DISCRETE SYMMETRIES

- SOME OF THE MOST SURPRISING RESULTS IN 20TH CENTURY PHYSICS

PARITY VIOLATION RIGHT-LEFT ASYMMETRY IN UNIVERSE AT MOST FUNDAMENTAL LEVEL

→ COMPLETE SURPRISE

TIME REVERSAL ASYMMETRY MAY BE ORIGIN OF ASYMMETRY BETWEEN MATTER & ANTIMATTER IN THE UNIVERSE

ALSO MAY BE CONNECTED TO THE OBSERVED 3 GENERATIONS OF QUARKS & LEPTONS

DISCRETE SYMMETRIES — PROPERTIES OF UNIVERSE

PARITY

- NOT VERY INTERESTING AT FIRST SIGHT

$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \xrightarrow{P} \begin{pmatrix} ct \\ -x \\ -y \\ -z \end{pmatrix}$$

P REVERSES 3 SPATIAL COORDINATES

— CANNOT BUILD FROM INFINITESSIMALS

— NOT A ROTATION IN SPACE

EXPECT THAT

$$[P, H] = 0$$

AND SINCE $P P \psi(\bar{x}) = \psi(\bar{x})$

PARITY OF $\psi(\bar{x})$ WILL BE A CONSERVED OBSERVABLE

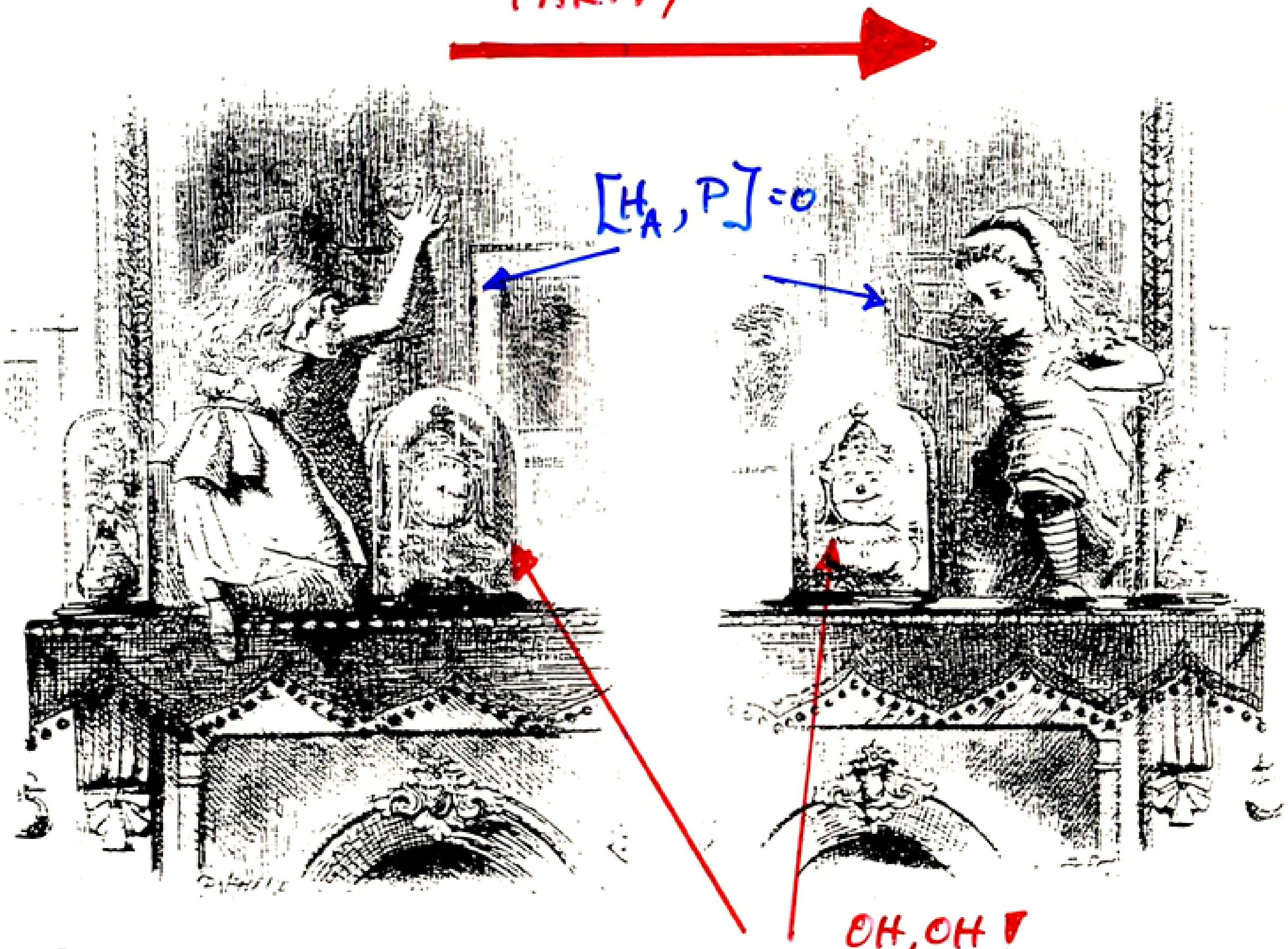
→ EXPERIMENTAL QUESTION

- OBSERVATIONS OF STRONG & ELECTROMAGNETIC DECAYS OF NUCLEI SHOWED ONLY TRANSITIONS BETWEEN STATES OF SAME PARITY

OK → PARITY CONSERVED AS EXPECTED

THROUGH THE LOOKING GLASS

PARITY



$[H_A, P] = 0$

306 The Weak Interaction

OH, OH!

$[H_{clock}, P] \neq 0$

PARITY OPERATOR

$$\bar{r} \longrightarrow -\bar{r}$$ VECTOR

$$\bar{p} = m\dot{\bar{r}} \longrightarrow -m\dot{\bar{r}} = -\bar{p}$$ VECTOR

$$|\bar{r}| = (\bar{r}\cdot\bar{r})^{\frac{1}{2}} \longrightarrow [(-\bar{r})\cdot(-\bar{r})]^{\frac{1}{2}} = |\bar{r}|$$ SCALAR

$$|\bar{p}| = (\bar{p}\cdot\bar{p})^{\frac{1}{2}} \longrightarrow [(-\bar{p})\cdot(-\bar{p})]^{\frac{1}{2}} = |\bar{p}|$$ SCALAR

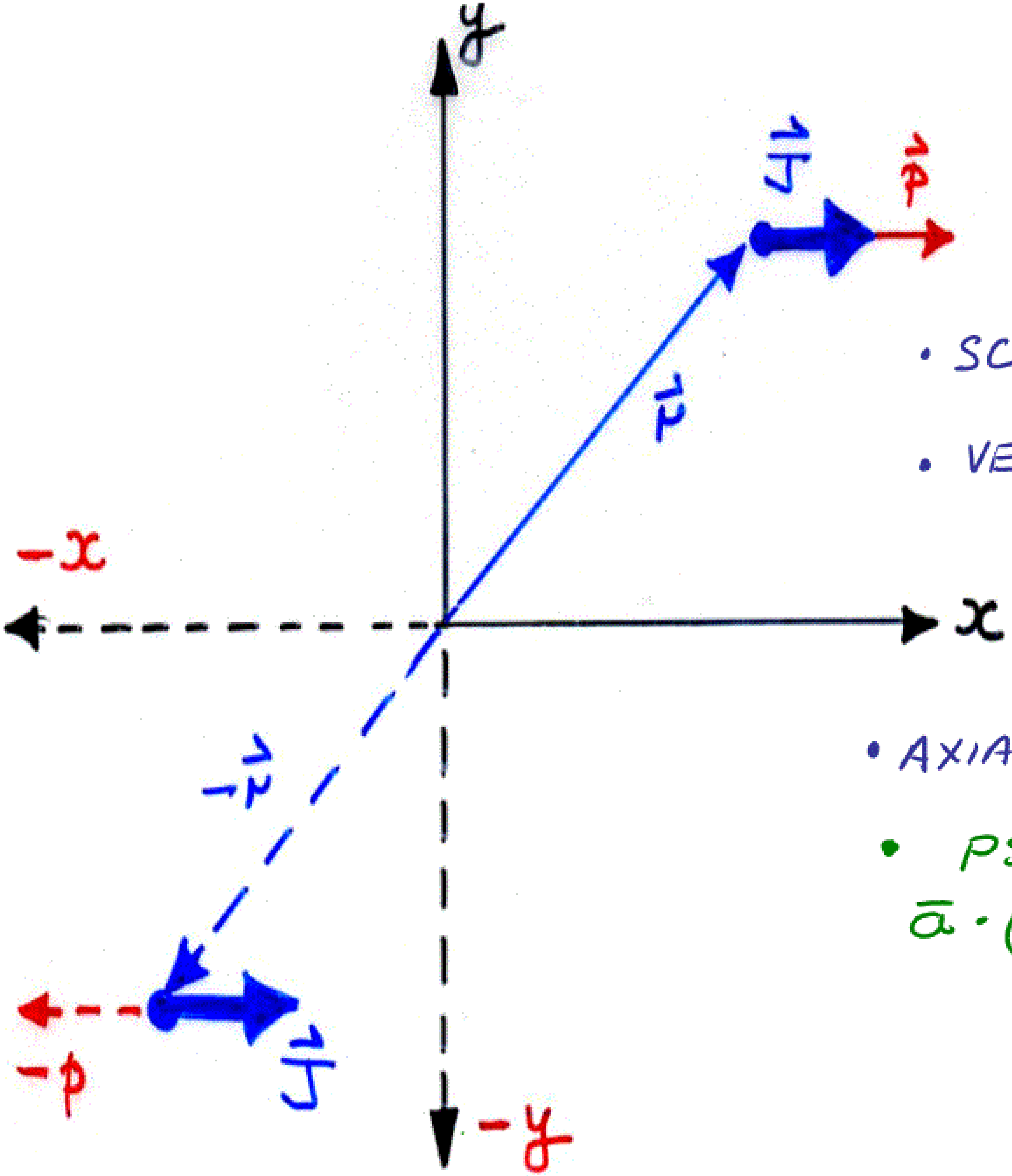
THIS DEFINES TRANSFORMATION OF VECTORS & SCALARS

ANGULAR MOMENTUM ?

$$\bar{L} = \bar{r}\times\bar{p} \longrightarrow (-\bar{r})\times(-\bar{p}) = \bar{r}\times\bar{p} = \bar{L}$$

DOES NOT TRANSFORM AS TRUE VECTOR

$$\left.\begin{array}{l}\text{AXIAL VECTOR}\\ \text{PSEUDOVECTOR}\end{array}\right\}$$



- SCALARS UNCHANGED
- VECTORS CHANGE SIGN

- AXIAL VECTORS UNCHANGED
- PSEUDSCALARS

$$\bar{a}\cdot(\bar{b}\times\bar{c}) \rightarrow -\bar{a}(\bar{b}\times\bar{c})$$

PARITY → UNITARY → SYMMETRY OF $\mathcal{H}$

$$P|\psi(\bar{x})\rangle \rightarrow |\psi(-\bar{x})\rangle$$

SECOND PARITY OPERATION RETURNS TO ORIGINAL

$$P.P|\psi(\bar{x})\rangle \propto P|\psi(-\bar{x})\rangle \propto |\psi(\bar{x})\rangle$$

$P^2 = I$ → HERMITIAN, OBSERVABLE

$$P^2|\psi(\bar{x})\rangle = \pi^2|\psi(\bar{x})\rangle$$

PARITY OPERATOR

PARITY EIGENVALUE

$$\pi = \begin{array}{ll} +1 & \text{EVEN} \\ -1 & \text{ODD} \end{array}$$

CONSERVATION OF PARITY

CONSERVATION OF PARITY IS A
MULTIPLICATIVE CONSERVATION LAW

$$a + b \to c + d$$

$$|\text{INITIAL}\rangle = |a\rangle |b\rangle |\text{RELATIVE MOTION}\rangle$$

$$P|i\rangle = P|a\rangle \, P|b\rangle \, P|\text{RELATIVE}\rangle$$

RADIAL $|\bar{r}| \to |\bar{r}| \to +1$

ANGULAR (ORBITAL) $\to (-1)^{\ell}$

$$P|a\rangle = \pi_a |a\rangle$$

INTRINSIC PARITY → AN ATTRIBUTE OF THE PARTICLE → cf SPIN, FLAVOR

RELATIVE MOTION → POLAR COORDINATES

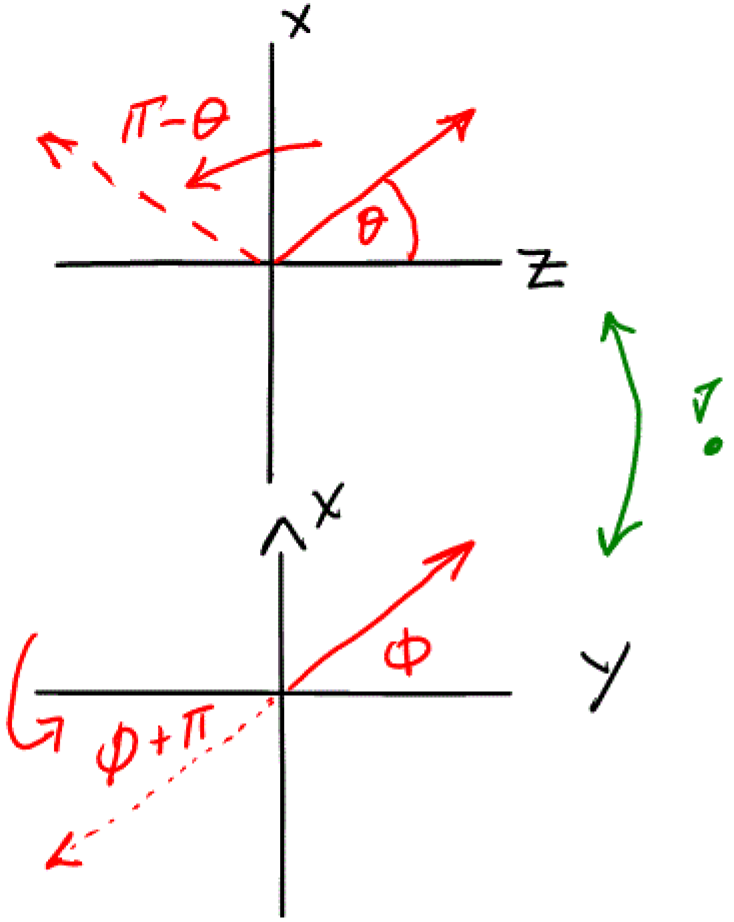
θ → POLAR ANGLE

ϕ → AZIMUTHAL ANGLE

PARITY OPERATION

IN

POLAR COORDINATES



FOR ROTATIONALLY INVARIANT 3-d SYSTEM

E.G. HYDROGEN ATOM

EIGENSTATES — ENERGY — SIMULTANEOUS
— ANGULAR MOMENTUM —

$$Y_{n\ell m}(\bar{r}) = \underbrace{R_{n\ell}(r)}_{\text{RADIAL}}\ \underbrace{Y_{\ell m}(\theta,\phi)}_{\text{ANGULAR}}$$

UNDER PARITY $\bar{r} \longrightarrow -\bar{r}$ EQUIV

$$\theta \to \pi - \theta$$
$$\phi \to \pi + \phi$$

$$Y_{\ell m}(\theta,\phi) \xrightarrow{P} Y_{\ell m}(\pi-\theta,\pi+\phi) = \underbrace{(-1)^{\ell}}_{\text{CONTRIBUTION TO PARITY FROM ORBITAL ANGULAR MOMENTUM}} Y_{\ell m}(\theta,\phi)$$

PARITY CONSERVATION → IF YOU LOOK AT AN INTERACTION IN A "PARITY MIRROR" AND SEE PHYSICALLY POSSIBLE STATE

WHETHER OR NOT PARITY IS CONSERVED IN A PARTICULAR INTERACTION → EXPERIMENT

IF PARITY IS CONSERVED IN DECAYS, ONE CAN MEASURE RELATIVE PARITIES OF VARIOUS STATES

OVERALL PHASE OF INTRINSIC PARITIES IS JUST A CONVENTION

$$\left.\begin{array}{l}\text{PROTON}\\ \text{NEUTRON}\\ \Lambda\end{array}\right\}\ \begin{array}{c}\text{INTRINSIC}\\ \text{PARITY}\\ +1\end{array}$$

Cf DEFINITION OF -VE & -VE ELECTRIC CHARGE

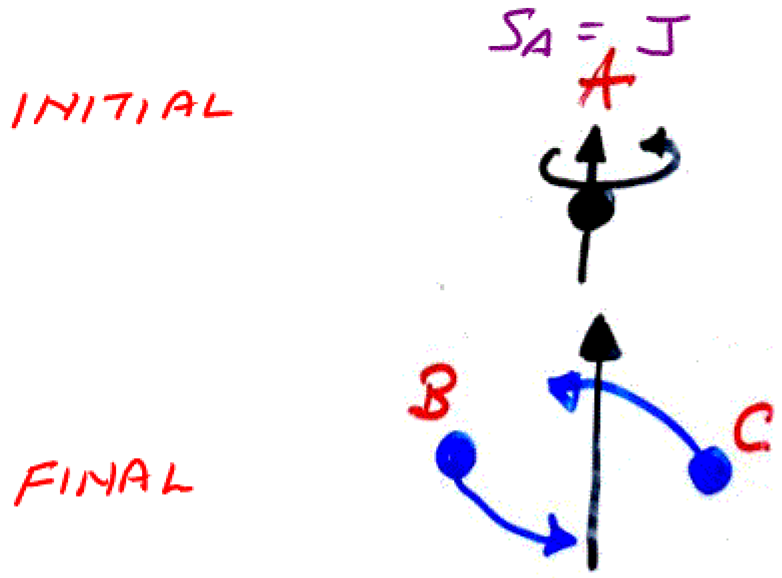
ASSUME PARITY IS CONSERVED IN

$A \rightarrow B + C$ STRONG
EM } DECAY

INTERPLAY OF PARITY & ANGULAR MOMENTUM CONSERVATION

(a) LIMIT DECAY FINAL STATES

(b) ALLOW DETERMINATION OF RELATIVE PARITIES OF INITIAL & FINAL STATES



ASSUME B, C SPIN 0
→ RELATIVE ORBITAL ANGULAR MOMENTUM MUST = SPIN of A

$J = \ell$

IN THIS SITUATION

$$A \rightarrow B + C$$

$J \rightarrow \ell$ ANGULAR MOMENTUM CONSERVED

$\pi \rightarrow \pi$ PARITY CONSERVED

$$\pi_A = \pi_B \cdot \pi_C (-1)^{\ell} = \pi_B \pi_C (-1)^{J}$$

PARITY CONSERVATION → J^P SELECTION RULES

EXAMPLE OF EVIDENCE FOR PARITY CONSERVE IN STRONG EM DECAYS

$0^+ \rightarrow 0^+ \, 0^+$

$0^+ \rightarrow 0^- \, 0^-$

$0^- \rightarrow 0^+ \, 0^-$

PARITY CONSERVED

$0^+ \not\rightarrow 0^+ \, 0^-$

$0^- \not\rightarrow 0^+ \, 0^+$

$0^- \not\rightarrow 0^- \, 0^-$

PARITY NOT CONSERVED

BREAKDOWN OF PARITY CONSERVATION

1950'S → 2 PARTICLES OBSERVERVED → SAME MASS
SAME SPIN
SAME LIFETIME

OBSERVED VIA THE WEAK DECAYS

$$\theta^+ \rightarrow \pi^+ \pi^0$$

HISTORICAL NAME → $\tau^+ \rightarrow \pi^+ \pi^- \pi^0$

NOT τ-LEPTON

(K^+ MESON)

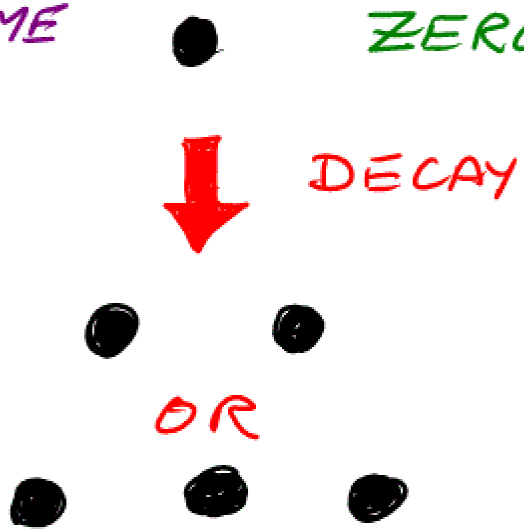
SINCE THEY HAD IDENTICAL PROPERTIES → EXPECT THEY ARE SAME PARTICLE

BUT τ, θ SEEM TO HAVE OPPOSITE PARITIES !

θ & τ OBSERVED TO HAVE SPIN-0

REST FRAME OF PARENT

ZERO ANGULAR MOMENTUM

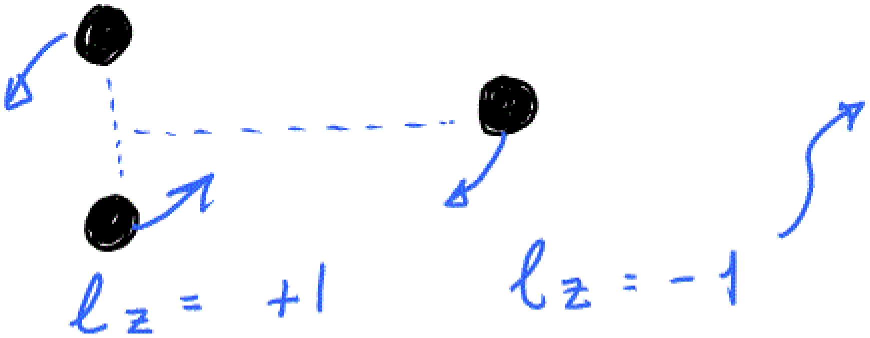


2π }
3π } π HAVE SPIN-0

2π

$\ell = 0$

3π



$\ell = 0$

NOW COMES A PUZZLE :

π → PSEUDOSCALAR $J^P = 0^-$

$\eta_{\pi^+} = \eta_{\pi^-} = \eta_{\pi^0}$ ← INTRINSIC PARITY

$$\left.\begin{array}{l} \eta_\theta = \eta_{\pi^+}\eta_{\pi^0} = +1 \\ \eta_\tau = \eta_{\pi^+}\eta_{\pi^+}\eta_{\pi^-} = -1 \end{array}\right\} \text{MULTIPLICATIVE}$$

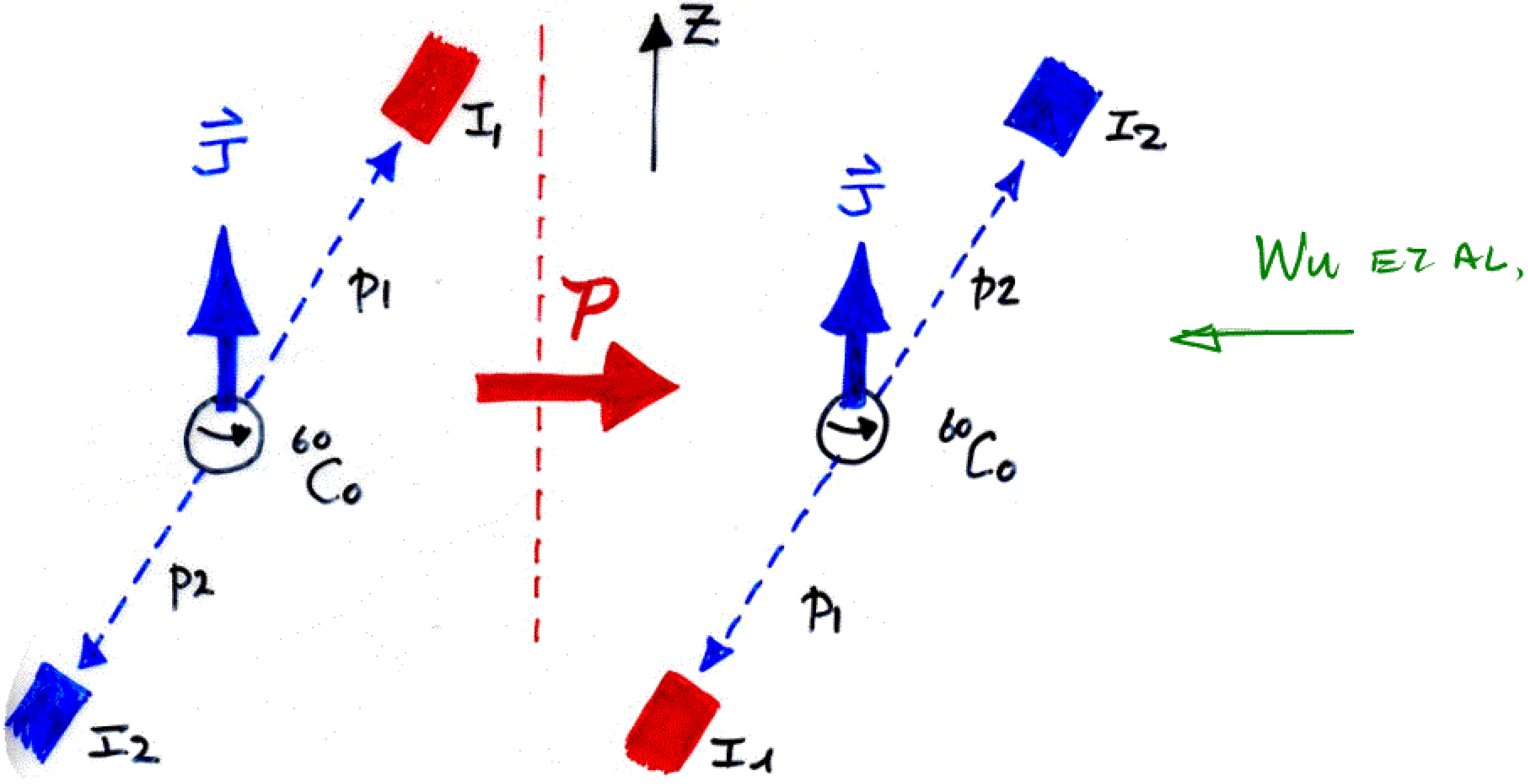
PARITY → INTRINSIC PROPERTY OF PARTICLES cf SPIN, MASS, LIFETIME

CAN τ, θ BE DIFFERENT PARTICLES ?

T. D. LEE & C.N. YANG NOTICED THAT PARITY CONSERVATION ONLY EXPERIMENTALLY CONFIRMED IN STRONG & ELECTROMAGNETIC DECAYS → THIS IS A WEAK DECAY

IS PARITY CONSERVED? → NO! → NOBEL PRIZE

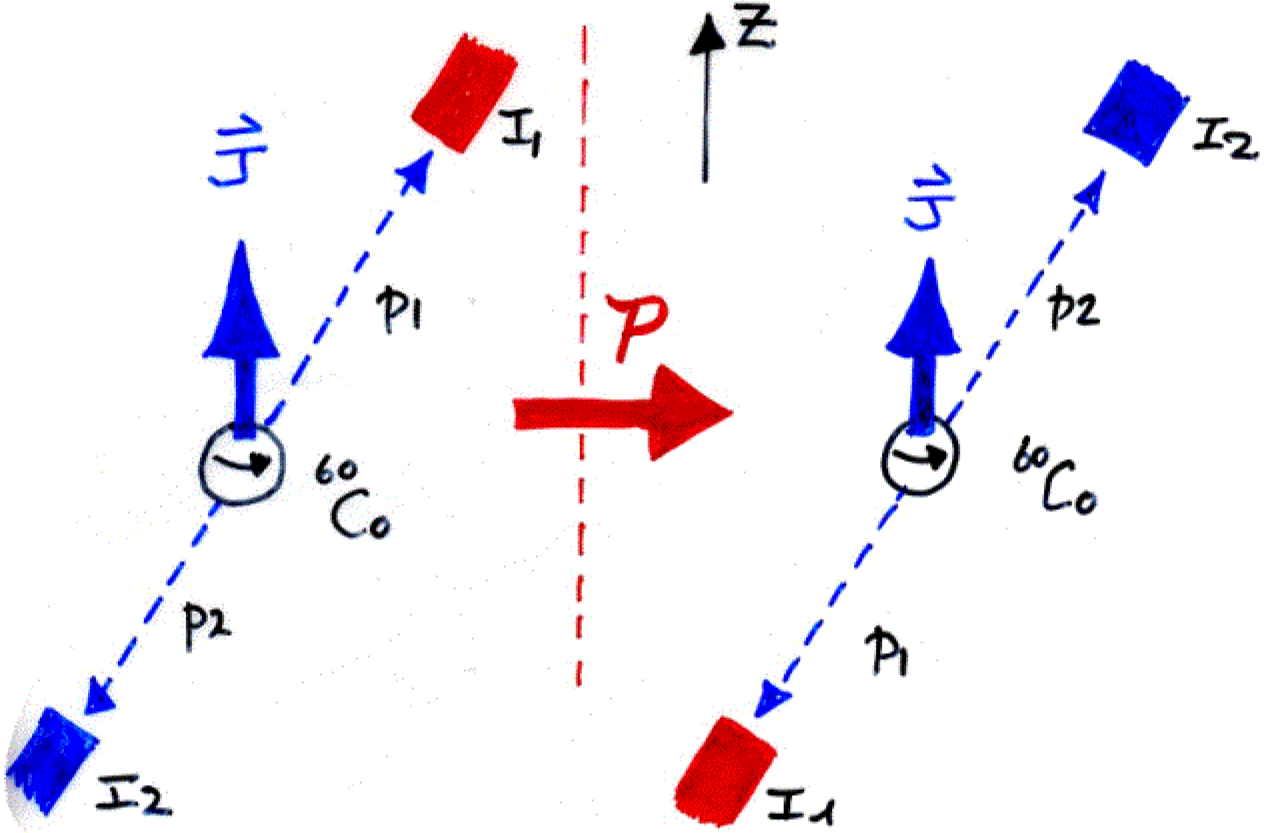
LEE & YANG → WEAK INTERACTION VIOLATES PARITY CONSERVATION



$$^{60}Co \rightarrow {}^{60}Ni\ e^-\ \bar{\nu}_e$$

↑ POLARIZED, $\vec{J}$ ALONG Z-AXIS

IS CONFIGURATION DRAWN ON RIGHT POSSIBLE IN NATURE?

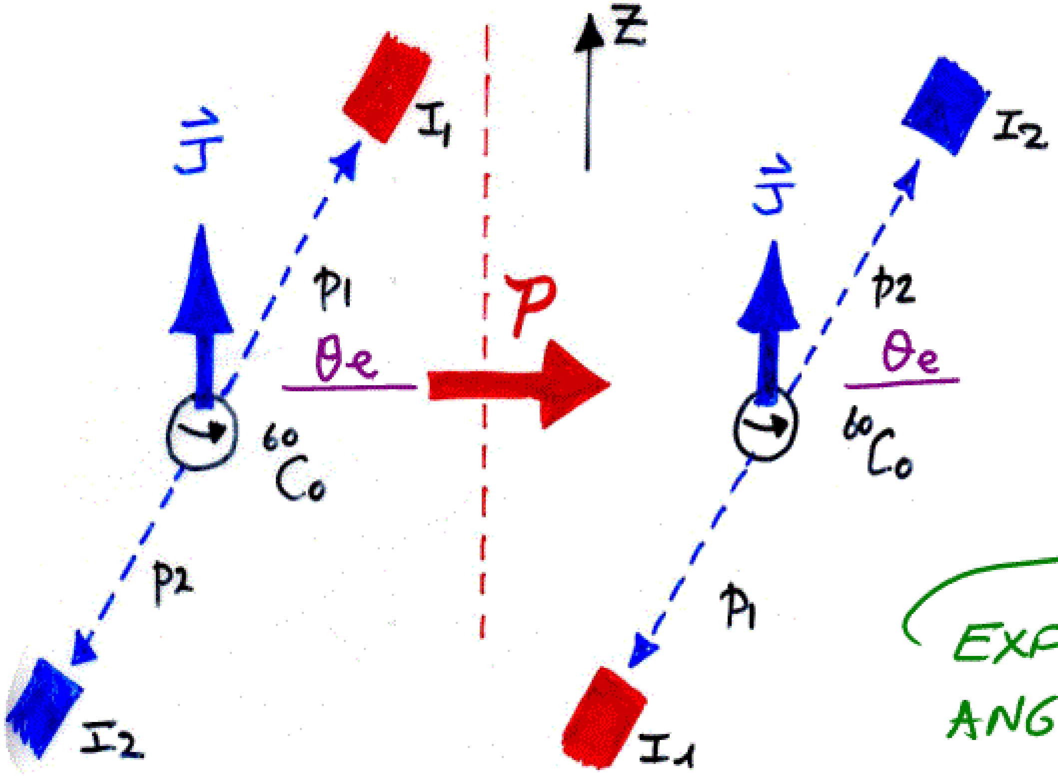


PARITY OPERATION →

$$\overline{J} \rightarrow \overline{J}$$
$$\overline{p}_1 \rightarrow -\overline{p}_1$$
$$\overline{p}_2 \rightarrow -\overline{p}_2$$

INVARIANCE UNDER PARITY OPERATION

$$I_1 = I_2$$



INVARIANCE UNDER PARITY

$I_1 = I_2$

ELECTRONS HAVE ANGULAR DISTRIBUTION θ_e

EXPERIMENT MEASURES THIS ANGULAR DISTRIBUTION

EXPECTATION VALUE OF $\cos\theta_e$

$$\langle \cos\theta_e \rangle = \left\langle \frac{\vec{J}\cdot\vec{p}}{|\vec{J}||\vec{p}|} \right\rangle$$

PSEUDO SCALAR – CHANGES SIGN UNDER PARITY

SCALAR DOES NOT CHANGE SIGN

PARITY INVARIANCE → $I_1 = I_2$ → $\langle \cos\theta_e \rangle$ UNCHANGED

$$\langle \cos\theta_e \rangle = \left\langle \frac{\bar{J}\cdot\bar{P}}{|\bar{J}||\bar{p}|} \right\rangle$$

BUT $\bar{J}\cdot\bar{p} \xrightarrow{P} -\bar{J}\cdot\bar{p}$

ONLY SOLUTION FOR PARITY INVARIANCE

$\bar{J}\cdot\bar{p} = 0 \rightarrow \langle \cos\theta_e \rangle = 0$ → NO UP-DOWN ASYMMETRY W.R.T Z-AXIS

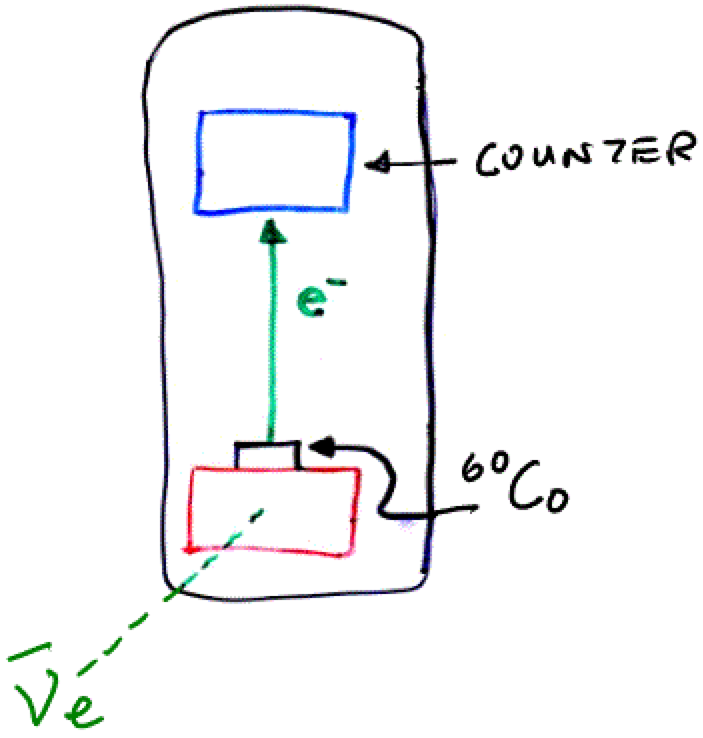
- EXPERIMENT OBSERVED LARGE UP-DOWN ASYMMETRY
- ELECTRONS PREFERENTIALLY EMITTED OPPOSITE TO NUCLEAR SPIN

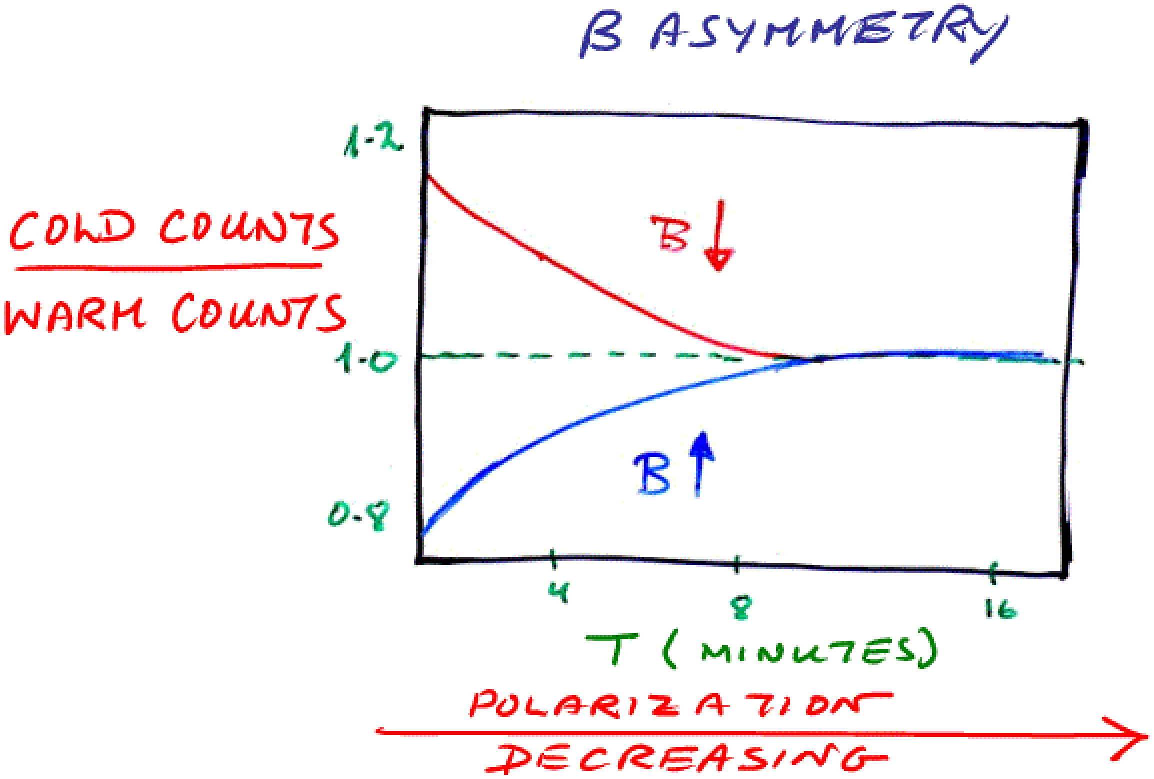
} PARITY INVARIANCE VIOLATED

$$\bar{J}\cdot\bar{p} \xrightarrow{P} -\bar{J}\cdot\bar{p}$$

EXPERIMENTALLY INVERT $\bar{J}$ BY REVERSING THE MAGNETIC FIELD POLARIZING ^{60}Co

EXPERIMENTAL TOUR DE FORCE → TEXT BOOK P.250
FEYNMAN LECTURES
ADIABATIC DEMAGNETIZATION





Mme Wu APPARATUS

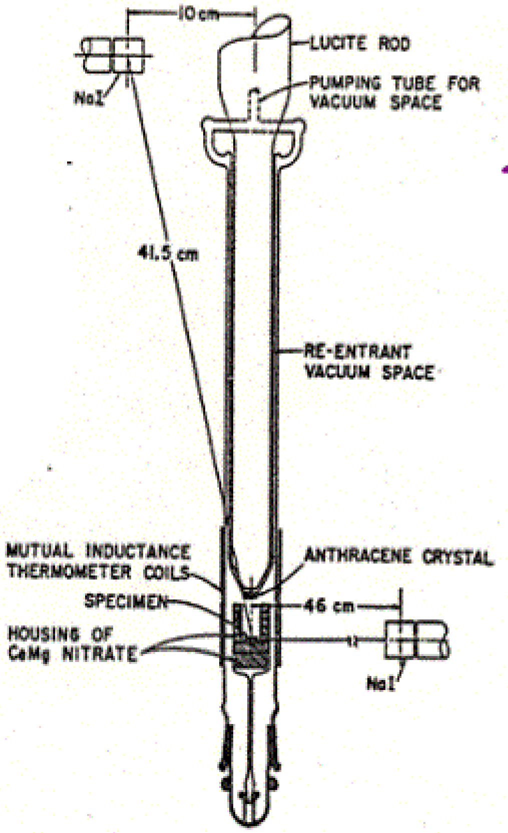


FIG. 1. Schematic drawing of the lower part of the cryostat.

EXPERIMENTAL RESULTS

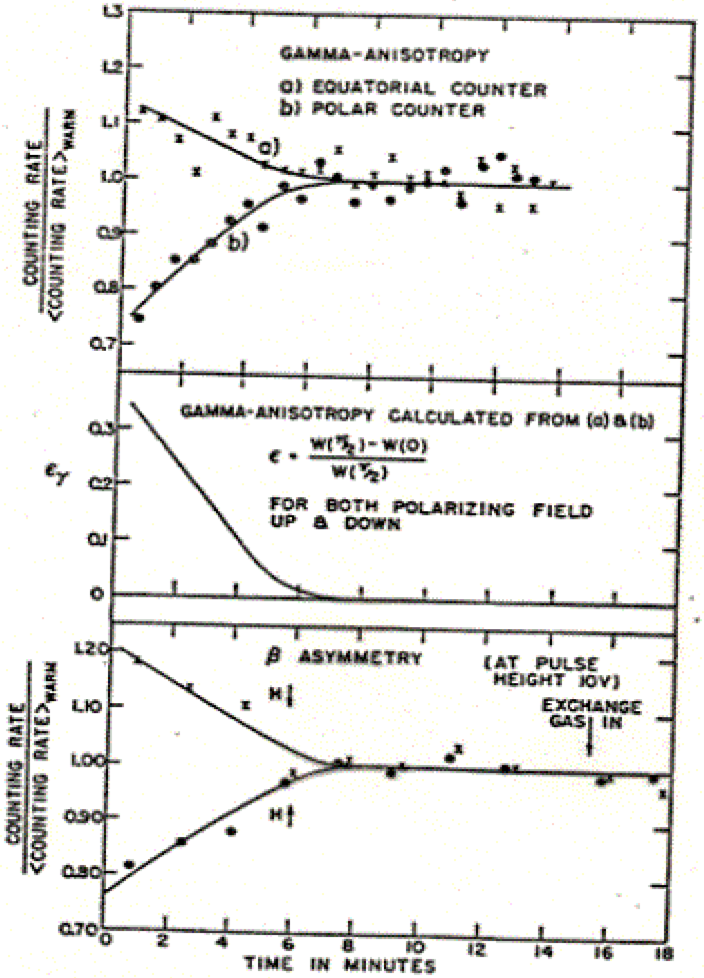
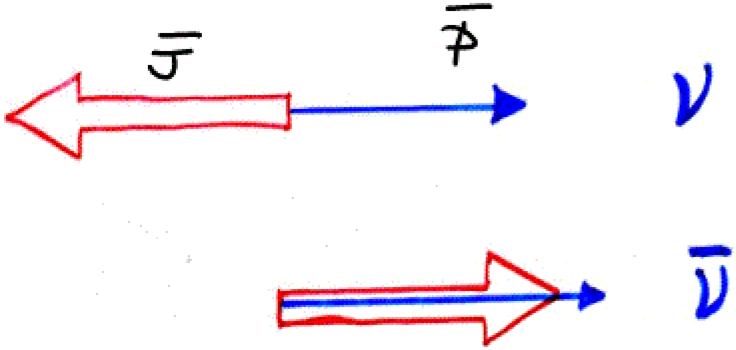


FIG. 2. Gamma anisotropy and beta asymmetry for polarizing field pointing up and pointing down.

NEUTRINOS → MANIFESTLY VIOLATE PARITY INVARIANCE



$$\text{HELICITY} = 2\,\frac{\bar{J}\cdot\hat{p}}{\hbar} = \begin{matrix} +1 & \nu \\ -1 & \bar{\nu} \end{matrix}$$

$\hat{p}$ IS A UNIT VECTOR IN THE DIRECTION OF THE MOMENTUM

MASSLESS ν PROPAGATE AT VELOCITY $= c$

PARITY TRANSFORM $\nu \rightarrow \bar{\nu}$ CANNOT BE PARITY EIGEN STATE

$$\begin{tikzcd} A \rightarrow B + C + \nu \arrow[d, "P"] \arrow[d, bend left, leftrightarrow, "\text{NOT SAME PHYSICAL PROCESS}"] \\ A \rightarrow B + C + \bar{\nu} \end{tikzcd}$$