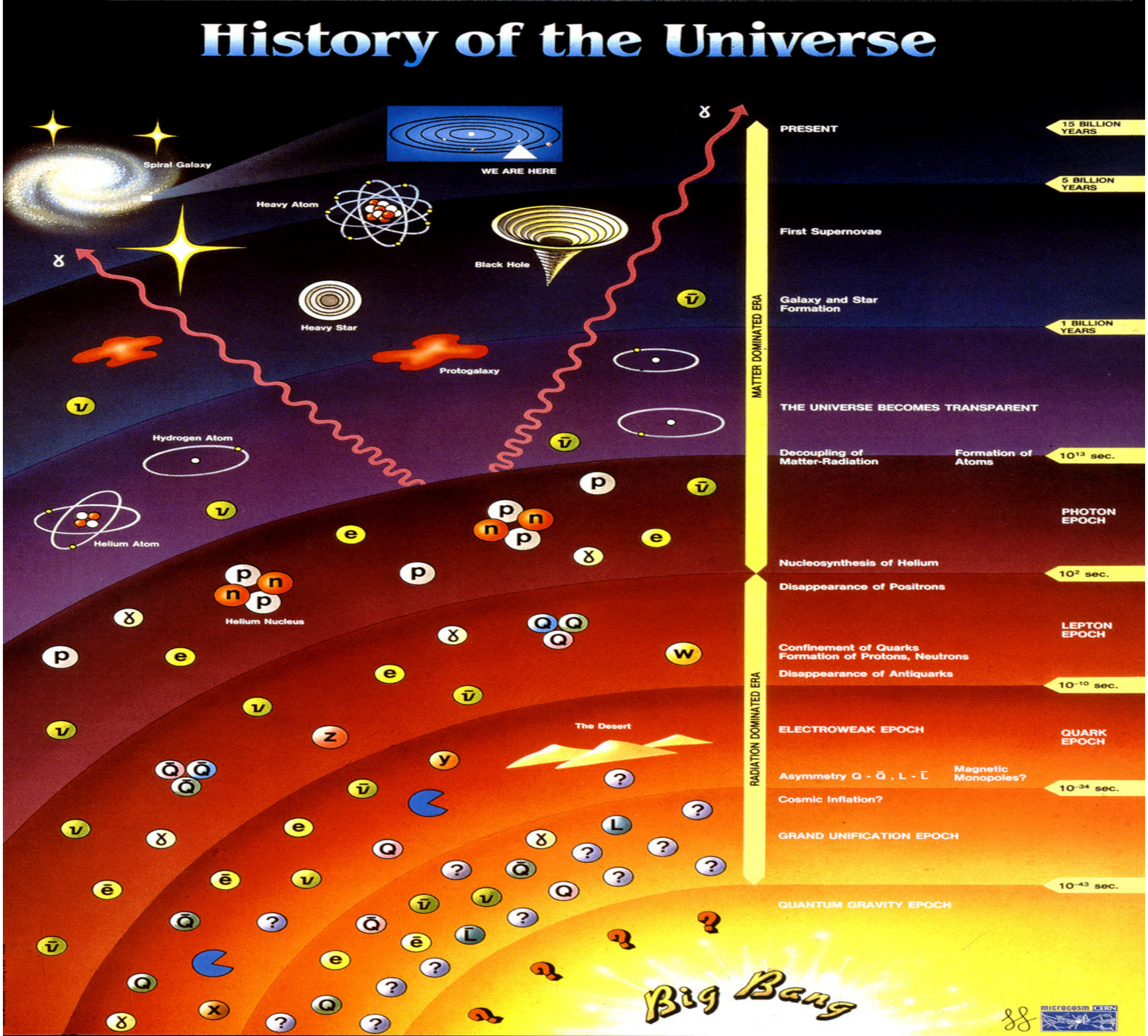
"Now the smallest Particles of Matter may cohere by the strongest Attractions and compose bigger Particles of weaker Virtue; and many of these may cohere and compose bigger Particles whose Virtue is still weaker, and so on for diverse successions, until the Progression ends in the biggest Particles on which the Operations in Chymistry and the Colours of natural Bodies depend, and which by cohering compose Bodies of a sensible Magnitude.

There are therefore Agents in Nature able to make the Particles of Bodies stick together by very strong Attractions. And it is the Business of Experimental Philosophy to find them out."

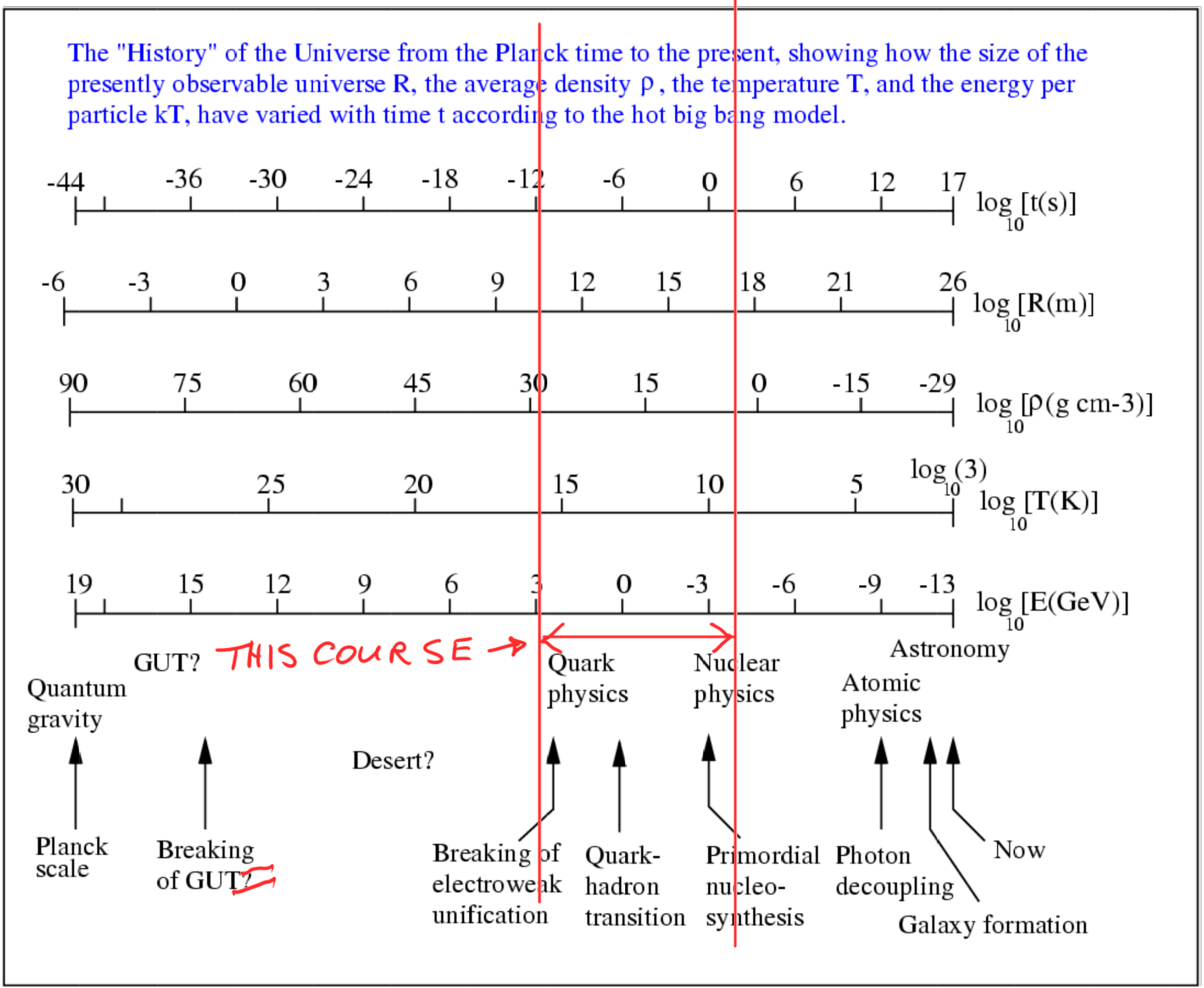
Isaac Newton, *Opticks*, 1704

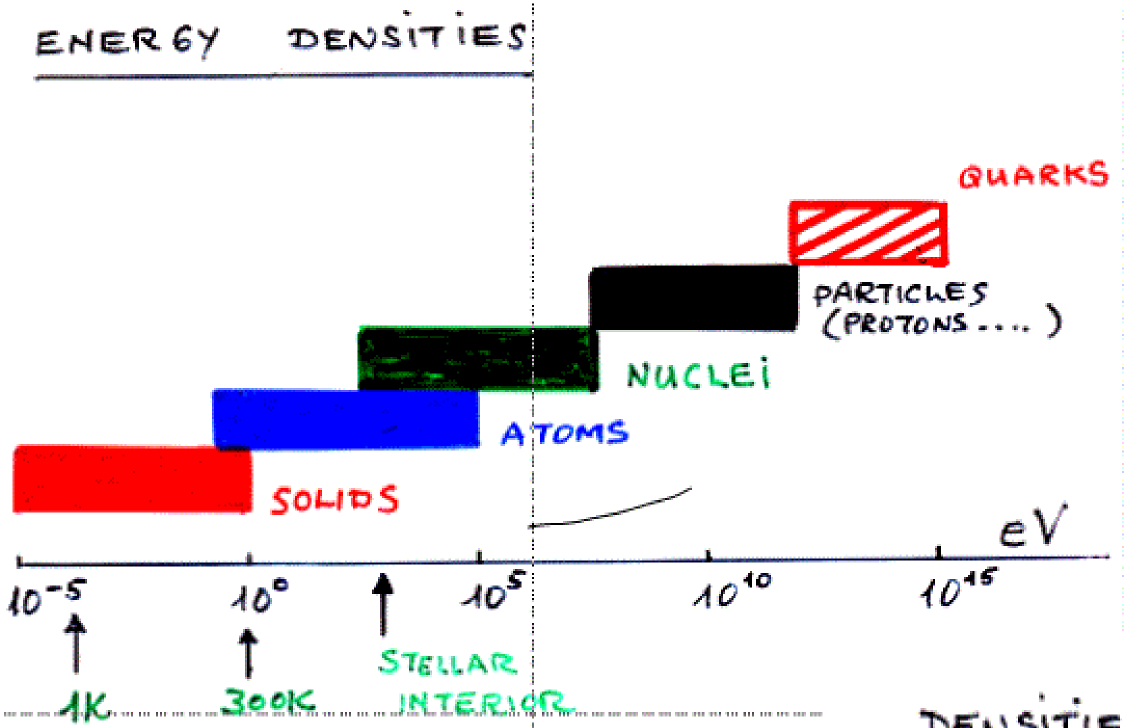
Is the Universe Made of These?

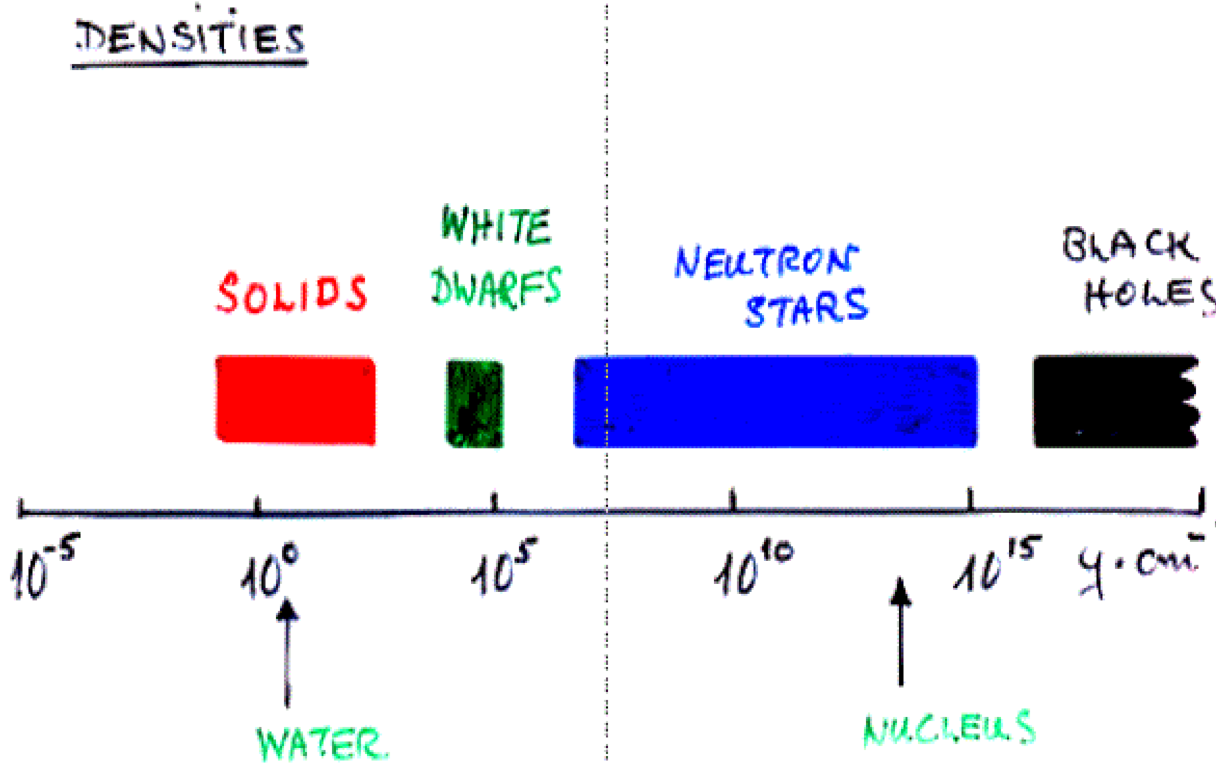
Proton = (u u d) – held together by gluons
Neutron = (u d d)


History of the Universe
PRESENT
15 BILLION YEARS
5 BILLION YEARS
First Supernovae
Galaxy and Star Formation
1 BILLION YEARS
THE UNIVERSE BECOMES TRANSPARENT
Decoupling of Matter-Radiation
Formation of Atoms
10^{13} sec.
PHOTON EPOCH
Nucleosynthesis of Helium
10^{2} sec.
Disappearance of Positrons
LEPTON EPOCH
Confinement of Quarks Formation of Protons, Neutrons
Disappearance of Antiquarks
10^{-10} sec.
ELECTROWEAK EPOCH
QUARK EPOCH
Asymmetry Q - $\bar{Q}$, L - $\bar{L}$
Magnetic Monopoles?
10^{-34} sec.
Cosmic Inflation?
GRAND UNIFICATION EPOCH
10^{-43} sec.
QUANTUM GRAVITY EPOCH
MATTER DOMINATED ERA
RADIATION DOMINATED ERA
Spiral Galaxy
WE ARE HERE
Heavy Atom
Black Hole
Heavy Star
Protogalaxy
Hydrogen Atom
Helium Atom
Helium Nucleus
The Desert
Big Bang
microcosm CERN

Space, Time and Energy Frontier




ENERGY DENSITIES
QUARKS
PARTICLES
(PROTONS ....)
NUCLEI
ATOMS
SOLIDS
eV
10^{-5}
10^{0}
10^{5}
10^{10}
10^{15}
1K
300K
STELLAR
INTERIOR


DENSITIES
SOLIDS
WHITE
DWARFS
NEUTRON
STARS
BLACK
HOLES
10^{-5}
10^{0}
10^{5}
10^{10}
10^{15}
$g \cdot cm^{-3}$
WATER
NUCLEUS

SPECIAL RELATIVITY

- TO REACH HIGH ENERGY DENSITIES AND SMALL DISTANCES USE PROBES $v \approx c$
- RESULTS OF MEASUREMENTS DEPEND ON:
  LORENTZ FRAME
- ANY SENSIBLE THEORY MUST BE:
  COVARIANT → LORENTZ INVARIANT
- BASED ON PROPERTIES / QUANTITIES THAT DO NOT DEPEND ON LORENTZ FRAME
  E.G. REST MASS
  PROPER LIFETIME
- FORMULATE IN TERMS OF 4-VECTORS

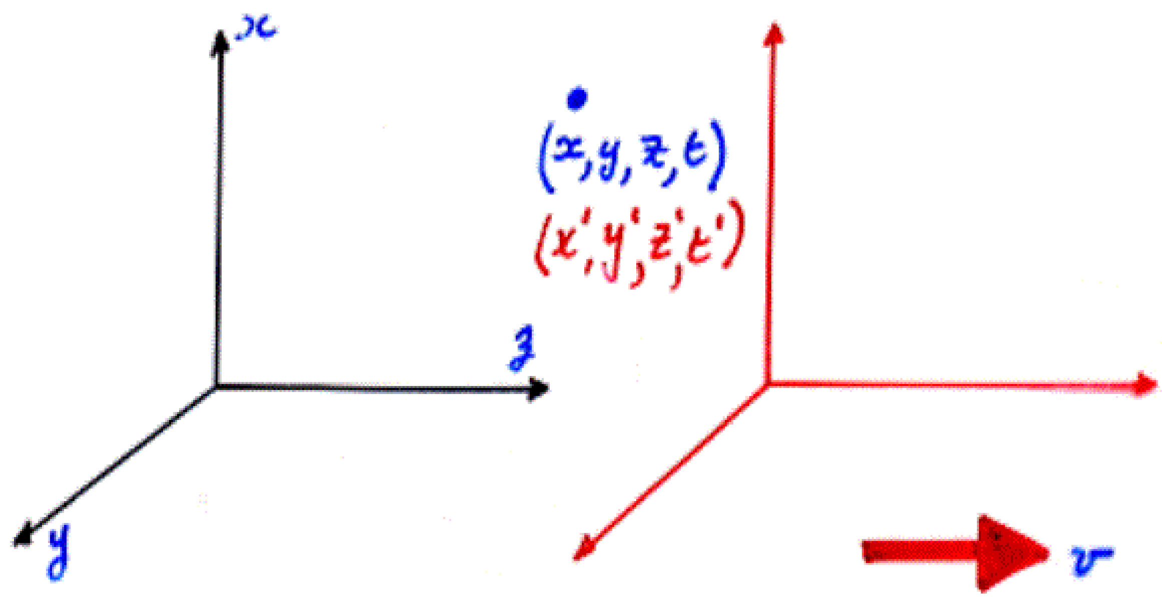
FIRST LET'S REMEMBER SIMPLE SPECIAL RELATIVITY

→ THEN MAKE IT A BIT MORE ELEGANT WITH 4-VECTORS.

SPECIAL RELATIVITY

ELECTRONS, PROTONS, NUCLEI HAVE ELECTRIC CHARGES.

MUCH OF WHAT WE TALK ABOUT WILL INVOLVE ELECTROMAGNETIC INTERACTION OF PARTICLE AND IT WILL BE RELATIVISTIC.



$$x' = x \;;\; y' = y$$

$$z' = \gamma(z - vt)$$

$$t' = \gamma\left(t - \frac{\beta}{c} \cdot z\right)$$

note mixing of z and t.

$$t' = \gamma \left(t - \beta/c \, z\right)$$

γ IS THE LORENTZ BOOST FACTOR

β IS VELOCITY IN UNITS WHERE $C=1$

$$\gamma = \frac{1}{(1-\beta^2)^{1/2}} \quad ; \quad \beta = v/c$$

$$\bar{p} = m \gamma \bar{v}$$

so $$v^2 = \frac{p^2}{m^2 \gamma^2} = \frac{p^2}{m^2}\left(1-\beta^2\right) = \frac{p^2}{m^2}\left(1 - v^2/c^2\right)$$

$$v^2 m^2 c^4 = c^4 p^2 - p^2 v^2 c^2$$

$$v^2 \big(\underbrace{m^2 c^4 + p^2 c^2}_{E^2}\big) = c^2 \cdot c^2 p^2$$

$$\frac{v^2}{c^2} E^2 = c^2 p^2 \longrightarrow \beta = \frac{cp}{E}$$

$$\boxed{\begin{array}{c} \beta \to 1 \\ cp \to E \end{array}}$$

$p \to E$ (UNITS $c=1$)

FOUR VECTORS

- 3 DIMENSIONAL VECTOR $\rightarrow$ (x, y, z) (p_x, p_y, p_z)
- VECTORS NOT FRAME INVARIANT
- SCALAR PRODUCTS ARE FRAME INVARIANT
  - $\hookrightarrow$ DISTANCE $(\bar{s} \cdot \bar{s}) \rightarrow$ FRAME INVARIANT
- CAN DEFINE 4-DIMENSIONAL VECTOR

get dimensions correct $\rightarrow$ $(ct, x, y, z) = (ct, \bar{x}) \rightarrow$ SPACE POINT

$(ct, \bar{x}) \cdot (ct, \bar{x}) \rightarrow$ INVARIANT INTERVAL

ANYTHING WHICH TRANSFORMS LIKE

$(ct, \bar{x})$

UNDER A LORENTZ TRANSFORM IS A

4-VECTOR

- 3 DIMENSIONS $\bar{x}$, $\bar{p}$ VECTORS

- 4 DIMENSIONS $(ct, \bar{x})$

$\left(\frac{E}{c}, \bar{p}\right)$ ENERGY MOMENTUM

$\left(\frac{E}{c}, \bar{p}\right) \cdot \left(\frac{E}{c}, \bar{p}\right) \rightarrow$ LORENTZ INVARIANT

$\rightarrow \; = \frac{E^2}{c^2} - p^2$ WHY?

KNOW $E^2 = p^2c^2 + m^2c^4 \rightarrow \frac{E^2}{c^2} - p^2 = m^2c^2$

$\rightarrow$ REST MASS

OBVIOUSLY UNITS WHERE $c=1$ CONVENIENT

$(E, \bar{p}) \cdot (E, \bar{p}) = E^2 - p^2 = m^2$

LORENTZ INVARIANT $\rightarrow$ SCALAR PRODUCT

4 - VECTOR DEFINITIONS

THE PHYSICS IS $E^2 = p^2c^2 + m^2c^4$ $[c=1]$

THIS IS WHAT DEFINES MULTIPLICATION RULE

$$(E, \bar{p}) \cdot (E, \bar{p}) = E^2 - p^2 = m^2$$

BETTER NOTATION MATHEMATICALLY

$$A \cdot B = g_{\mu\nu} A^\mu B^\nu = g^{\mu\nu} A_\mu B_\nu = A^\mu B_\mu$$

$\mu = 0, 1, 2, 3$

CONTRAVARIANT $A^\mu = (A^0, \vec{A})$

COVARIANT $A_\mu = (A^0, -\vec{A})$

METRIC

$$g_{\mu\nu} = \begin{pmatrix} +1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix}$$

LORENTZ INVARIANT $\rightarrow$ UPPER & LOWER INDICES BALANCE

$$(E, \bar{p}) = p^\mu = p$$

$$p \cdot p = p^\mu p_\mu = E^2 - p^2 = m^2$$

MORE ON 4-VECTORS

FIRST OF ALL HAVE TO UNDERSTAND

SUMMATION CONVENTION

$$A^{\mu} B_{\mu} = \sum_{\mu=0}^{3} A^{\mu} B_{\mu}$$

REPEATED INDICES SUMMED OVER

$$g_{\mu\nu} = \begin{pmatrix} +1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix}$$

SO $g^{\mu\alpha} x_{\alpha} = x^{\mu}$

$$\begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} = \begin{pmatrix} +1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$x^0 = g^{00} x_0 + \cancel{g^{01} x_1} + \cancel{g^{02} x_2} + \cancel{g^{03} x_3} = x_0$$

$$x^1 = \cancel{g^{10} x_0} + g^{11} x_1 + \cancel{g^{12} x_2} + \cancel{g^{13} x_3} = -x_1$$

IF WE SAY $x_\mu = (x_0, x_1, x_2, x_3)$

$$x^\mu = (x_0, -x_1, -x_2, -x_3)$$

$$x^\mu x_\mu = (\quad)(\quad) = x_0^2 + x_1^2 + x_2^2 + x_3^2$$

$$g_{\mu\nu} A^\nu B^\mu = g^{\mu\nu} A_\nu B_\mu = A^\mu B_\mu$$

$$= A^0 B_0 - A^1 B_1 - A^2 B_2 - A^3 B_3 = A \cdot B$$

MINKOWSKI NOTATION

SOMETIMES SEE FOLLOWING NOTATION (e.g. PERKINS)

3 REAL "SPACE" COMPONENTS

1 IMAGINARY "TIME" COMPONENT

$$p_x \quad p_y \quad p_z \quad E$$

$$\mu = 1, 2, 3, 4$$

$$p_1 = p_x, \quad p_2 = p_y, \quad p_3 = p_z, \quad p_4 = i E$$

$$p = (\bar{p}, i E)$$

PHYSICS IS THE SAME

$$E^2 = p^2 + m^2$$

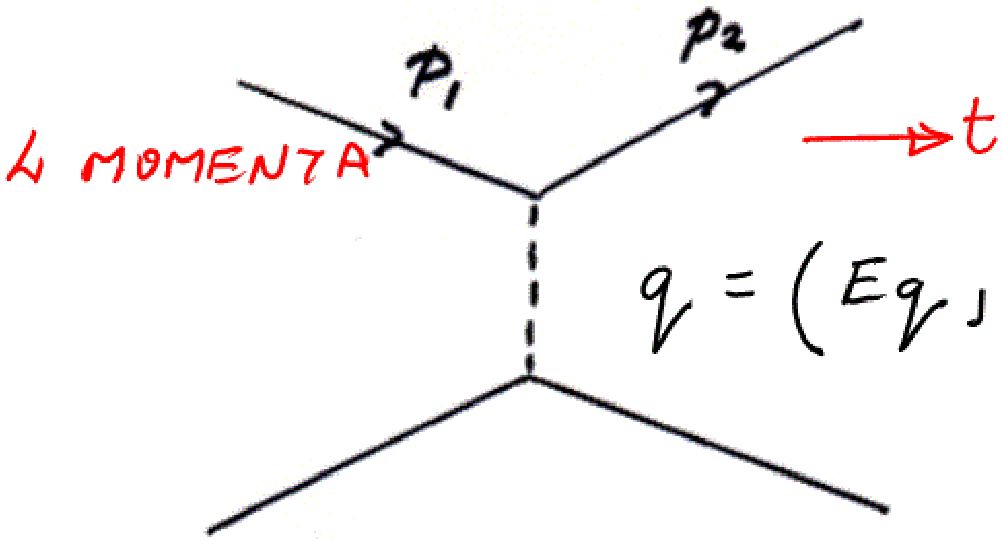
$$p^2 = \sum_\mu p_\mu^2 = p_1^2 + p_2^2 + p_3^2 + p_2^4$$

$$= \bar{p}^2 - E^2 = -m^2$$

$$p^2 = -m^2$$

JUST DEFINITION OF p^2

FOR A SCATTERING PROCESS



$$q = (E_q, \bar{p}_q)$$

EXCERCISE

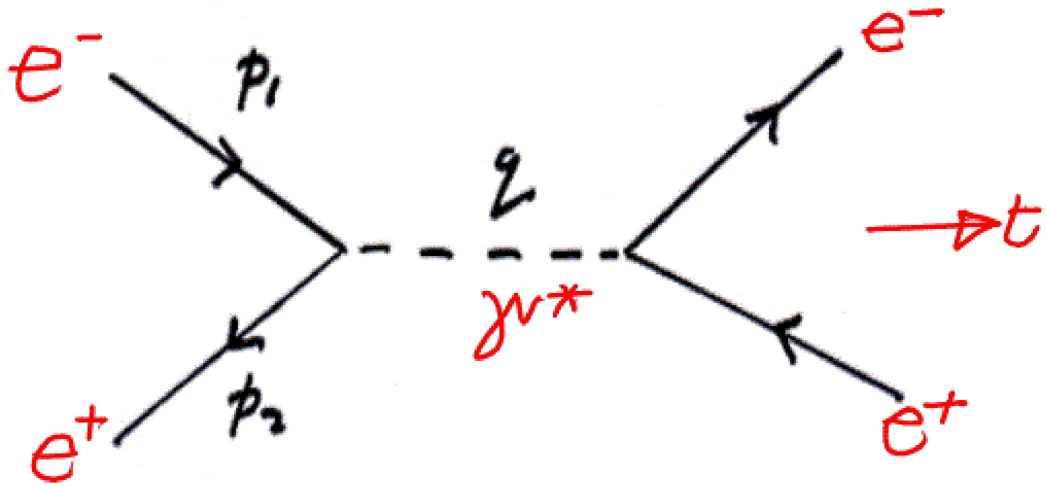
$$q^2 = (p_1 - p_2)^2 = -ve$$

$$= E_q^2 - \underline{\bar{p}_q^2}$$

SPACE LIKE

SPACE COMPONENT

FOR AN ANNIHILATION PROCESS



$$q^2 = (p_1 + p_2)^2 = +ve$$

$$= E_q^2 - p_q^2 = m_{\gamma^*}^2$$

"TIME" COMPONENT

TIME LIKE

SIGNS REVERSED FOR MINKOWSKI

LORENTZ TRANSFORMATION

$$p'_\mu = \sum \alpha_{\mu\nu}\, p_\nu$$

FOR A BOOST ALONG x-AXIS OF β

MINKOWSKI

$$\begin{pmatrix} p'_x \\ p'_y \\ p'_z \\ iE' \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & i\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\beta\gamma & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} p_x \\ p_y \\ p_z \\ iE \end{pmatrix}$$

"MODERN" METRIC

$$\begin{pmatrix} E' \\ p'_x \\ p'_y \\ p'_z \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix}$$

$$E' = \gamma(E - \beta p_x)$$

$$p_x = \gamma(p_x - \beta E)$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

C = 1 ON THIS PAGE