

TIME REVERSAL

IN QUANTUM FIELD THEORY, ALL SENSIBLE HAMILTONIANS HAVE THE SYMMETRY:

CHARGE CONJUGATION [CPT, H]
PARTICLES \rightarrow ANTIPARTICLES

↑
PARITY

←
TIME REVERSAL

WEAK INTERACTION \rightarrow VIOLATES P $\therefore \rightarrow$ ALSO SOME COMBINATION OF CT

$$t \xrightarrow{\quad \pi \quad} -t$$

$$\vec{r} \xrightarrow{\quad \quad} \vec{r}$$

$$\vec{p} = m \dot{\vec{r}} \xrightarrow{\quad \quad} -m \dot{\vec{r}} = -\vec{p}$$

$$\vec{L} = \vec{r} \times \vec{p} \xrightarrow{\quad \quad} \vec{r} \times (-\vec{p}) = -\vec{L}$$

CLASSICALLY

$$m \frac{d^2 \vec{r}}{dt^2} = \vec{F}$$

MANIFESTLY INVARIANT
UNDER $t \rightarrow -t$

2ND ORDER DIFFERENTIAL \rightarrow ALL MICROSCOPIC
CLASSICAL SYSTEMS
ARE T INVARIANT

USUALLY "ARROW OF TIME" ASSERTED TO
COME FROM ENTROPY \rightarrow READ R. PENROSE.

IN QUANTUM MECHANICS SCHRÖDINGER SHOWS
HOW TIME EVOLUTION DEPENDS ON HAMILTONIAN

$$i\hbar \frac{\partial \psi(\vec{r}, t)}{\partial t} = H \psi(\vec{r}, t)$$

1ST ORDER IN $t \rightarrow$

\therefore NOT T INVARIANT \rightarrow

$$\psi(\vec{r}, t) \xrightarrow{\quad \uparrow \quad} \psi(\vec{r}, -t)$$

IF T IS A SYMMETRY OF THE HAMILTONIANS

$$[H, T] = 0 \quad \text{AND IF} \quad \psi(-t) = T \psi(t)$$

THEN

$$T \left(i\hbar \frac{\partial \psi}{\partial t} = H\psi \right) \rightarrow i\hbar \frac{\partial [T\psi]}{\partial (-t)} = HT\psi$$

SINCE

$$T \psi(t) \rightarrow \psi(-t)$$

$$-i\hbar \frac{\partial \psi(-t)}{\partial t} = H\psi(-t)$$

THIS IS NOT SAME AS $i\hbar \frac{\partial \psi(t)}{\partial t} = H\psi(t)$

$\psi(-t)$ AND $\psi(t)$ OBEY DIFFERENT DYNAMICS

CAN MAINTAIN TIME REVERSAL SYMMETRY

IF DEFINE! $\psi(\vec{r}, t) \xrightarrow{T} \psi^*(\vec{r}, -t)$

TIME REVERSED SCHRÖDINGER

$$i\hbar \frac{\partial [T\psi]}{\partial (-t)} = H T\psi$$

$$\text{IS } -i\hbar \frac{\partial \psi^*(-t)}{\partial t} = H \psi^*(-t)$$

TAKE COMPLEX CONJUGATE

$$i\hbar \frac{\partial \psi(-t)}{\partial t} = H \psi(-t)$$

THIS IS SAME
SCHRÖDINGER, IF

$$\psi(t) \xrightarrow{T} \psi^*(-t)$$

NOTE THAT

$$\psi(t) \psi^*(t) = \psi^*(-t) \psi(-t)$$

TRANSFORMATION $\psi(t) \xrightarrow{\mathbb{T}} \psi^*(-t)$

IS OK, SINCE PHYSICS $\rightarrow \psi\psi^*$

FOR PARITY, CAN HAVE EIGENVALUE EQUATION

$$P|\psi_p\rangle = \mathbb{P} |\psi_p\rangle$$

CANNOT HAVE EIGENVALUES OF \mathbb{T}

$$\mathbb{T}|\psi\rangle \rightarrow \langle\psi(-t)|$$

\mathbb{T} IS A SYMMETRY OF THE HAMILTONIAN
DOES NOT CORRESPOND TO AN OBSERVABLE

\mathbb{T}

NOT UNITARY, HERMITIAN

ANTI UNITARY, ANTI LINEAR

TIME REVERSAL IS MOTION REVERSAL

FREELY PROPAGATING PLANE WAVE STATE

$$\Psi(\vec{r}, t) = e^{i(\vec{p} \cdot \vec{r} - Et)/\hbar}$$

$$\begin{aligned} T \Psi(\vec{r}, t) &= \Psi^*(\vec{r}, -t) \\ &= e^{-i(\vec{p} \cdot \vec{r} + E \cdot t)/\hbar} \\ &= e^{i(-\vec{p} \cdot \vec{r} - Et)/\hbar} \end{aligned}$$

cf

THIS DEFINITION OF TIME REVERSAL IS
SAME AS REVERSING MOMENTUM

→ EXACTLY SAME AS CLASSICAL IDEA OF
"RUNNING THE MOVIE BACKWARDS"

$$T |\vec{p}, \vec{j}\rangle \rightarrow |-\vec{p}, -\vec{j}\rangle$$

DIRECT EXPERIMENTAL TESTS OF T INVARIANCE

TIME REVERSAL INVARIANCE IMPLIES

$$\langle i | M | f \rangle = \langle f | M | i \rangle$$

↑ MATRIX ELEMENT INITIAL → FINAL
IN STRONG INTERACTIONS TESTED BY COMPARING



SEEMS GOOD TO ~ 0.3% ← NOT VERY INTERESTING

→ NEUTRON ELECTRIC DIPOLE MOMENT

ELECTRIC DIPOLE MOMENT OF NEUTRON

THIS WOULD INDICATE T-NONINVARIANCE

IF NEUTRON HAS SPIN \vec{S}
ELECTRIC DIPOLE MOMENT $\vec{\mu}$

$$\text{THEN } \vec{\mu} = \alpha \vec{S}$$

INTERACTION ENERGY WITH ELECTRIC FIELD

$$U = \vec{\mu} \cdot \vec{E} = \alpha \vec{S} \cdot \vec{E}$$

UNDER T

$$\vec{E} \rightarrow \vec{E}$$

$$\vec{S} \rightarrow -\vec{S}$$



T INVARIANCE

$$TU = -U \neq U$$

ONLY TRUE IF $U = 0 \rightarrow \vec{\mu} = 0$

ELECTRIC DIPOLE MOMENT VIOLATES T INVARIANCE

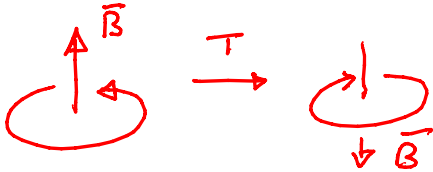
WHAT ABOUT MAGNETIC DIPOLE MOMENT?

$$\bar{\mu}_B = \beta \bar{S}$$

INTERACTION ENERGY

$$U_B = \bar{\mu}_B \cdot \bar{B} = \beta \bar{S} \cdot \bar{B}$$

HOW DOES \bar{B} TRANSFORM? \bar{B} COMES FROM MOVING CHARGE



UNDER T

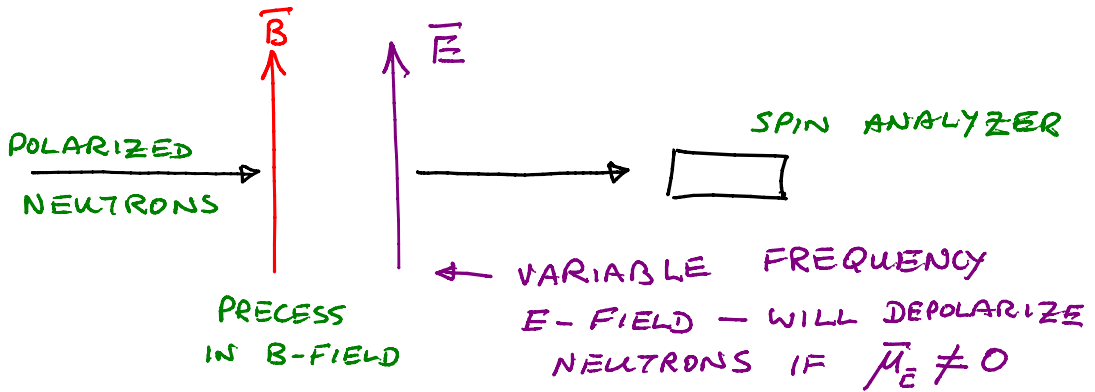
$$\begin{aligned} \bar{S} &\rightarrow -\bar{S} \\ \bar{B} &\rightarrow -\bar{B} \end{aligned}$$

$$T U_B = U_B = U_B$$

MAGNETIC DIPOLE MOMENT DOES NOT VIOLATE T INVARIANCE

SEARCH FOR NEUTRON ELECTRIC DIPOLE MOMENT

REMEMBER MAGNETIC DIPOLE MOMENT
PRECESSES IN MAGNETIC FIELD — LARMOR



IF T-INVARIANCE WAS VIOLATED?

WHAT WOULD ONE EXPECT FOR $\bar{\mu}_E$

$$\bar{\mu}_E \sim \text{CHARGE} \times \text{DISTANCE}$$

$$\sim e \times \text{SIZE OF NEUTRON}$$

$$\sim e \times 10^{-13} \text{ cm}$$

EXPERIMENTALLY $\bar{\mu}_E < e \times 3 \times 10^{-26} \text{ cm}$

T INVARIANCE IS VERY GOOD FOR
ELECTROMAGNETIC INTERACTIONS

AN ELECTRIC DIPOLE MOMENT ALSO
VIOLATES PARITY INVARIANCE

INTERACTION ENERGY WITH ELECTRIC FIELD

$$U = \vec{\mu} \cdot \vec{E} = \alpha \vec{S} \cdot \vec{E}$$

UNDER P $\vec{E} \rightarrow \vec{E}$ VECTOR
 $\vec{S} \rightarrow -\vec{S}$ AXIAL VECTOR

$$P U = P \alpha \vec{S} \cdot \vec{E} = -\alpha \vec{S} \cdot \vec{E}$$

SAME ARGUMENT

EXPECT WEAK INTERACTION EFFECTS TO
GENERATE ELECTRIC DIPOLE MOMENT

IN STANDARD MODEL $\vec{\mu}_{EW} \sim 10^{-32} e \text{ cm}$
TINY!

CHARGE CONJUGATION SYMMETRY

P & $T \rightarrow$ SPACE-TIME SYMMETRIES

$C \rightarrow$ CHANGE THE ELECTRIC CHARGE OF ALL PARTICLES

IN QUANTUM FIELD THEORY THIS MEANS INTERCHANGING PARTICLES \leftrightarrow ANTI PARTICLES

ELECTROMAGNETISM

$$\begin{array}{l} Q \xrightarrow{C} -Q \\ \vec{E} \rightarrow -\vec{E} \\ \vec{B} \rightarrow -\vec{B} \end{array}$$

INTUITIVELY ELECTROMAGNETISM IS INVARIANT UNDER SUCH A TRANSFORMATION

QUANTUM MECHANICALLY

$$|\psi(q, \vec{p}, t)\rangle \xrightarrow{C} |\psi(-q, \vec{p}, t)\rangle$$

A PARTICLE CAN BE AN EIGENSTATE OF C

$$C |\pi^0\rangle \rightarrow |\pi^0\rangle$$

FOR COMPOSITE PARTICLES, INTERNAL QUANTUM NUMBERS MAY CHANGE \rightarrow EVEN FOR NEUTRAL PARTICLES

$$C |n\rangle = C |ddu\rangle \rightarrow |d\bar{d}\bar{u}\bar{u}\rangle = |\bar{n}\rangle$$

$$C |K^0\rangle = C |d\bar{s}\rangle \rightarrow |\bar{d}s\rangle = |\bar{K}^0\rangle$$

JUST AS WITH PARITY, 2 SUCCESSIVE C OPERATIONS LEAVE A STATE UNCHANGED

C HAS EIGENVALUES ± 1

$$C|\bar{E}\rangle \rightarrow -|\bar{E}\rangle \quad \rightarrow C|\gamma^{\mu}\rangle = -|\gamma^{\mu}\rangle$$

$$C|\bar{B}\rangle \rightarrow -|\bar{B}\rangle$$

γ^{μ} HAS -VE "C-PARITY"

$$\text{IF } [C, H] = 0$$

TOTAL C-PARITY IS CONSERVED IN INTERACTIONS

$$\pi^0 \rightarrow \gamma \gamma$$

$$\eta_c(\pi^0) = \eta_c(\gamma) \cdot \eta_c(\gamma)$$

$$= (-1) \cdot (-1) = +1$$

OBSERVABLE

CONSEQUENCE

$$\pi^0 \not\rightarrow \begin{matrix} 3\gamma \\ \gamma\gamma \\ \vdots \end{matrix}$$

$$C = -1$$

CP SYMMETRY

CPT \rightarrow GOOD SYMMETRY FOR ALL ~~H~~

EXPECT T SYMMETRY GOOD

\therefore CP SHOULD ALSO BE A GOOD SYMMETRY

WE KNOW WEAK INTERACTIONS VIOLATES P
SO IT MUST VIOLATE C FOR CP GOOD

$| \nu_L \rangle \xrightarrow{C} | \bar{\nu}_L \rangle$ $\left. \begin{array}{l} \text{DOESN'T EXIST} \\ \therefore \text{NO C SYMMETRY} \end{array} \right\}$

$| \nu_L \rangle \xrightarrow{C} | \bar{\nu}_L \rangle \xrightarrow{P} | \bar{\nu}_R \rangle$ $\left. \begin{array}{l} \text{DOES EXIST} \end{array} \right\}$

\therefore LOOKS LIKE CP IS A GOOD SYMMETRY
OF THE WEAK INTERACTIONS

Table 3.1. *Tests of the CPT theorem*

Measured quantity	Limit or value
$(M_{K^0} - M_{\bar{K}^0})/(M_{K^0} + M_{\bar{K}^0})$	$< 10^{-19}$
$(M_{e^+} - M_{e^-})/(M_{e^+} + M_{e^-})$	$< 4 \times 10^{-8}$
$(M_{\Lambda} - M_{\bar{\Lambda}})/(M_{\Lambda} + M_{\bar{\Lambda}})$	$(-5 \pm 5) \times 10^{-6}$
$(Q_p - Q_{\bar{p}})/e$	$< 2 \times 10^{-5}$
$\left(\frac{Q_p}{M_p} - \frac{Q_{\bar{p}}}{M_{\bar{p}}}\right) / \left(\frac{Q_p}{M_p} + \frac{Q_{\bar{p}}}{M_{\bar{p}}}\right)$	$(8 \pm 6) \times 10^{-10}$
$(\mu_{e^+} - \mu_{e^-})/(\mu_{e^+} + \mu_{e^-})$	$-(3 \pm 5) \times 10^{-13}$
$(\tau_{\mu^+} - \tau_{\mu^-})/(\tau_{\mu^+} + \tau_{\mu^-})$	$< 10^{-4}$

Table 3.2. *Effect of T and P operations*

Quantity		Effect of T	Effect of P
position	r	r	-r
momentum	p	-p	-p
spin	σ , axial vector (r × p)	-σ	σ
electric field	E (= -∇V)	E	-E
magnetic field	B , axial vector	-B	B
magnetic dipole moment	$\sigma \cdot B$	$\sigma \cdot B$	$\sigma \cdot B$
electric dipole moment	$\sigma \cdot E$	$-\sigma \cdot E$	$-\sigma \cdot E$
longitudinal polarisation	$\sigma \cdot p$	$\sigma \cdot p$	$-\sigma \cdot p$
transverse polarisation	$\sigma \cdot (p_1 \times p_2)$	$-\sigma \cdot (p_1 \times p_2)$	$\sigma \cdot (p_1 \times p_2)$

