TIME REVERSAL

IN QUANTUM FIELD THEORY, ALL SENSIBLE HAMILTONIANS HAVE THE SYMMETRY!

$$[CPT, H]$$

CHARGE CONJUGATION

PARTICLES → ANTIPARTICLES

PARITY

TIME REVERSAL

WEAK INTERACTION → VIOLATES P ∴ → ALSO SOME COMBINATION OF CT

$$t \xrightarrow{T} -t$$

$$\bar{r} \longrightarrow \bar{r}$$

$$\bar{p} = m\dot{\bar{r}} \longrightarrow -m\dot{\bar{r}} = -\bar{p}$$

$$\bar{L} = \bar{r} \times \bar{p} \longrightarrow \bar{r} \times (-\bar{p}) = -\bar{L}$$

CLASSICALLY

$$m\frac{d^2\bar{r}}{dt^2} = \bar{F}$$

MANIFESTLY INVARIANT UNDER $t \rightarrow -t$

2ND ORDER DIFFERNTIAL → ALL MICROSCOPIC CLASSICAL SYSTEMS ARE T INVARIANT

USUALLY "ARROW OF TIME" ASSERTED TO COME FROM ENTROPY → READ R. PENROSE.

IN QUANTUM MECHANICS SCHRÖDINGER SHOWS HOW TIME EVOLUTION DEPENDS ON HAMILTONIAN

$$i\hbar\frac{\partial\psi(\bar{r},t)}{\partial t} = H\,\psi(\bar{r},t)$$

1ST ORDER IN t →

∴ NOT T INVARIANT →

$$\psi(\bar{r},t) \xrightarrow{\;T\;} \psi(\bar{r},-t)$$

IF T IS A SYMMETRY OF THE HAMILTONIAN

$[H, T] = 0$ AND IF $\psi(-t) = T\psi(t)$

THEN

$$T\left(i\hbar \frac{\partial \psi}{\partial t} = H\psi \right) \rightarrow i\hbar \frac{\partial [TH]}{\partial(-t)} = HT\psi$$

SINCE $T\psi(t) \longrightarrow \psi(-t)$

$$-i\hbar \frac{\partial \psi(-t)}{\partial t} = H\psi(-t)$$

THIS IS NOT SAME AS $i\hbar \frac{\partial \psi(t)}{\partial t} = H\psi(t)$

$\psi(-t)$ AND $\psi(t)$ OBEY DIFFERENT DYNAMICS

CAN MAINTAIN TIME REVERSAL SYMMETRY IF DEFINE:

$$\psi(\vec{r},t) \xrightarrow{T} \psi^*(\vec{r},-t)$$

TIME REVERSED SCHRÖDINGER

$$i\hbar \frac{\partial[T\psi]}{\partial(-t)} = HT\psi$$

IS

$$-i\hbar \frac{\partial \psi^*(-t)}{\partial t} = H\psi^*(-t)$$

TAKE COMPLEX CONJUGATE

$$i\hbar \frac{\partial \psi(-t)}{\partial t} = H\psi(-t)$$

THIS IS SAME SCHRÖDINGER, IF

$$\psi(t) \xrightarrow{T} \psi^*(-t)$$

NOTE THAT

$$\psi(t)\,\psi^*(t) = \psi^*(-t)\,\psi(-t)$$

TRANSFORMATION $\psi(t) \xrightarrow{T} \psi^*(-t)$

IS OK, SINCE PHYSICS $\rightarrow \psi\psi^*$

FOR PARITY, CAN HAVE EIGENVALUE EQUATION

$$P|\psi_p\rangle = \pi\ |\psi_p\rangle$$

CANNOT HAVE EIGENVALUES OF T

$$T|\psi\rangle \rightarrow \langle\psi(-t)|$$

T IS A SYMMETRY OF THE HAMILTONIAN

DOES NOT CORRESPOND TO AN OBSERVABLE

T NOT UNITARY, HERMITIAN

ANTI UNITARY, ANTI LINEAR

TIME REVERSAL IS MOTION REVERSAL

FREELY PROPAGATING PLANE WAVE STATE

$$\psi(\bar{r},t) = e^{i(\bar{p}\cdot\bar{r}-Et)/\hbar}$$

$$T\,\psi(\bar{r},t) = \psi^*(\bar{r},-t)$$

$$= e^{-i(\bar{p}\cdot\bar{r}+E\cdot t)/\hbar}$$

$$= e^{i(-\bar{p}\cdot\bar{r}-Et)}$$

cf

THIS DEFINITION OF TIME REVERSAL IS SAME AS REVERSING MOMENTUM

→ EXACTLY SAME AS CLASSICAL IDEA OF "RUNNING THE MOVIE BACKWARDS"

$$T\,|\bar{p},\bar{J}\rangle \longrightarrow |-\bar{p},-\bar{J}\rangle$$

DIRECT EXPERIMENTAL TESTS OF T INVARIANCE

TIME REVERSAL INVARIANCE IMPLIES

$$\langle i | M | f \rangle = \langle f | M | i \rangle$$

MATRIX ELEMENT INITIAL → FINAL

IN STRONG INTERACTIONS TESTED BY COMPARING

$$p + {}^{37}Al \rightarrow \alpha + {}^{24}Mg$$

$$\alpha + {}^{24}Mg \rightarrow p + {}^{27}Al$$

SEEMS GOOD TO $\sim$ 0.3‰ ← NOT VERY INTERESTING

→ NEUTRON ELECTRIC DIPOLE MOMENT

ELECTRIC DIPOLE MOMENT OF NEUTRON

THIS WOULD INDICATE T- NONINVARIANCE

IF NEUTRON HAS SPIN $\bar{S}$

ELECTRIC DIPOLE MOMENT $\bar{\mu}$

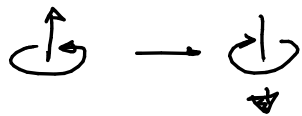
THEN $\bar{\mu} = \alpha \bar{S}$

INTERACTION ENERGY WITH ELECTRIC FIELD

$$\mathcal{U} = \bar{\mu} \cdot \bar{E} = \alpha \bar{S} \cdot \bar{E}$$

UNDER T

$$\bar{E} \to \bar{E}$$
$$\bar{S} \to -\bar{S}$$



T INVARIANCE $T\mathcal{U} = -\mathcal{U} \neq \mathcal{U}$

ONLY TRUE IF $\mathcal{U} = 0 \to \bar{\mu} = 0$

ELECTRIC DIPOLE MOMENT VIOLATES T INVARIANCE

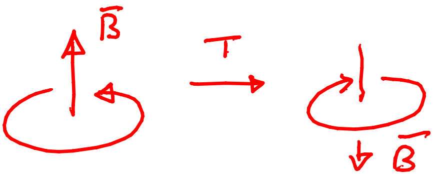
WHAT ABOUT MAGNETIC DIPOLE MOMENT?

$$\bar{\mu}_B = \beta \bar{S}$$

INTERACTION ENERGY

$$U_B = \bar{\mu}_B \cdot \bar{B} = \beta \bar{S} \cdot \bar{B}$$

HOW DOES $\bar{B}$ TRANSFORM? $\bar{B}$ COMES FROM MOVING CHARGE



UNDER T

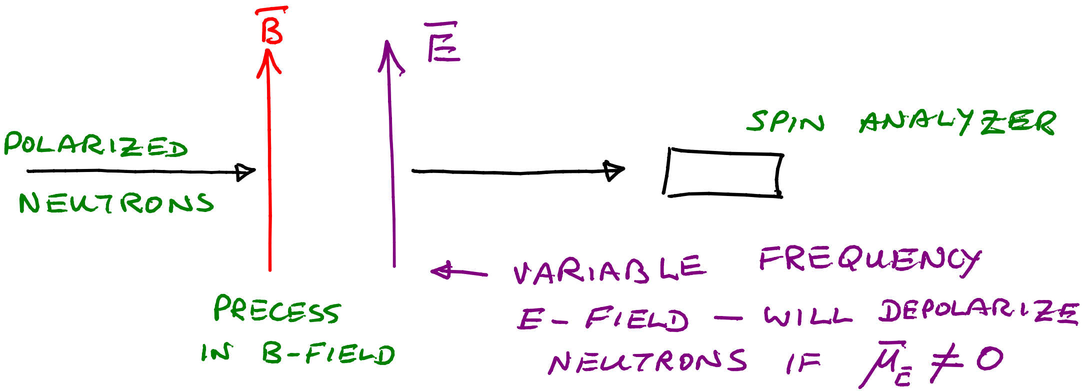
$$\bar{S} \to -\bar{S}$$
$$B \to -\bar{B}$$

$$T\, U_B = U_B = U_B$$

MAGNETIC DIPOLE MOMENT DOES NOT VIOLATE T INVARIANCE

SEARCH FOR NEUTRON ELECTRIC DIPOLE MOMENT

REMEMBER MAGNETIC DIPOLE MOMENT
PRECESSES IN MAGNETIC FIELD — LARMOR



IF T-INVARIANCE WAS VIOLATED?

WHAT WOULD ONE EXPECT FOR $\vec{\mu}_E$

$$\vec{\mu}_E \sim \text{CHARGE} \times \text{DISTANCE}$$

$$\sim e \times \text{SIZE OF NEUTRON}$$

$$\sim e \times 10^{-13}\ \text{cm}$$

EXPERIMENTALLY $\vec{\mu}_E < e \times 3 \times 10^{-26}\ \text{cm}$

T INVARIANCE IS VERY GOOD FOR ELECTROMAGNETIC INTERACTION

AN ELECTRIC DIPOLE MOMENT ALSO VIOLATES PARITY INVARIANCE

INTERACTION ENERGY WITH ELECTRIC FIELD

$$\mathcal{U} = \bar{\mu} \cdot \bar{E} = \alpha \, \bar{S} \cdot \bar{E}$$

UNDER P

$$\bar{E} \rightarrow \bar{E} \qquad \text{VECTOR}$$

$$\bar{S} \rightarrow -\bar{S} \qquad \text{AXIAL VECTOR}$$

$$P\,\mathcal{U} = P\,\alpha\,\bar{S}\cdot\bar{E} = -\alpha\,\bar{S}\cdot\bar{E}$$

SAME ARGUMENT

EXPECT WEAK INTERACTION EFFECTS TO GENERATE ELECTRIC DIPOLE MOMENT

IN STANDARD MODEL $\bar{\mu}_{EW} \sim 10^{-32}\,e\ \mathrm{cm}$

TINY !

CHARGE CONJUGATION SYMMETRY

P & T $\rightarrow$ SPACE-TIME SYMMETRIES

C $\rightarrow$ CHANGE THE ELECTRIC CHARGE OF ALL PARTICLES

IN QUANTUM FIELD THEORY THIS MEANS INTERCHANGING PARTICLES $\leftrightarrow$ ANTIPARTICLES

ELECTROMAGNETISM

$$Q \xrightarrow{C} -Q$$

$$\vec{E} \rightarrow -\vec{E}$$

$$\vec{B} \rightarrow -\vec{B}$$

INTUITIVELY ELECTROMAGNETISM IS INVARIANT UNDER SUCH A TRANSFORMATION

QUANTUM MECHANICALLY

$$|\psi(Q, \vec{r}, t)\rangle \xrightarrow{C} |\psi(-Q, \vec{r}, t)\rangle$$

A PARTICLE CAN BE AN EIGENSTATE OF C

$$C\,|\pi^0\rangle \rightarrow |\pi^0\rangle$$

FOR COMPOSITE PARTICLES, INTERNAL QUANTUM NUMBERS MAY CHANGE → EVEN FOR NEUTRAL PARTICLES

$$C\,|n\rangle = C\,|ddu\rangle \rightarrow |\bar{d}\bar{d}\bar{u}\rangle = |\bar{n}\rangle$$

$$C\,|K^0\rangle = C\,|d\bar{s}\rangle \rightarrow |\bar{d}s\rangle = |\bar{K}^0\rangle$$

JUST AS WITH PARITY, 2 SUCCESSIVE C OPERATIONS LEAVE A STATE UNCHANGED

C HAS EIGENVALUES ± 1

$C|\vec{E}\rangle \to -|\vec{E}\rangle$

$C|\vec{B}\rangle \to -|\vec{B}\rangle$

$\to C|\gamma\rangle = -|\gamma\rangle$

γ HAS -VE "C-PARITY"

IF $[C, H] = 0$ TOTAL C-PARITY IS CONSERVED IN INTERACTIONS

$$\pi^0 \to \gamma\gamma$$

$$\eta_c(\pi^0) = \eta_c(\gamma)\cdot\eta_c(\gamma)$$

$$= (-1)\cdot(-1) = +1$$

OBSERVABLE CONSEQUENCE

$$\pi^0 \not\to 3\gamma \qquad C = -1$$

$$5\gamma$$

CP SYMMETRY

CPT → GOOD SYMMETRY FOR ALL H

EXPECT T SYMMETRY GOOD

∴ CP SHOULD ALSO BE A GOOD SYMMETRY

WE KNOW WEAK INTERACTION VIOLATES P
SO IT MUST VIOLATE C FOR CP GOOD

$$|\nu_L\rangle \xrightarrow{C} |\bar{\nu}_L\rangle$$

DOESN'T EXIST
∴ NO C SYMMETRY

$$|\nu_L\rangle \xrightarrow{C} |\bar{\nu}_L\rangle \xrightarrow{P} |\bar{\nu}_R\rangle$$

DOES EXIST

∴ LOOKS LIKE CP IS A GOOD SYMMETRY
OF THE WEAK INTERACTION

Table 3.1. *Tests of the CPT theorem*

Measured quantity	Limit or value
$(M_{K^0} - M_{\bar{K}^0})/(M_{K^0} + M_{\bar{K}^0})$	$< 10^{-19}$
$(M_{e^+} - M_{e^-})/(M_{e^+} + M_{e^-})$	$< 4 \times 10^{-8}$
$(M_\Lambda - M_{\bar{\Lambda}})/(M_{\bar{\Lambda}} + M_\Lambda)$	$(-5 \pm 5) \times 10^{-6}$
$(Q_p - Q_{\bar{p}})/e$	$< 2 \times 10^{-5}$
$\left(\frac{Q_p}{M_p} - \frac{Q_{\bar{p}}}{M_{\bar{p}}}\right) \Big/ \left(\frac{Q_p}{M_p} + \frac{Q_{\bar{p}}}{M_{\bar{p}}}\right)$	$(8 \pm 6) \times 10^{-10}$
$(\mu_{e^+} - \mu_{e^-})/(\mu_{e^+} + \mu_{e^-})$	$-(3 \pm 5) \times 10^{-13}$
$(\tau_{\mu^+} - \tau_{\mu^-})/(\tau_{\mu^+} + \tau_{\mu^-})$	$< 10^{-4}$

Table 3.2. *Effect of T and P operations*

Quantity		Effect of T	Effect of P
position	$\mathbf{r}$	$\mathbf{r}$	$-\mathbf{r}$
momentum	$\mathbf{p}$	$-\mathbf{p}$	$-\mathbf{p}$
spin	$\boldsymbol{\sigma}$, axial vector $(\mathbf{r} \times \mathbf{p})$	$-\boldsymbol{\sigma}$	$\boldsymbol{\sigma}$
electric field	$\mathbf{E}\ (= -\nabla V)$	$\mathbf{E}$	$-\mathbf{E}$
magnetic field	$\mathbf{B}$, axial vector	$-\mathbf{B}$	$\mathbf{B}$
magnetic dipole moment	$\boldsymbol{\sigma} \cdot \mathbf{B}$	$\boldsymbol{\sigma} \cdot \mathbf{B}$	$\boldsymbol{\sigma} \cdot \mathbf{B}$
electric dipole moment	$\boldsymbol{\sigma} \cdot \mathbf{E}$	$-\boldsymbol{\sigma} \cdot \mathbf{E}$	$-\boldsymbol{\sigma} \cdot \mathbf{E}$
longitudinal polarisation	$\boldsymbol{\sigma} \cdot \mathbf{p}$	$\boldsymbol{\sigma} \cdot \mathbf{p}$	$-\boldsymbol{\sigma} \cdot \mathbf{p}$
transverse polarisation	$\boldsymbol{\sigma} \cdot (\mathbf{p}_1 \times \mathbf{p}_2)$	$-\boldsymbol{\sigma} \cdot (\mathbf{p}_1 \times \mathbf{p}_2)$	$\boldsymbol{\sigma} \cdot (\mathbf{p}_1 \times \mathbf{p}_2)$