WHY ARE SOME QUANTITIES CONSERVED

&

WHY ARE WE INTERESTED IN THEM?

IN SOLVING PHYSICS PROBLEMS → USE CONSERVED QUANTITIES

ENERGY, MOMENTUM, ELECTRIC CHARGE, LEPTON FLAVOR

THESE CONSERVED QUANTITIES CHARACTERIZE THE SYSTEM

CONSERVATION LAWS ↔ SYMMETRIES OF EQUATIONS DESCRIBING THE SYSTEM

CONSERVED QUANTITY	SYMMETRY OF HAMILTONIAN =	INVARIANCE UNDER
ENERGY		TIME TRANSLATIONS
LINEAR MOMENTUM		LINEAR TRANSLATION
ANGULAR MOMENTUM		ROTATIONS
ELECTRIC CHARGE CONSERVATION		GAUGE SYMMETRY OF MAXWELL'S EQUATIONS

NEW CONSERVED QUANTITIES → SYMMETRIES OF HAMILTONIAN DESCRIBING PARTICLE INTERACTIONS

FLAVOUR

BARYON NUMBER

⋮

CONSERVED QUANTITIES IN QUANTUM MECHANICS

CONSERVED QUANTITY $\longleftrightarrow$ SYMMETRY

ANY OPERATOR WHICH COMMUTES WITH THE HAMILTONIAN CORRESPONDS TO A CONSERVED QUANTITY

$$[F, H] = 0$$

F: CONSERVED

H: HAMILTONIAN

NÖTHERS' THEOREM IN CLASSICAL MECHANICS

Some Results From Quantum Mechanics

$$\langle F \rangle = \int d^3x \, \psi_a^* \, F \, \psi_a$$

→ Expectation value of operator F

↖ will obey classical equations of motio

If $[H, F] = 0$ → $\frac{d}{dt}\langle F \rangle = 0$

↳ conserved

↓ can simultaneously measure the eigenstates of H & F

$$H\psi = E\psi$$

$$F\psi = f\psi$$

SYMMETRY OPERATION

SYMMETRY OPERATOR → TRANSFORMATION → LEAVES HAMILTONIANS UNCHANGED

$$\psi'(\vec{x},t) = \mathcal{U}\,\psi(\vec{x},t)$$

NEW STATE

ORIGINAL STATE

$$i\hbar\frac{d}{dt}\psi = H\psi \quad \text{AND} \quad i\hbar\frac{d}{dt}\mathcal{U}\psi = H\mathcal{U}\psi$$

IF $\mathcal{U}\mathcal{U}^{+} = 1$ → UNITARY

→ $H = \mathcal{U}^{+} H \mathcal{U}$ → $[H, \mathcal{U}] = 0$

$[H, U]$

SYMMETRY OPERATOR ONLY NEEDS TO BE UNITARY

$$U^+ U = 1$$

ONLY HERMITIAN OPERATORS CORRESPOND TO OBSERVABLES

$$A = A^+$$

GENERALLY SYMMETRY TRANSFORMATIONS DO NOT CORRESPOND TO OBSERVABLES

↳ BUT A SYMMETRY TRANSFORMATION OPERATOR IS ALWAYS RELATED TO SOME OTHER OPERATOR WHICH IS AN OBSERVABLE

TWO DISTINCT KINDS OF TRANSFORMATIONS

- CONTINUOUS → DEPEND ON SOME CONTINUOUS PARAMETER. CAN DIFFER FROM UNITY (= DO NOTHING) BY AN ARBITRARILY SMALL AMOUNT

  EXAMPLE → ROTATION IN SPACE

- DISCRETE → EITHER HAPPEN OR NOT

  REFLECTION, TIME REVERSAL, PARTICLE ↓ ANTIPARTICLE

SOME DISCRETE TRANSFORMATIONS → OBSERVABLES

SPATIAL REFLECTION → PARITY = $\mathcal{U}_P$

$$\psi(\vec{x}) \to \psi(-\vec{x}) \quad ; \quad \psi(-\vec{x}) = \mathcal{U}_P \, \psi(\vec{x})$$

$\mathcal{U}_P \mathcal{U}_P = 1$ → HERMITIAN → MUST CORRESPOND TO AN OBSERVABLE

UNITARY