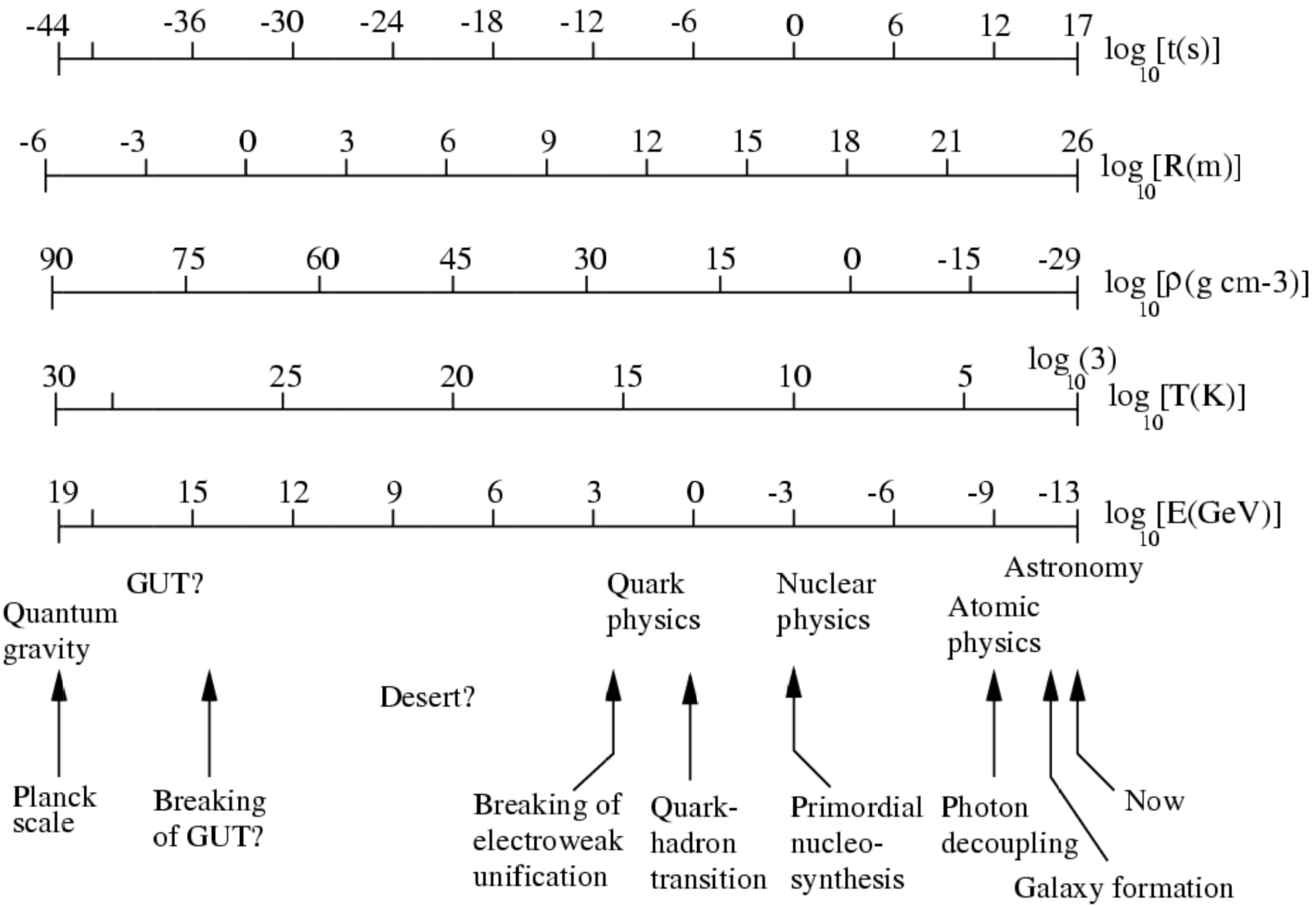
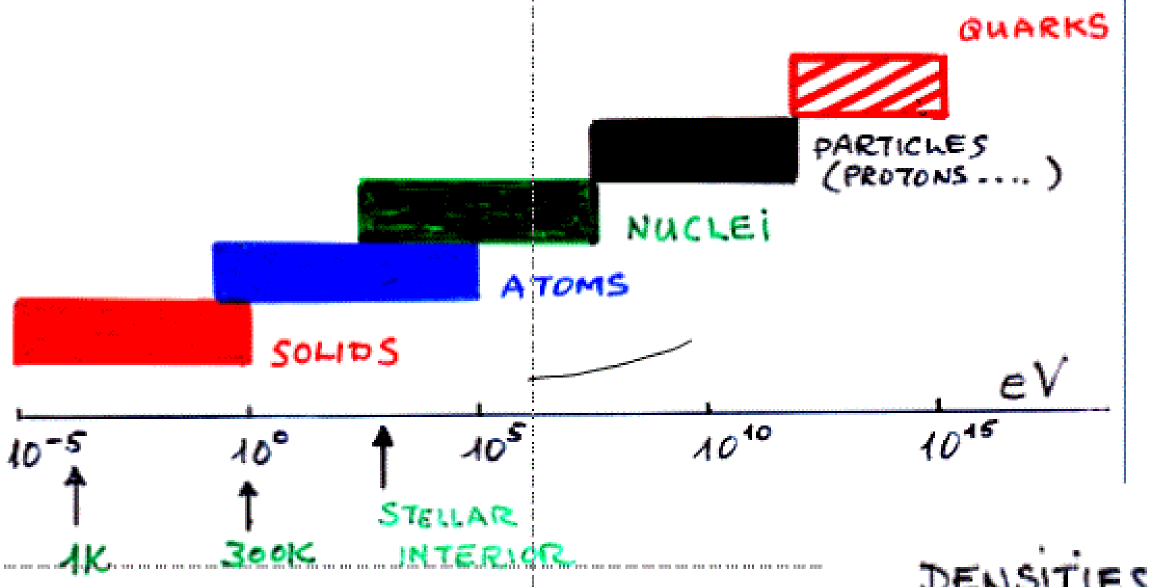
Space, Time and Energy Frontier

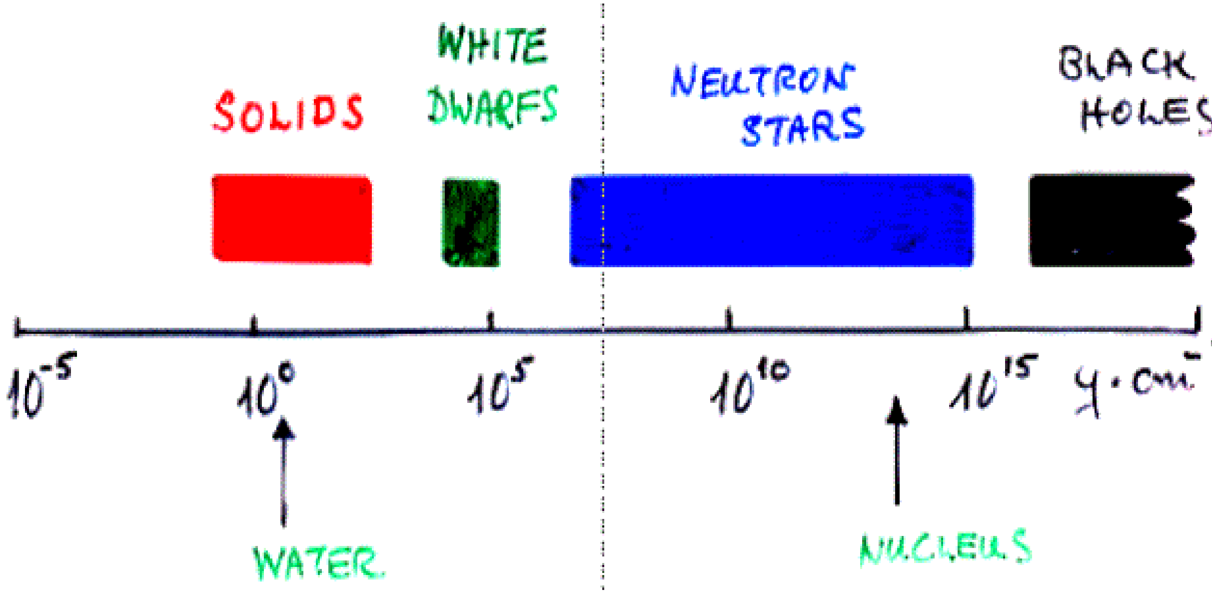
The "History" of the Universe from the Planck time to the present, showing how the size of the presently observable universe R, the average density ρ, the temperature T, and the energy per particle kT, have varied with time t according to the hot big bang model.



ENERGY DENSITIES



DENSITIES



SPECIAL RELATIVITY

- TO REACH HIGH ENERGY DENSITIES AND SMALL DISTANCES USE PROBES $v \approx c$
- RESULTS OF MEASUREMENTS DEPEND ON !

  LORENTZ FRAME
- ANY SENSIBLE THEORY MUST BE !

  COVARIANT → LORENTZ INVARIANT
- BASED ON PROPERTIES / QUANTITIES THAT DO NOT DEPEND ON LORENTZ FRAME

  E.G. REST MASS

  PROPER LIFETIME
- FORMULATE IN TERMS OF 4-VECTORS

FIRST LET'S REMEMBER SIMPLE SPECIAL RELATIVITY

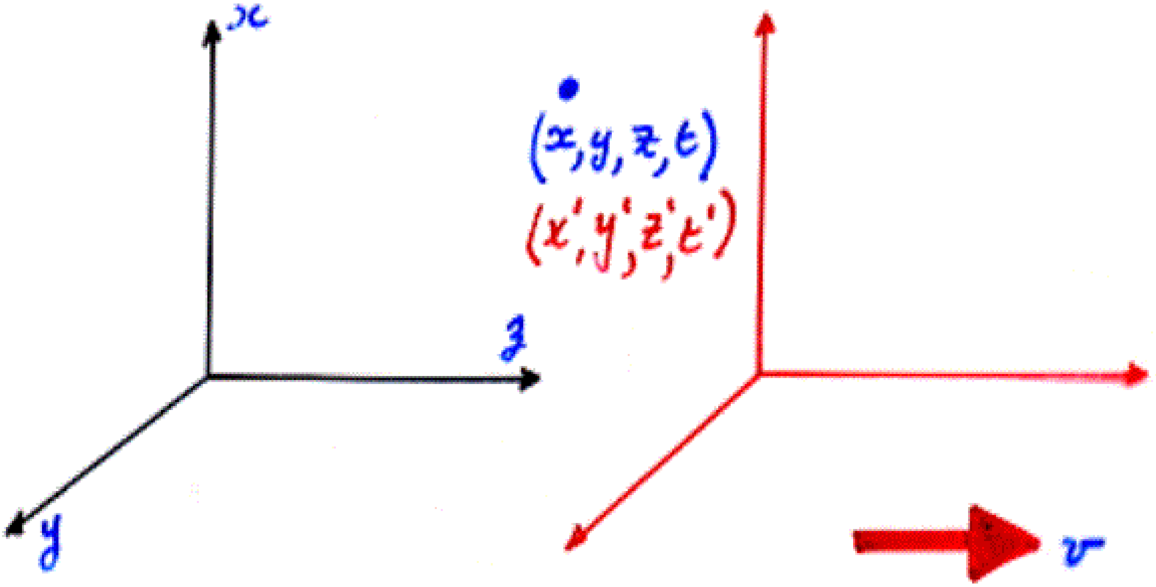
→ THEN MAKE IT A BIT MORE ELEGANT

WITH 4-VECTORS.

SPECIAL RELATIVITY

ELECTRONS, PROTONS, NUCLEI HAVE ELECTRIC CHARGES.

MUCH OF WHAT WE TALK ABOUT WILL INVOLVE ELECTRO MAGNETIC INTERACTION OF PARTICLE AND IT WILL BE RELATIVISTIC.



$$x' = x \; ; \; y' = y$$

$$z' = \gamma(z - vt)$$

$$t' = \gamma\left(t - \frac{\beta}{c} \cdot z\right)$$

note mixing of z and t.

$$t' = \gamma\left(t - \beta/c\, z\right)$$

γ IS THE LORENTZ BOOST FACTOR

β IS VELOCITY IN UNITS WHERE $C=1$

$$\gamma = \frac{1}{(1-\beta^2)^{1/2}} \quad ; \quad \beta = v/c$$

$$\bar{p} = m\gamma\bar{v}$$

so $$v^2 = \frac{p^2}{m^2\gamma^2} = \frac{p^2}{m^2}\left(1-\beta^2\right) = \frac{p^2}{m^2}\left(1 - v^2/c^2\right)$$

$$v^2 m^2 c^4 = c^4 p^2 - p^2 v^2 c^2$$

$$v^2 \left(\underbrace{m^2 c^4 + p^2 c^2}_{E^2}\right) = c^2 \cdot c^2 p^2$$

$$\frac{v^2}{c^2} E^2 = c^2 p^2 \longrightarrow \beta = \frac{cp}{E}$$

$\beta \to 1$

$cp \to E$

$p \to E$ (UNITS $C=1$)

FOUR VECTORS

- 3 DIMENSIONAL VECTOR $\longrightarrow$ (x, y, z) (p_x, p_y, p_z)
- VECTORS NOT FRAME INVARIANT
- SCALAR PRODUCTS ARE FRAME INVARIANT
  - $\hookrightarrow$ DISTANCE $(\bar{s} \cdot \bar{s}) \longrightarrow$ FRAME INVARIANT

- CAN DEFINE 4-DIMENSIONAL VECTOR

get dimensions correct

$$(ct, x, y, z) = (ct, \bar{x}) \longrightarrow \text{SPACE POINT}$$

$$(ct, \bar{x}) \cdot (ct, \bar{x}) \longrightarrow \text{INVARIANT INTERVAL}$$

ANYTHING WHICH TRANSFORMS LIKE

$$(ct, \bar{x})$$

UNDER A LORENTZ TRANSFORM IS A

4-VECTOR

- 3 DIMENSIONS $\left.\begin{matrix}\bar{x}\\ \bar{p}\end{matrix}\right\}$ VECTORS

- 4 DIMENSIONS $(ct, \bar{x})$

$$\left(\frac{E}{c}, \bar{p}\right)$$

ENERGY MOMENTUM

$$\left(\frac{E}{c}, \bar{p}\right)\cdot\left(\frac{E}{c}, \bar{p}\right) \to$$ LORENTZ INVARIANT

$$\to \; = \frac{E^2}{c^2} - p^2$$

WHY?

KNOW $E^2 = p^2c^2 + m^2c^4 \to \frac{E^2}{c^2} - p^2 = m^2c^2$

→ REST MASS

OBVIOUSLY UNITS WHERE $C=1$ CONVENIENT

$$(E, \bar{p})\cdot(E, \bar{p}) = E^2 - p^2 = m^2$$

LORENTZ INVARIANT → SCALAR PRODUCT

4-VECTOR DEFINITIONS

THE PHYSICS IS $E^2 = p^2 c^2 + m^2 c^4$ $[c = 1]$

THIS IS WHAT DEFINES MULTIPLICATION RULE

$$(E, \bar{p}) \cdot (E, \bar{p}) = E^2 - p^2 = m^2$$

BETTER NOTATION MATHEMATICALLY

$$A \cdot B = g_{\mu\nu} A^\mu B^\nu = g^{\mu\nu} A_\mu B_\nu = A^\mu B_\mu$$

$\mu = 0, 1, 2, 3$

CONTRAVARIANT $A^\mu = (A^0, \vec{A})$

COVARIANT $A_\mu = (A^0, -\vec{A})$

METRIC

$$g_{\mu\nu} = \begin{pmatrix} +1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix}$$

LORENTZ INVARIANT → UPPER & LOWER INDICES BALANCE

$$(E, \bar{p}) = p^\mu = p$$

$$p \cdot p = p^\mu p_\mu = E^2 - p^2 = m^2$$

MINKOWSKI NOTATION

SOMETIMES SEE FOLLOWING NOTATION (e.g. PERKINS)

3 REAL "SPACE" COMPONENTS

1 IMAGINARY "TIME" COMPONENT

$$p_x \quad p_y \quad p_z \quad E$$

$$\mu = 1, 2, 3, 4$$

$$p_1 = p_x, \quad p_2 = p_y, \quad p_3 = p_z, \quad p_4 = i E$$

$$p = (\bar{p}, i E)$$

$$p^2 = \sum_\mu p_\mu^2 = p_1^2 + p_2^2 + p_3^2 + p^4_2$$

$$= \bar{p}^2 - E^2 = -m^2$$

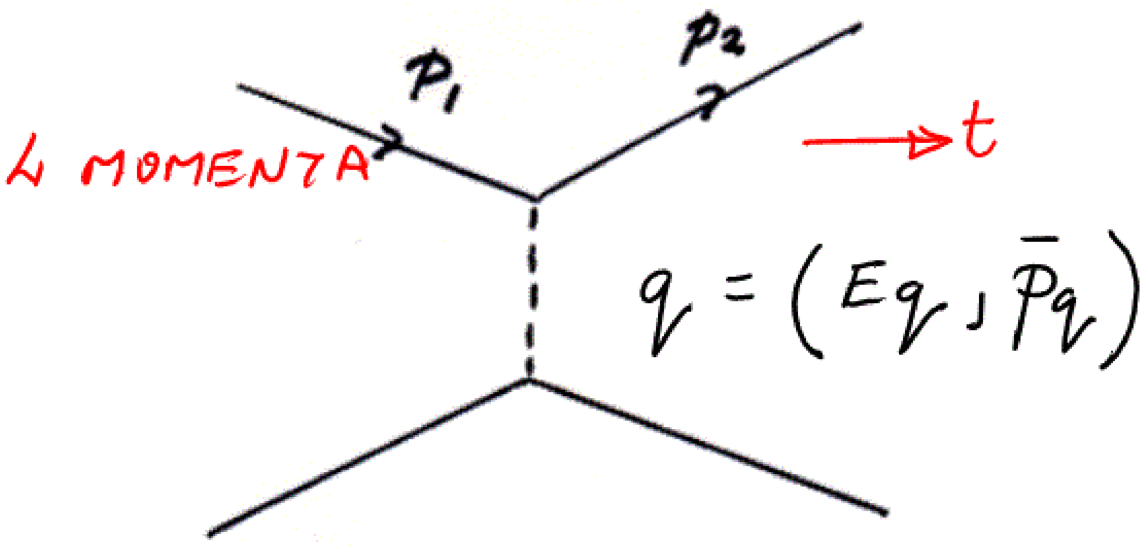
PHYSICS IS THE SAME

$$E^2 = p^2 + m^2$$

$$p^2 = -m^2$$

JUST DEFINITION OF p^2

FOR A SCATTERING PROCESS



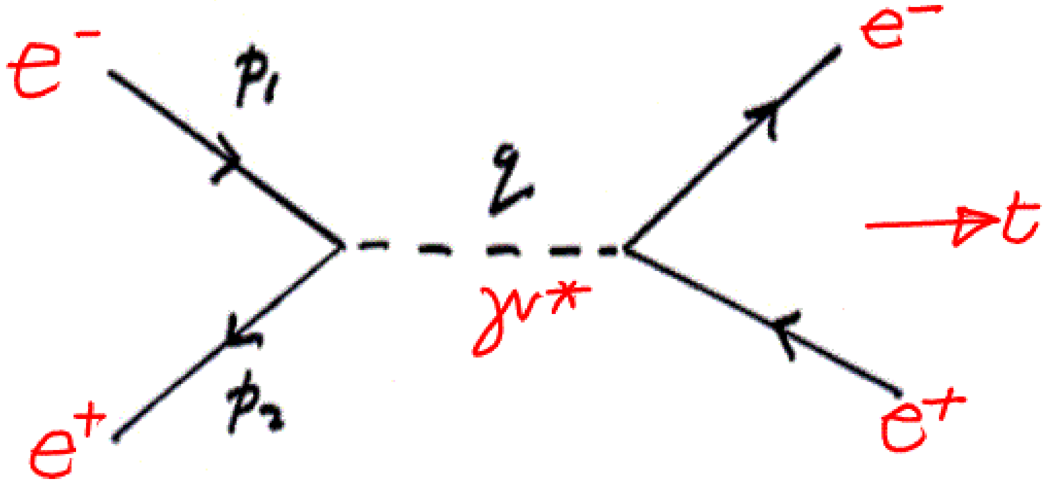
$$q = (E_q , \bar{p}_q)$$

$$q^2 = (p_1 - p_2)^2 = \overset{\text{EXCERCISE}}{-ve}$$

$$= E_q^2 - \underset{\uparrow \ \text{SPACE COMPONENT}}{\underline{\bar{p}_q}}{}^2$$

SPACE LIKE

FOR AN ANNIHILATION PROCESS



$$q^2 = (p_1 + p_2)^2 = +ve$$

$$= \underset{\text{"TIME" COMPONENT}}{E_q^2} - p_q^2 = m_{\gamma^*}^2$$

TIME LIKE

SIGNS REVERSED FOR MINKOWSKI

LORENTZ TRANSFORMATION

$$p'_\mu = \sum \alpha_{\mu\nu} \, p_\nu$$

FOR A BOOST ALONG x-AXIS OF β

MINKOWSKI

$$\begin{pmatrix} p'_x \\ p'_y \\ p'_z \\ iE' \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & i\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\beta\gamma & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} p_x \\ p_y \\ p_z \\ iE \end{pmatrix}$$

"MODERN" METRIC

$$\begin{pmatrix} E' \\ p'_x \\ p'_y \\ p'_z \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix}$$

$$E' = \gamma(E - \beta p_x)$$

$$p_x = \gamma(p_x - \beta E)$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

C = 1 ON THIS PAGE

UNITS IN SUBATOMIC PHYSICS

- PHYSICS & TECHNOLOGY → SI UNITS USED
- RELATIVISTIC PHYSICS CGS / GAUSSIAN

  NATURAL SYSTEM → NO "$4\pi\varepsilon_0$" IN EM

LENGTH	METER	m
TIME	SECOND	s
ENERGY	ELECTRON VOLT	eV
MASS	"	eV/c²
MOMENTUM	"	eV/c

- SCALE OF COLOR FORCE
- SCALE OF ELECTROWEAK FORCE

10^6 eV = MeV — BINDING ENERGY OF NUCLEI

10^9 eV = GeV — MASS ENERGY OF PROTON

10^{12} eV = TeV — MASS ENERGY OF HIGGS BOSON

- 10^{-15} m = FEMTOMETER

  = 1 FERMI

  = DIAMETER OF PROTON

- TIME FOR LIGHT SIGNAL TO CROSS PROTON

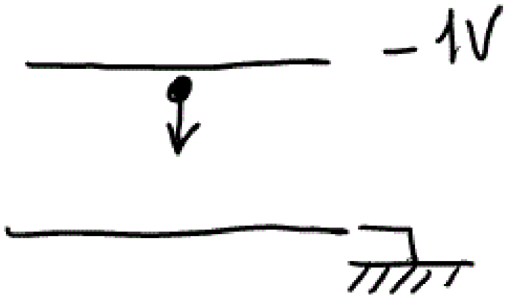
$$\approx \frac{10^{-15}\,[m]\,[s]}{3\times 10^{8}\,[m]} \approx 10^{-23}\,\text{sec}$$

- TIME SCALE OF STRONG INTERACTION

WHY USE ELECTRON VOLTS?

- CONVENIENT ENERGY UNIT
- SUBATOMIC PHYSICS EXPERIMENTS DONE BY ACCELERATING BEAMS OF PARTICLES IN ELECTRIC FIELDS, AND SCATTERING OFF TARGET

- ONE ELECTRON VOLT (eV) IS THE KINETIC ENERGY AN ELECTRON GAINS BY ACCELERATING THRU A POTENTIAL OF ONE VOLT



$$1\,\text{eV} = e \times 1\,\text{VOLT}$$

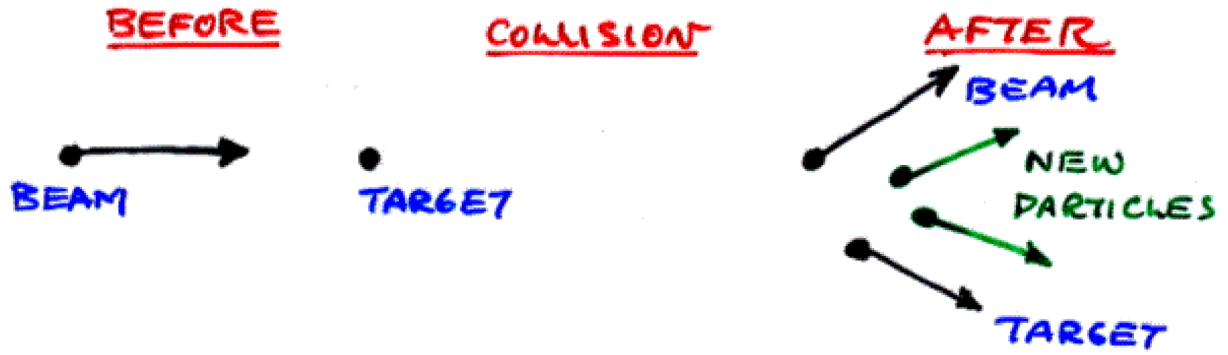
$$= 1.60\times10^{-19}(\text{COULOMB})\times 1\,\text{VOLT}$$

$$= 1.60 \times 10^{-19}\ \text{JOULES}$$

$$= 1.60 \times 10^{-12}\ \text{ERGS}$$

WHY ELECTRON VOLTS FOR MASSES ?

WHEN A BEAM OF RELATIVISTIC PARTICLES SCATTERS FROM A TARGET, SOME OF THE KINETIC ENERGY CAN APPEAR AS MASS → NEW PARTICLES



- CONVENIENT TO MEASURE MASS IN SAME UNITS AS ENERGY

$$E^2 = p^2 c^2 + m^2 c^4$$

TOTAL RELATIVISTIC ENERGY

MOMENTUM

VELOCITY OF LIGHT

MASS SOMETIMES CALLED "REST MASS"

$$E^2 = p^2 c^2 + m^2 c^4$$

- FOR A PARTICLE WITH NO MASS

PHOTON — γ
NEUTRINO — ν (?)

$$E = p \cdot c \longrightarrow p = \frac{E}{c} = \left.\frac{[eV]}{c}\right\}$$

UNIT OF MOMENTUM

MASSLESS PARTICLE $E = 1eV \longrightarrow p = 1eV/c$

- FOR A MASSIVE PARTICLE AT REST — IN ITS REST FRAME

$$E = mc^2 \longrightarrow m = \frac{E}{c^2} = \left.\frac{[eV]}{c^2}\right\}$$

UNIT OF MASS

PARTICLE OF MASS $1eV/c^2$ AT REST
HAS A TOTAL ENERGY OF $1eV$

$$E^2 = p^2c^2 + m^2c^4 \quad \text{———} \quad ①$$

$$E^2 = p^2 + m^2 \qquad [\text{WHERE } C = 1]$$

EACH TERM OF ① HAS DIMENSIONS $[E]^2$

EXAMPLE – PARTICLE MASS = $1000 \frac{MeV}{c^2}$, $p = 1000 \frac{MeV}{c}$

$$E^2 = 10^6 \frac{MeV^2}{c^2} \cdot c^2 + 10^6 \frac{MeV^2}{c^4} \cdot c^4$$

$$E^2 = 2 \times 10^6 \; MeV^2$$

THE PARTICLE HAS A TOTAL RELATIVISTIC ENERGY

$$E = \sqrt{2} \times 10^3 \; MeV$$

Table 1.1. *Units in high energy physics*

(a)

Quantity	High energy unit	Value in SI units
length	1 fm	10^{-15} m
energy	1 GeV = 10^9 eV	1.602×10^{-10} J
mass, E/c^2	1 GeV/c^2	1.78×10^{-27} kg
$\hbar = h/(2\pi)$	6.588×10^{-25} GeV s	1.055×10^{-34} J s
c	2.998×10^{23} fm s^{-1}	2.998×10^8 m s^{-1}
$\hbar c$	0.1975 GeV fm	3.162×10^{-26} J m

PROTON MASS

(b)

natural units, $\hbar = c = 1$	
mass, Mc^2/c^2	1 GeV
length, $\hbar c/(Mc^2)$	1 GeV^{-1} = 0.1975 fm
time, $\hbar c/(Mc^3)$	1 GeV^{-1} = 6.59×10^{-25} s
Heaviside–Lorentz units, $\epsilon_0 = \mu_0 = \hbar = c = 1$	
fine structure constant	$\alpha = e^2/(4\pi) = 1/137.06$

Relations between energy units

1 MeV = 10^6 eV	1 GeV = 10^3 MeV	1 TeV = 10^3 GeV

$\hbar c = 1 = 0.1975$ GeV. fm

$\hbar = 1 = 6.59 \times 10^{-25}$ GeV.S

CHOOSE $\hbar c = 1 \left[\begin{matrix}\text{NATURAL}\\ \text{UNITS}\end{matrix}\right] = 0.1975\,[\text{GeV}\cdot\text{fm}]$

CAN CONVERT BETWEEN UNITS

LENGTH

$[\text{GeV}\cdot\text{fm}]$

$\left[\frac{\text{GeV}}{c^2}\cdot c^2\right]$

$$\frac{\hbar c}{mc^2} = \frac{\hbar c}{\text{GeV}}[L] = 0.1975\left[\frac{\text{GeV}\cdot\text{fm}}{\text{GeV}}\right]$$

$$\boxed{1\,\text{GeV}^{-1} = 0.1975\,\text{fm}}$$

TIME

$$\frac{\hbar c}{mc^3}[T] = \frac{\hbar c}{\text{GeV}} = 6.588\times10^{-25}\left[\frac{\text{GeV}\cdot\text{s}}{\text{GeV}}\right]$$

$$\boxed{1\,\text{GeV}^{-1} = 6.588\times10^{-25}\,\text{s}}$$

$$\boxed{\hbar = c = 1}$$

RANDOMNESS OF DECAYS

- QUANTUM MECHANICS → IN AN ENSEMBLE OF UNSTABLE PARTICLES, ANY ONE MAY DECAY AT RANDOM IN A SMALL TIME INTERVAL.
- EACH PARTICLE DECAYS AFTER A RANDOM TIME → ENSEMBLE CHARACTERIZED BY MEAN LIFETIME
- NUMBER DECAYING IN TIME INTERVAL dt:

$$dN = -\omega N(t)\, dt$$

TIME INTERVAL

DECAYING

PROBABILITY PER UNIT TIME FOR DECAY → DECAY CONSTANT TRANSITION RATE

UNDECAYED AT START OF TIME INTERVAL

$$\frac{dN}{N(t)} = -\omega\, dt$$

$$dN = -\omega\, N(t)\, dt \rightarrow \frac{dN}{N} = -\omega\, dt$$

$$\int_{N(0)}^{N(t)} dN/N = -\omega \int_0^t dt \rightarrow \ln N(t) - \ln N(0) = -\omega t$$

$$\boxed{N(t) = N(0)\, e^{-\omega t}}$$

SURVIVAL EQUATION

INTENSITY OF RADIATION = ACTIVITY

$$I(t) = \frac{-dN(t)}{dt} = \underbrace{N(0)\,\omega}\, e^{-\omega t}$$

$$\boxed{I(t) = I(0)\, e^{-\omega t}}$$

INTENSITY OF EMITTED RADIATION

INITIAL ACTIVITY OR INTENSITY

UNITS OF RADIOACTIVITY

- CURIE (Ci) AMOUNT OF RADIOACTIVE MATERIAL IN WHICH NUMBER OF DISINTEGRATIONS PER SECOND = 1g OF RADIUM

$3 \cdot 7 \times 10^{10} \, s^{-1}$

- BECQUEREL (Bq) ONE DISINTEGRATION PER SECOND

STARTED WHOLE SUBJECT BY DISCOVERING RADIOACTIVITY IN 1896

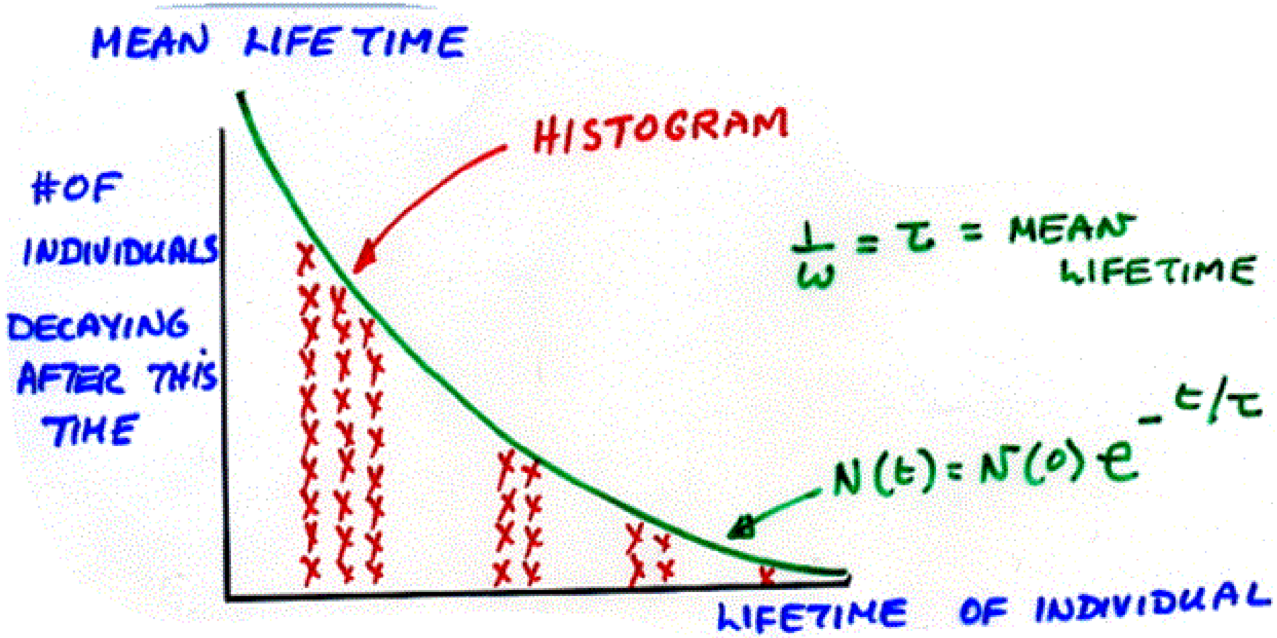
? HOW MUCH RADIOACTIVITY DID CHERNOBYL RELEASE

1000'S OF CURIES !

— WHAT ABOUT FUKUSHIMA → FIND OUT!

http://en.wikipedia.org/wiki/Comparison_of_Fukushima_and_Chernobyl_nuclear_accidents

MEAN LIFETIME



- INDIVIDUAL DECAY RANDOM
- POPULATION CHARACTERIZED BY MEAN LIFETIME

ENORMOUS RANGE OF LIFETIMES

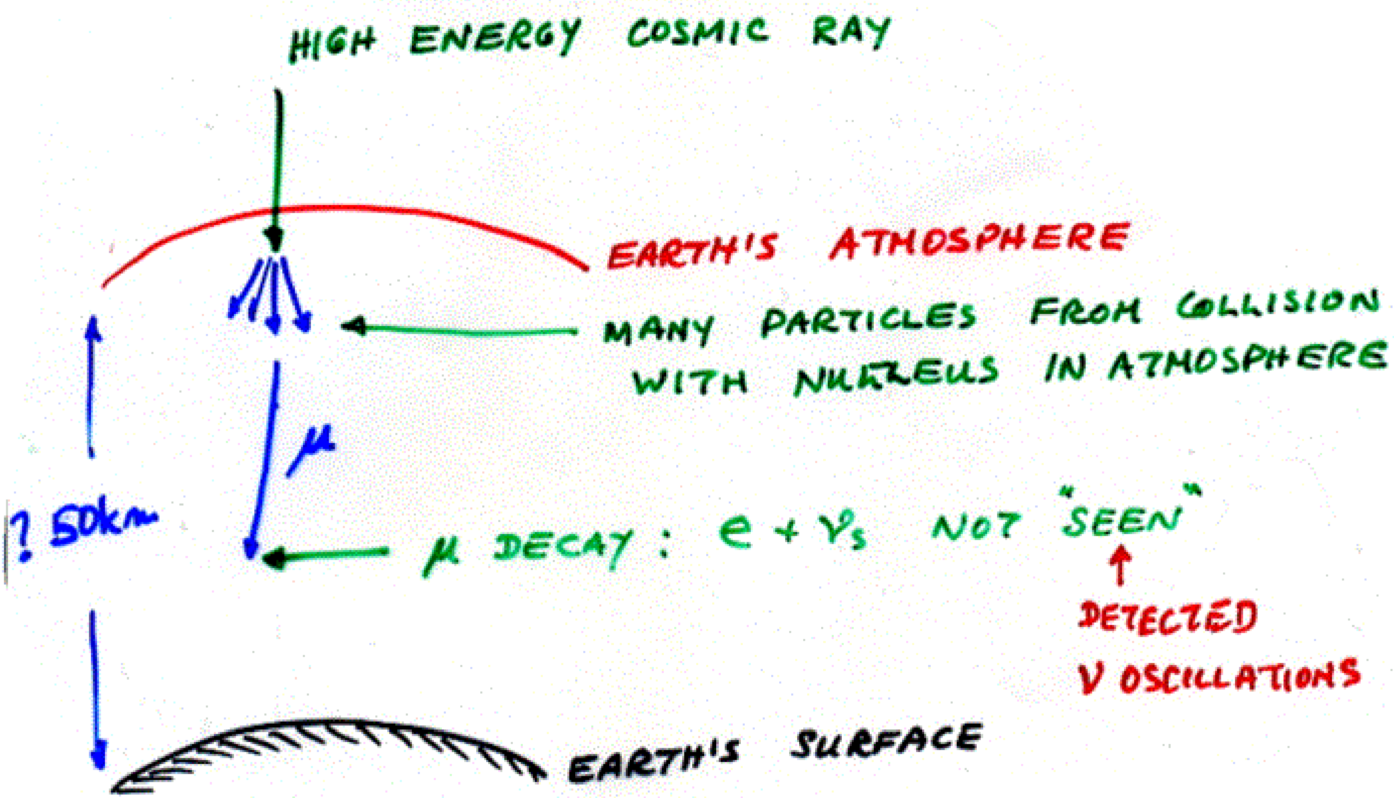
PROTON DECAY	$> 10^{33}$ YEARS
$^{238}U \rightarrow {}^{234}Th + \alpha$	6.5×10^{9} YEARS
$^{215}Po \rightarrow {}^{211}Pb + \alpha$	2.0×10^{3} s
$\mu^{+} \rightarrow e^{+} \nu_e \bar{\nu}_\mu$	2.2×10^{-6} s
$\pi^{0} \rightarrow \gamma\gamma$	8.3×10^{-17} s
$\Delta^{++} \rightarrow p\pi^{+}$	6×10^{-24} s

DECAY RATE GOVERNED BY FORCE (INTERACTION) CAUSING THE DECAY PROCESS

SPECIAL RELATIVITY & LIFETIME

LIFETIME ONLY MAKES SENSE IN SPECIAL FRAME

- REST FRAME OF PARTICLE



eg $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$

$\tau = 2{\cdot}2 \times 10^{-6} s$

- ASSUME μ TRAVELLING AT SPEED OF LIGHT

MEAN DISTANCE TO DECAY: $c\tau$

$$= 3 \times 10^8 \times 2 \times 10^{-6} \frac{[m][s]}{[s]}$$

$$\approx 600 \, m.$$

- TIME TO REACH EARTH'S SURFACE FROM 50km

$$t_{50} = \frac{50\times 10^{3}}{3\times 108} = \frac{50}{3}\times 10^{-5} \approx 2\times 10^{-4}\,s$$

- HOW MANY UNDECAYED AFTER THIS TIME?

$$\text{FRACTION SURVIVING} = \frac{N(t_{50})}{N(0)} = e^{-t_{50}/\tau_{LAB}}$$

$$= \exp -\left\{\frac{2\times 10^{-4}}{2\times 10^{-6}}\right\} \sim 10^{-44} \sim 0$$

- WHY DO WE SEE ANY MUONS AT EARTH'S SURFACE?

$$\tau_{LAB} \neq \tau_{\mu} \Rightarrow \quad \tau^{REST} = t_2^{REST} - t_1^{REST}$$

DEFINES τ IN REST FRAME

SO!
$$\tau_{LAB} = \gamma t_2^{REST} - \gamma t_1^{REST}$$

$$\tau_{LAB} = \gamma \tau_{\mu}^{REST}$$

THAT'S MORE SENSIBLE!

ASSUME μ HAVE ENERGY OF 100 GeV

$$\boxed{E = pc/\beta} \quad \text{AND} \quad \gamma^2 = \frac{1}{1-\beta^2}$$

$$\gamma^2 = \left(1-\frac{p^2c^2}{E^2}\right)^{-1} = E^2/(E^2-p^2c^2)$$

$$\therefore \quad \gamma^{\mu} = E/mc^2$$

MASS OF μ IS $m_\mu = 106$ MeV$/c^2$

$$\gamma^{\mu} = \frac{10^5\,[\text{MeV}]}{10^2\,\frac{[\text{MeV}]}{[c^2]}\cdot c^2} \approx 10^3 \longrightarrow \tau_{LAB} = 10^3\times 2.2\times 10^{-6}$$

$$= 2.2\times 10^{-3}$$

- FRACTION LEFT AT SEA LEVEL

$$= \exp-\left\{\frac{t_{50}}{\tau_{LAB}}\right\} = \exp-\left\{\frac{2\times 10^{-4}}{2\times 10^{-3}}\right\} = \exp-\left\{10^{-1}\right\} = .905$$

- FRACTION LEFT AT SEA LEVEL $\sim 90\%$