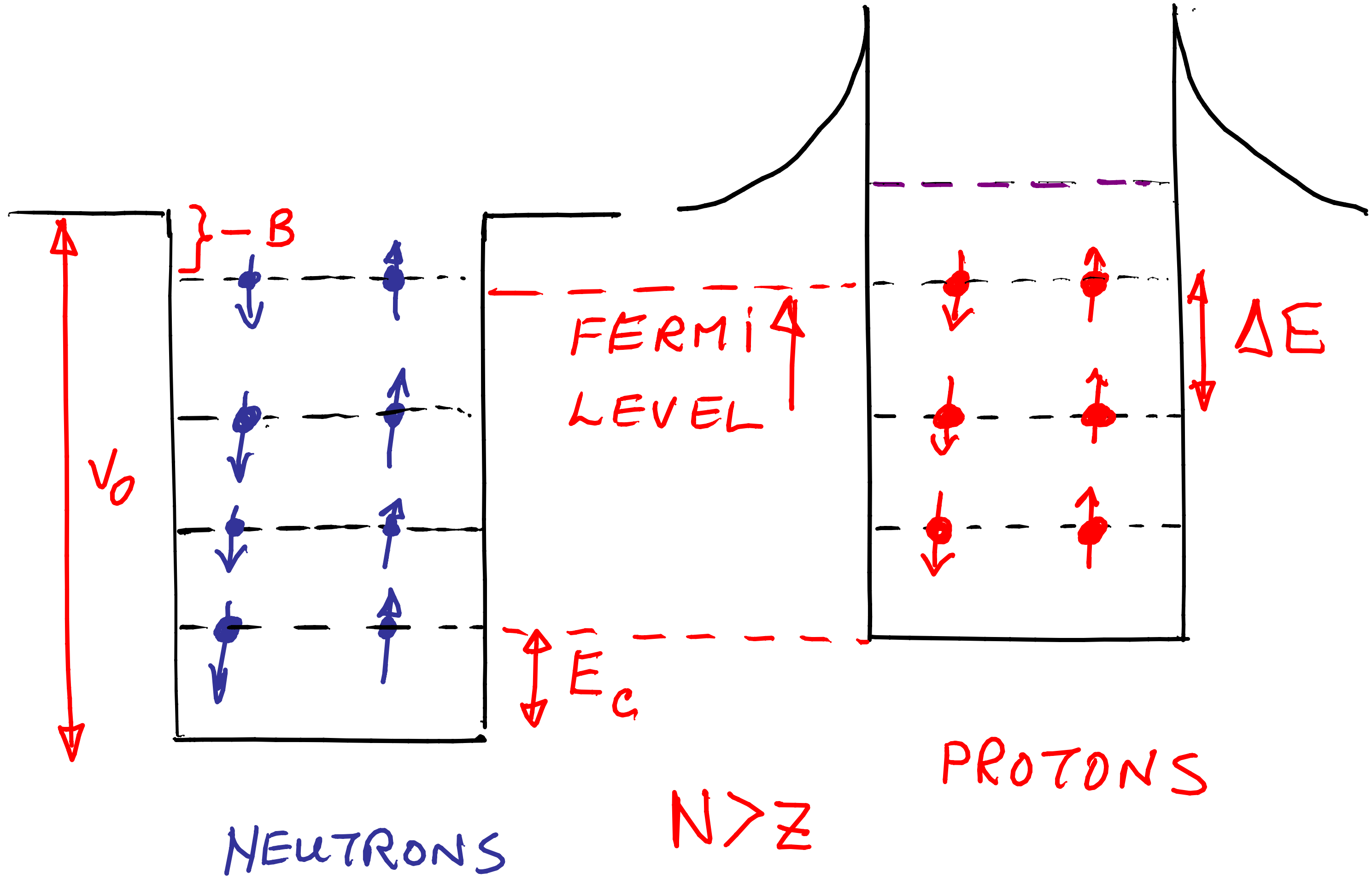
FERMI GAS MODEL



NEUTRON AND PROTON POTENTIAL WELL DEPTHS $\longrightarrow$ DIFFERENT.

IF THEY WERE THE SAME IN HEAVY NUCLEI WHERE $N > Z \rightarrow E_F^{NEUTRON} > E_F^{PROTON}$

THEN BINDING ENERGY OF LAST NUCLEON WOULD BE CHARGE DEPENDANT

$\rightarrow$ NOT TRUE
CHARGE INDEPENDENCE

- ALSO IF $E_F^N > E_F^P$ NEUTRONS WOULD BE ENERGETICALLY FAVOURED TO DECAY TO PROTONS VIA β-DECAY $\rightarrow$ PROTONS

$\rightarrow$ $N > Z$ UNSTABLE $\rightarrow$ NOT TRUE

NON-RELATIVISTICALLY, THE FERMI ENERGY

$$E_F = \frac{p_F^2}{2M}$$ NUCLEON MASS

VOLUME IN MOMENTUM SPACE

$$V_{p_F} = \frac{4\pi}{3} p_F^3$$

PHASE SPACE = CONFIGURATION SPACE × MOMENTUM SPACE

$$V_{TOT} = \frac{4\pi}{3} r_0^3 A \cdot \frac{4\pi}{3} p_F^3$$

$$= \left(\frac{4\pi}{3}\right)^2 A \cdot (r_0 p_F)^3$$

$\propto$ NUMBER OF QUANTUM STATES OF SYSTEM

NUMBER OF FERMIONS THAT CAN FILL STATES UP TO THE FERMI LEVEL

$$n_F = \frac{2\,V_{TOT}}{(2\pi\hbar)^3} = \frac{4}{9\pi} A \left(\frac{r_0 p_F}{\hbar}\right)^3$$

SPIN UP/DOWN

FOR $N = Z = A/2 \rightarrow$ ALL STATES FILLED

$$N = Z = A/2 = 4/9\pi \cdot A \left(\frac{r_0 p_F}{\hbar}\right)^3$$

$$p_F = \frac{\hbar}{r_0}\left(\frac{9\pi}{8}\right)^{1/3}$$

DOES NOT DEPEND ON NUMBER OF NUCLEONS

$$p_F = \frac{\hbar}{r_0}\left(\frac{9\pi}{8}\right)^{1/3}$$

$$E_F = \frac{p_F^2}{2M} = \frac{1}{2M}\left(\frac{\hbar}{r_0}\right)^2\left(\frac{9\pi}{8}\right)^{2/3} \sim 33 \text{ MeV}$$

NOW $B/A \sim -8 \text{ MeV}$

SO THE DEPTH OF THE POTENTIAL WELL

$$V_0 = E_F + B \approx 40 \text{ MeV}$$

THE NUCLEONS ARE NON-RELATIVISTIC

AVERAGE KINETIC ENERGY

$$\langle E \rangle = \frac{\int_0^{p_F} E \, d^3p}{\int_0^{p_F} d^3p}$$

$$= \frac{\int_0^{p_F} \frac{p_F^2}{2M} \cdot p^2 \sin\theta \, d\theta \, d\phi \, dp}{\int_0^{p_F} p^2 \sin\theta \, d\theta \, d\phi \, dp}$$

$$= \frac{p_F^5}{5} \frac{1}{2M} \cdot \frac{3}{p_F^2} = \frac{3}{5} \frac{p_F^2}{2M}$$

$$\approx 24 \, \mathrm{MeV}$$

TOTAL AVERAGE KINETIC ENERGY

$$\langle E(Z,N)\rangle = N\langle E_N\rangle + Z\langle E_Z\rangle$$

$$= \frac{3}{10M}\left(N p_N^2 + Z p_Z^2\right)$$

USE $\quad N = Z = \frac{A}{2} = \frac{4}{9\pi} A \left(\frac{r_0 p_F}{\hbar}\right)^3$

$$\langle E(Z,N)\rangle = \frac{3}{10M}\,\frac{\hbar^2}{r_0^2}\left(\frac{9\pi}{4}\right)^{2/3}\frac{N^{5/3}+Z^{5/3}}{A^{2/3}}$$

FOR A GIVE VALUE OF A

$\langle E(Z,N)\rangle$ MINIMUM WHEN $N = Z = \dfrac{A}{2}$

LOOK AT BEHAVIOUR AROUND MINIMUM

PUT $\quad Z - N = \varepsilon \quad , \quad Z + N = A$ FIXED

$$Z = \frac{A}{2}\left(1 + \varepsilon/A\right), \quad N = \frac{A}{2}\left(1 - \varepsilon/A\right)$$

IF ASSUME $\quad \varepsilon/A \ll 1$

$$Z^{5/3} = \frac{A}{2}^{5/3}\left(1 + \frac{\varepsilon}{A}\right)^{5/3} = \frac{A^{5/3}}{2}\left(1 + \frac{5}{3}A + \frac{5/3(5/3-1)}{2}A^2 + \cdots\cdots\right.$$

$$N = \frac{A^{5/3}}{2}\left(1 - 5/3\,A + \frac{5/3(5/3-1)A^2}{2} + \cdots\cdots\right)$$

INSERT THESE EXPASIONS INTO

$$\langle E(Z,N)\rangle = \frac{3}{10M}\frac{\hbar^2}{r_0^2}\left(\frac{9\pi}{4}\right)^{2/3}\left(\frac{N^{5/3}+Z^{5/3}}{A^{2/3}}\right)$$

AND PUT $\varepsilon \to N-Z$

$$\langle E(Z,N)\rangle = \frac{3}{10M}\frac{\hbar^2}{r_0^2}\left(\frac{9\pi}{8}\right)^{2/3}\left(A+\frac{5}{9}\frac{(Z-N)^2}{A}\right)$$

ASYMMETRY TERM — $\frac{\langle E\rangle}{A} \propto \frac{(Z-N)^2}{A^2}$