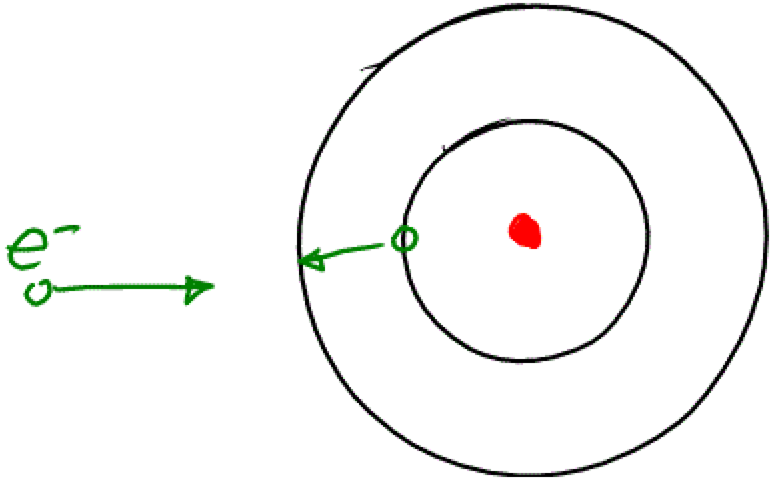
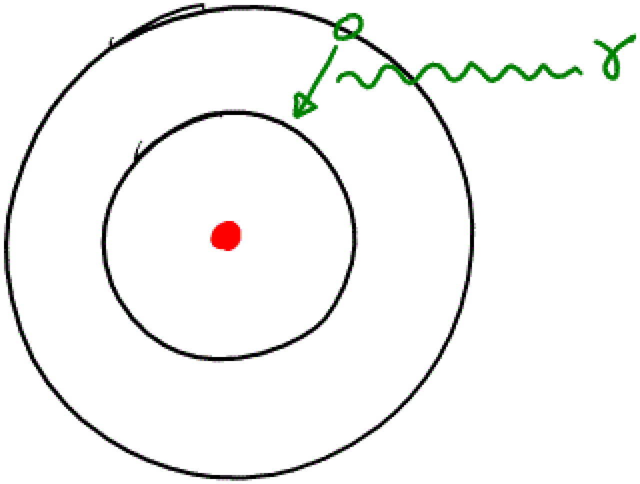
NUCLEAR & HADRONIC (π, p, K ....) STRUCTURE

- ATOMIC STRUCTURE UNDERSTOOD FROM :-

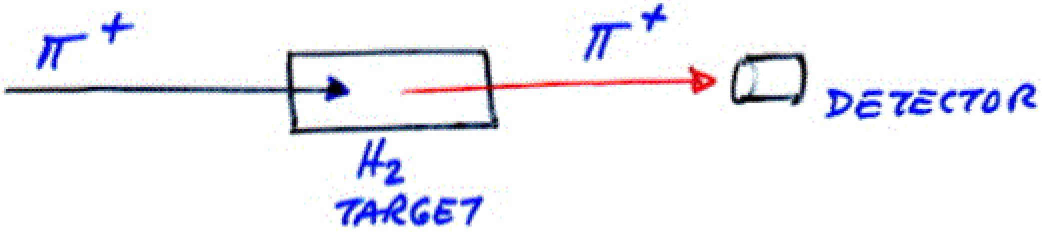
SCATTERING
EG. FRANK - HERTZ

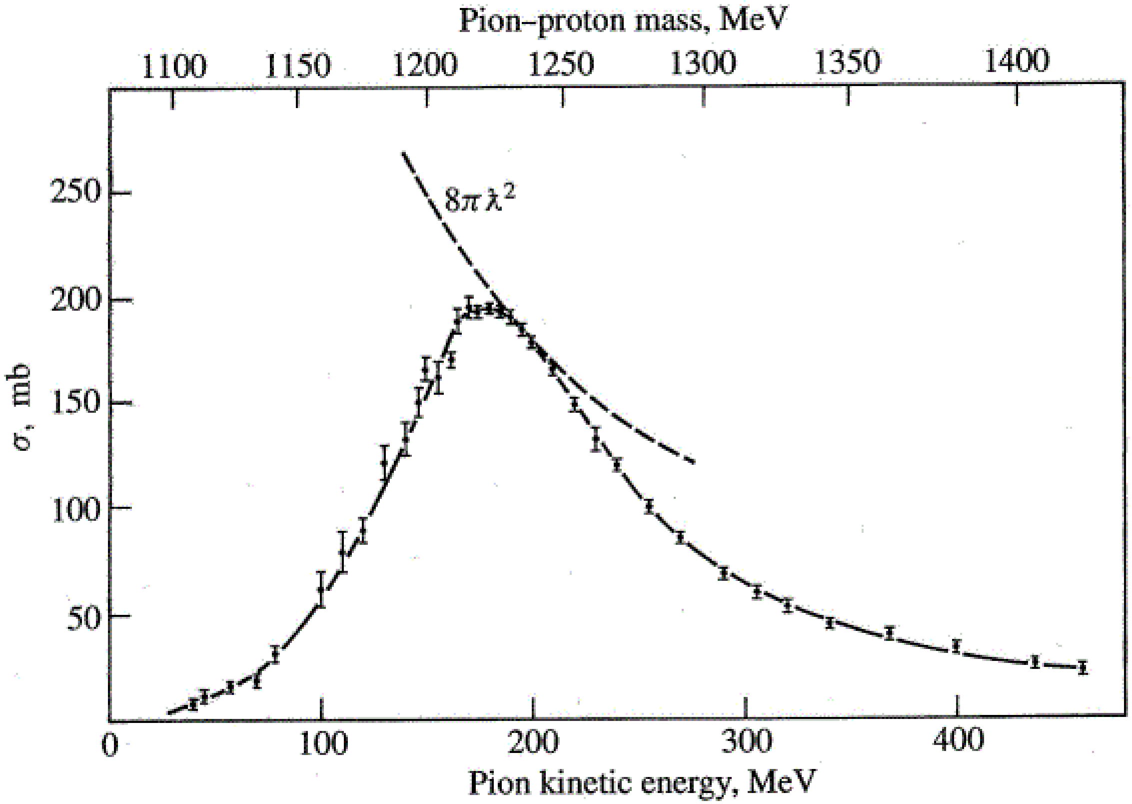


EMISSION
SPECTRA



ALREADY TALKED ABOUT $\pi^+ p$ RESONANT SCATTERING

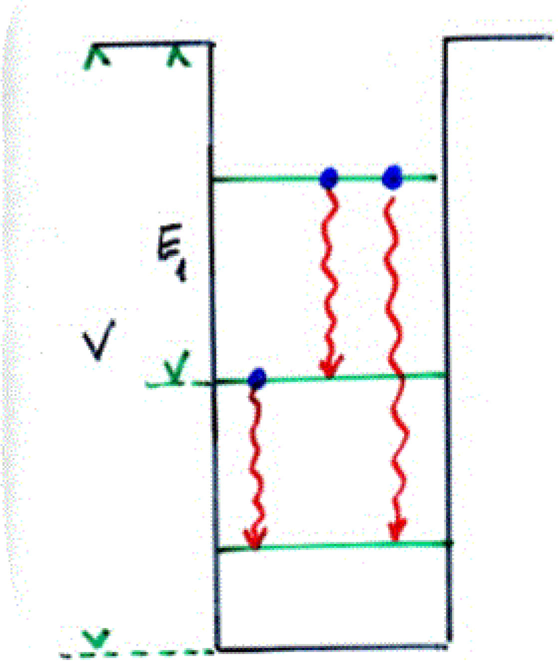


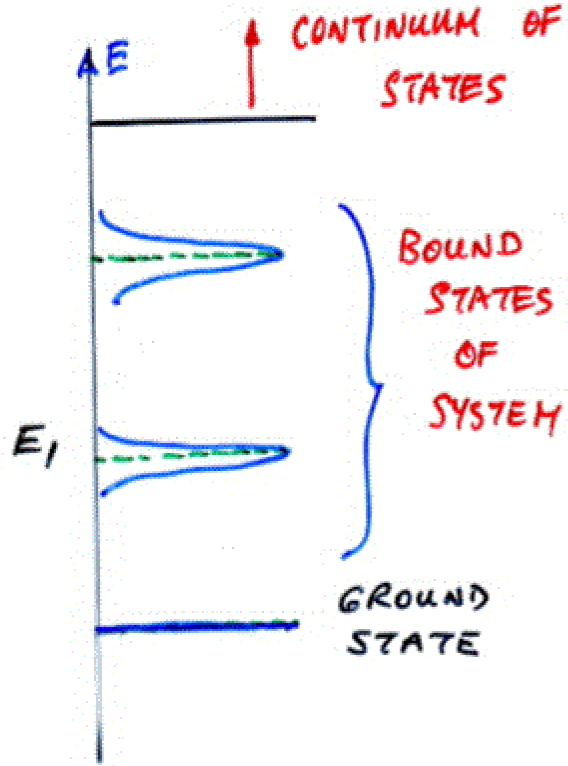


$\pi^+ p \to \Delta^{++} \to \pi^+ p$

EXAMPLES FROM NON-RELATIVISTIC QUANTUM MECHANICS

SIMPLE SQUARE WELL POTENTIAL





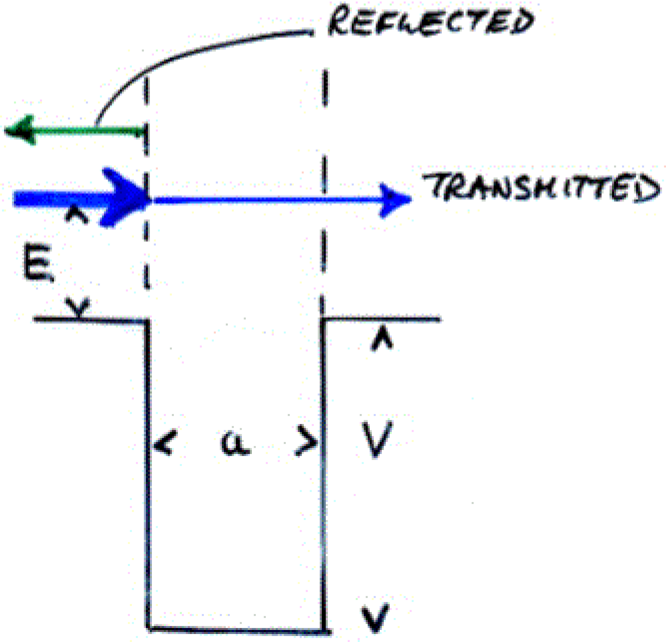
$\mathcal{H}\Psi = E\Psi$ → SPECTRUM OF BOUND STATES

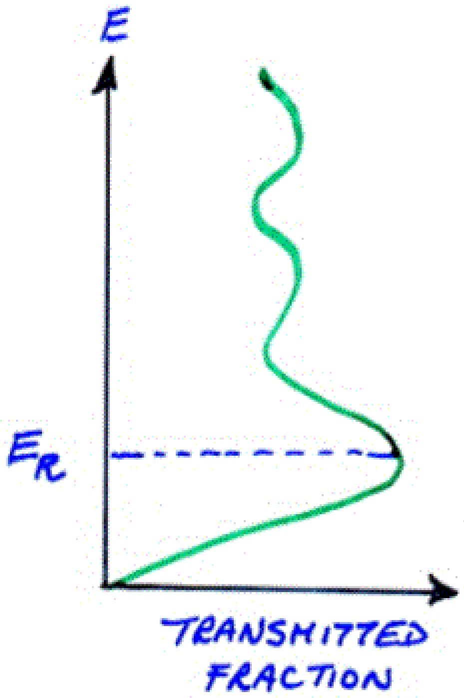
→ DETERMINES POTENTIAL

TRANSMISSION RESONANCES

— FORM OF POTENTIAL

— INTERNAL STRUCTURE

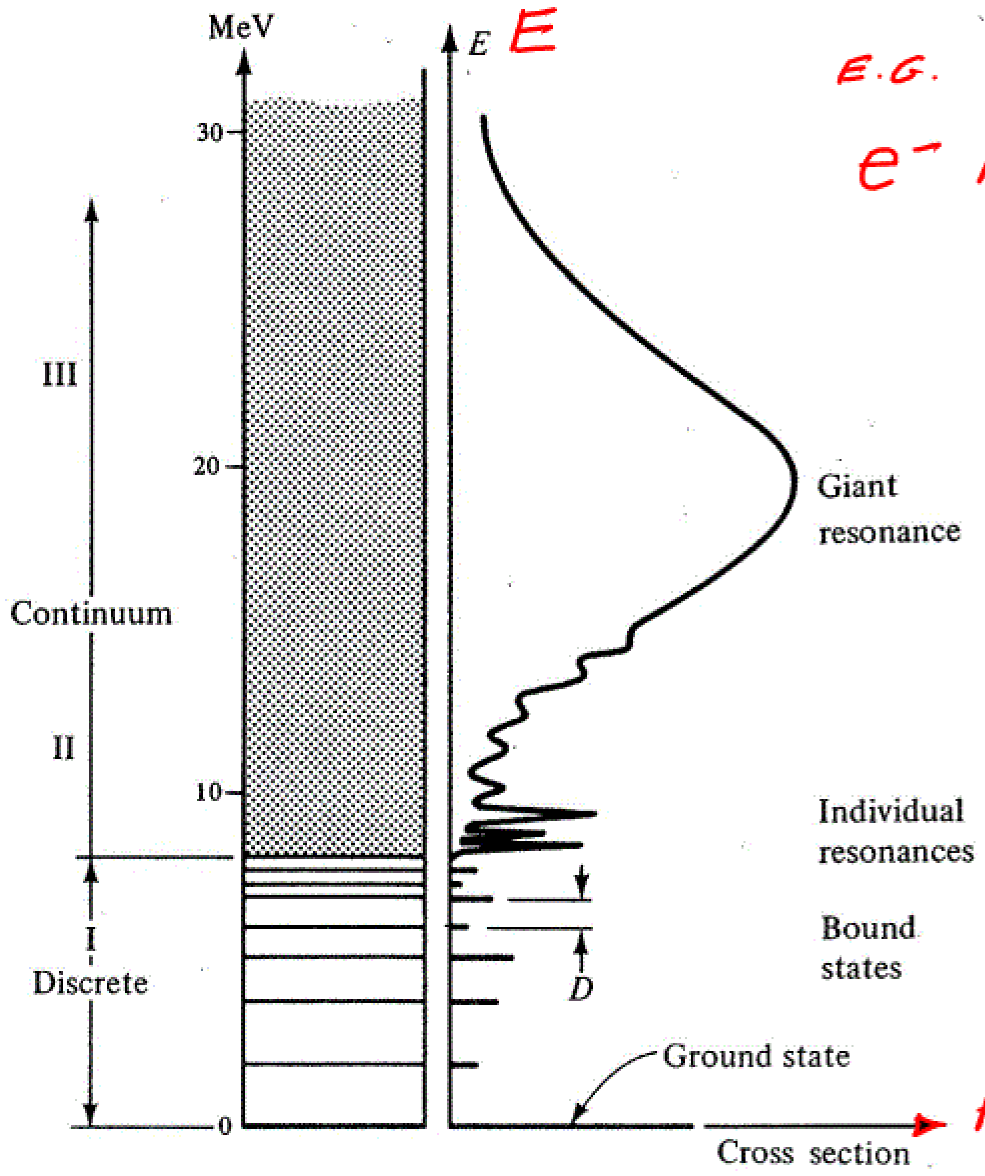




$$\left(\begin{array}{c}\text{TRANSMITTED}\\ \text{FRACTION}\end{array}\right)^{-1} = 1 + \frac{V^2}{4E\left(E+|V|\right)}\sin^2 ka$$

INFER POTENTIAL FROM RESONANCE STRUCTURE

NUCLEAR STRUCTURE FROM SCATTERING

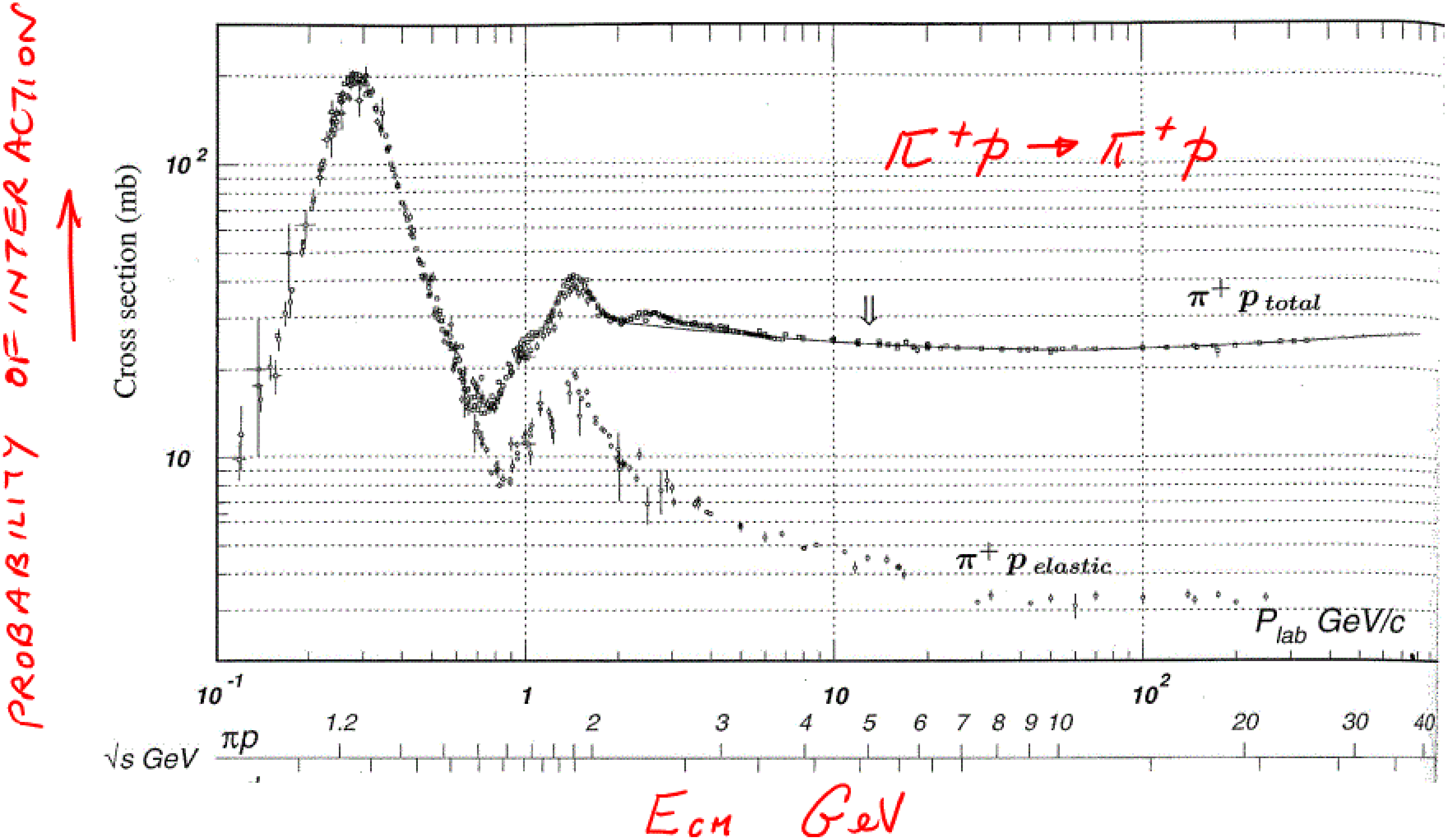


E.G.

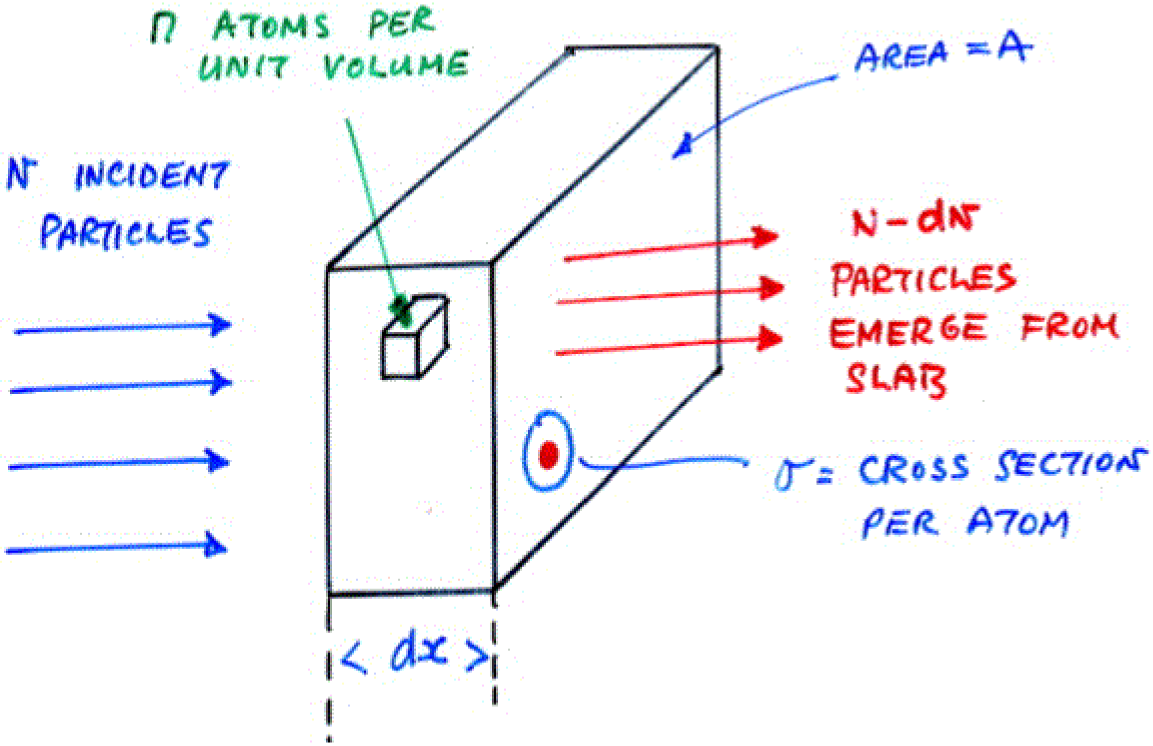
$$e^- N \rightarrow N^* e^-$$

$$\hookrightarrow \gamma N$$

PROTON STRUCTURE FROM SCATTERING



SCATTERING CROSS SECTION



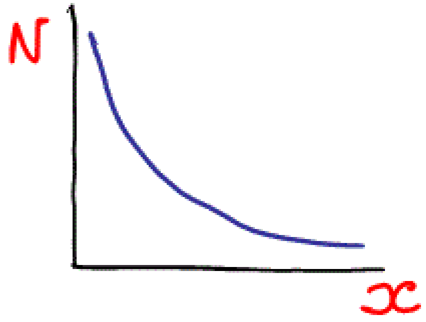
$$\frac{\text{\# INTERACTING PARTICLES}}{\text{\# INCIDENT PARTICLES}} = \frac{\text{AGGREGATE CROSS SECTION}}{\text{TOTAL TARGET AREA}}$$

$$-\frac{dN}{N} = \frac{n \cdot \sigma \cdot A\,dx}{A} = n\,\sigma\,dx$$

FOR A THIN SLAB

$$-\frac{dN}{N} = n\sigma\, dx$$ FOR A THIN SLAB

FOR A THICK TARGET



$$\int_{N_0}^{N} \frac{dN}{N} = -n\sigma \int_0^x dx$$

$$\ln N - \ln N_0 = -n\sigma x \rightarrow N = N_0\, e^{-n\sigma x}$$

$$= N_0\, e^{-x/\lambda}$$

MEAN FREE PATH

NUMBER OF PARTICLE INTERACTING

$$(N_0 - N) = N_0\left(1 - e^{-n\sigma x}\right)$$

- CAN CALCULATE CROSS SECTION FROM:
  - NUMBER OF SCATTERED PARTICLES
  - BEAM ATTENUATION

MEAN FREE PATH

$$N = N_0 e^{-x/\lambda} \quad \text{COMPARE} \quad N = N_0 e^{-t/\tau}$$

↑ MEAN LIFETIME

- λ AVERAGE DISTANCE A PARTICLE GOES BEFORE INTERACTING
- f IS PROBABILITY OF INTERACTING IN Δx

$$f = n \cdot \sigma \, \Delta x$$

- NUBER OF SLABS Δx BEFORE INTERACTING

$$H = \frac{1}{n\sigma \, \Delta x}$$

← MUST INTERACT EVENTUALLY

← PROB OF INTERACT IN SLAB

- AVERAGE DISTANCE BEFORE INTERACTING

$$H \cdot \Delta x = \frac{1}{n\sigma} \quad \longrightarrow \quad \lambda = \frac{1}{n\sigma}$$

MEAN FREE PATH

INTERACTION IN A THIN TARGET

$$(N_0 - N) = N_0\left(1 - e^{-n\sigma x}\right)$$

$e^x = 1 + x + \frac{x^2}{2!} + \cdots\cdots$ FOR SMALL x $\quad 1 - e^{-n\sigma x} \approx n\sigma x$

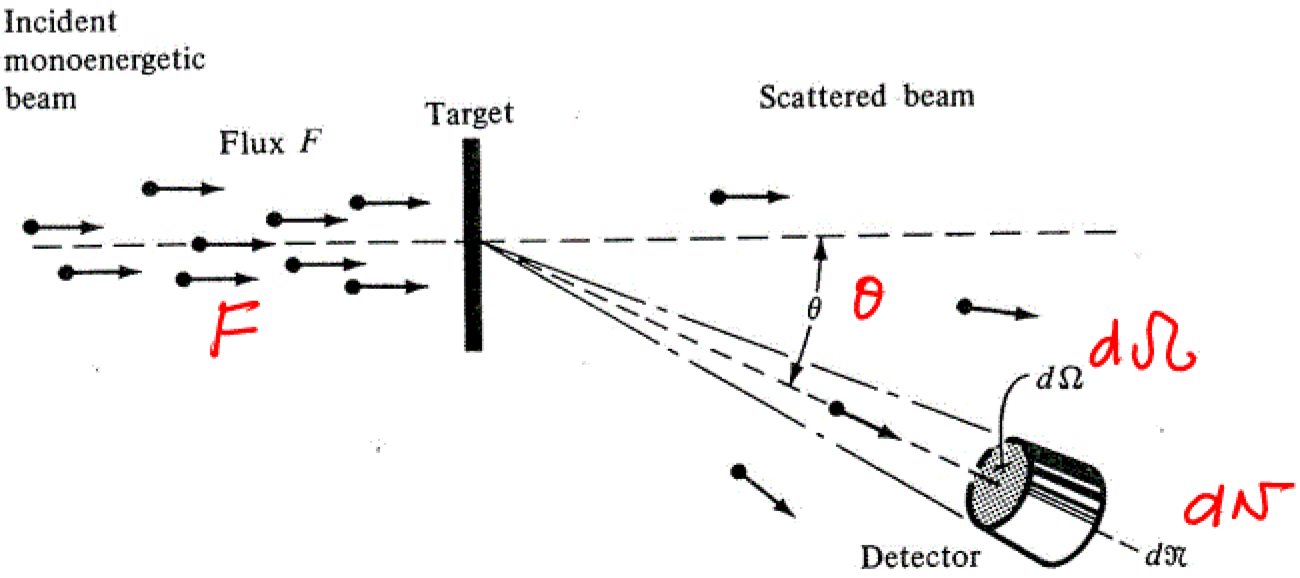
SO, THE NUMBER OF INTERACTIONS IN A THIN TARGET

$$= N_0 n\sigma x = N_0 x / \lambda$$

FOR A SINGLE PARTICLE PROBABILITY ON INTERACTING IN x

$$= \left(1 - \underbrace{e^{-x/\lambda}}_{\text{SURVIVAL PROBABILITY}}\right)$$

DIFFERENTIAL CROSS SECTION



- FIXED TARGET EXPERIMENT – "LOOK" AT TARGET WITH A DETECTOR AT AN ANGLE θ WHICH SUBTENDS A SOLID ANGLE $d\Omega$

- EXPERIMENTAL MEASUREMENT :

FOR FLUX F

HOW MANY PARTICLES PER UNIT TIME dN

SCATTER INTO θ AT $d\Omega$

NUMBER OF PARTICLES PER UNIT TIME
SCATTERED INTO SOLID ANGLE $d\Omega$
AT SCATTERING ANGLE θ

$$dN_s = F \; N \; \sigma(\theta) \; d\Omega$$

FLUX

NUMBER
OF SCATTERING
CENTERS IN TARGE

SOLID ANGLE

DEFINITION OF DIFFERENTIAL
SCATTERING CROSS SECTION

$$\sigma(\theta) = \frac{d\sigma(\theta)}{d\Omega}$$

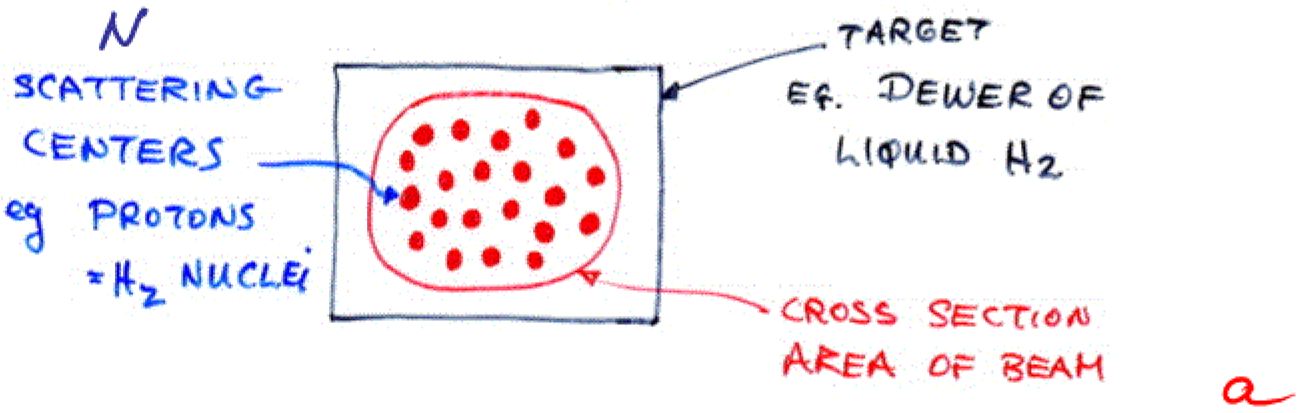
TOTAL CROSS SECTION ⇒ TOTAL NUMBER OF PARTICLES
SCATTERED PER UNIT TIME

$$N_S = F N \sigma_{TOT} \longrightarrow \sigma_{TOT} = \int \sigma(\theta) \, d\Omega$$

$$\downarrow$$

$$\sigma = \int \frac{d\sigma}{d\Omega} \, d\Omega$$

BEAM'S EYE VIEW

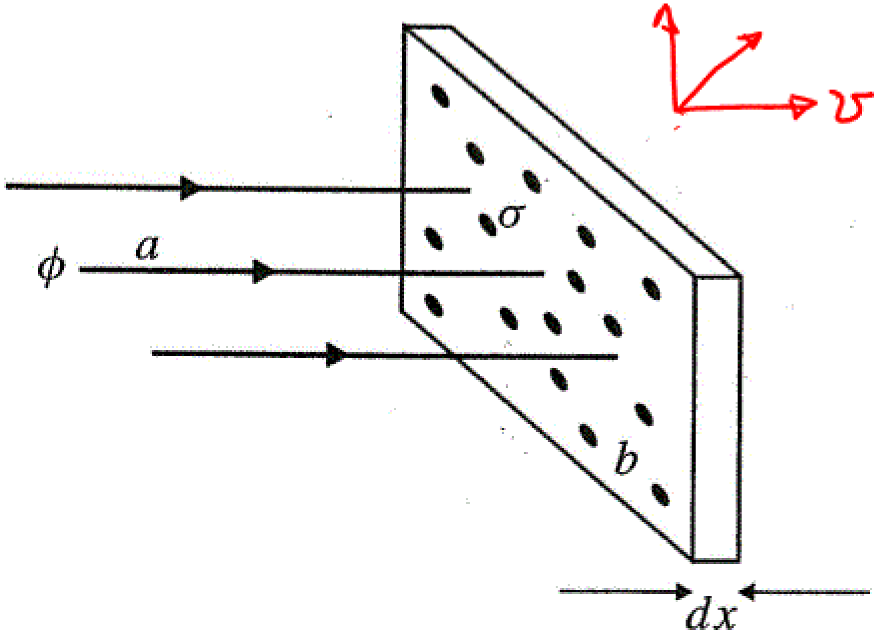


- AREA a OF TARGET INTERCEPTED BY BEAM CONTAINS N SCATTERING CENTER

- TOTAL NUMBER OF INCIDENT PARTICLES / UNIT TIME

$$\mathcal{N}_{IN} = \overset{\text{FLUX}}{F} \cdot a$$

$$\frac{\mathcal{N}_{SCAT}}{\mathcal{N}_{IN}} = \frac{\overset{\text{LAST SLIDE}}{F\, N\, \sigma_{TOT}}}{F \cdot a} = \frac{N\, \sigma_{TOT}}{a} = \frac{\text{SCATTERING AREA}}{\text{TOTAL AREA}}$$



NOTICE THAT CROSS SECTION IS A LORENTZ INVARIANT QUANTITY

Table 2.3. *Units of cross-section and energy*

Cross-section σ	Energy E
1 barn $= 10^{-28}$ m^2	1 MeV $= 10^6$ eV
1 millibarn = 1 mb $= 10^{-3}$ b	1 GeV $= 10^3$ MeV
1 microbarn = 1 μb $= 10^{-6}$ b	1 TeV $= 10^3$ GeV
1 nanobarn = 1 nb $= 10^{-9}$ b	
1 picobarn = 1 pb $= 10^{-12}$ b ($= 10^{-40}$ m^2)	

- SOLID OR LIQUID → LARGE NUMBER OF SCATTERING CENTRES

  – CAN MEASURE VERY SMALL CROSS SECTIONS

- CROSS SECTION FOR A ν SCATTERING FROM A NUCLEUS IS $\sim 10^{-42} cm^2$ MUCH SMALLER THAN GEOMETRICAL (OR ELECTRIC "SIZE" OF NUCLEUS)

MEASURABLE INTERACTION RATE

PRACTICAL CALCULATIONS

- NO OF SCATTERING CENTRES / UNIT VOLUME n
  - TARGET THICKNESS d
    - AREA INTERCEPTED BY BEAM a

TOTAL NUMBER OF SCATTERING CENTERS

$$N = a \cdot n \cdot d$$

AVOGADRO

DENSITY

$$n = \frac{N_A \cdot \rho}{A}$$

ATOMIC WEIGHT

$$\frac{\text{ATOMS}}{\text{VOLUME}} = \frac{\frac{\text{ATOM}}{\text{MOLE}} \times \frac{\text{MASS}}{\text{VOL}}}{\frac{\text{MASS}}{\text{MOLE}}}$$

COLLIDING BEAMS

BEAMS OF PARTICLES HAVE ORDERS OF MAGNITUDE LOWER DENSITY OF SCATTERING CENTERS THAN SOLID OR LIQUID TARGETS

NUMBER OF SCATTERS PER SECOND

$$R = \sigma_{TOT} \cdot \mathcal{L}$$

DEFINITION OF LUMINOSITY

NO. OF PARTICLES IN EACH BEAM

$$\mathcal{L} = \frac{N_1 N_2}{A} \cdot f$$

FREQUENCY OF COLLISION

OVERLAP AREA OF BEAMS

LHC

10^{34} cm^{-2} sec^{-1}

$$R = \sigma_{TOT} \cdot \frac{N_1 N_2}{A} \cdot f$$

PARTIAL CROSS SECTIONS

BEAM HITTING TARGET (OR COLLIDING BEAMS) CAN HAVE VARIOUS KINDS OF INTERACTIONS

- ELASTIC SCATTERING σ_e
- INELASTIC SCATTERING σ_i
- ABSORPTION σ_a

$$\sigma_{TOT} = \sigma_e + \sigma_i + \sigma_a$$

eg PROBABILITY OF INELASTIC SCATTERING

$$= \frac{\sigma_i}{\sigma}\left(\underbrace{1 - e^{-n\overbrace{\sigma}^{\sigma_{TOT}}x}}_{\text{N OF SCATTERS } N = N_0 e^{-x/\lambda}}\right)$$

FOR $x \ll \lambda \rightarrow \sigma_i n x$