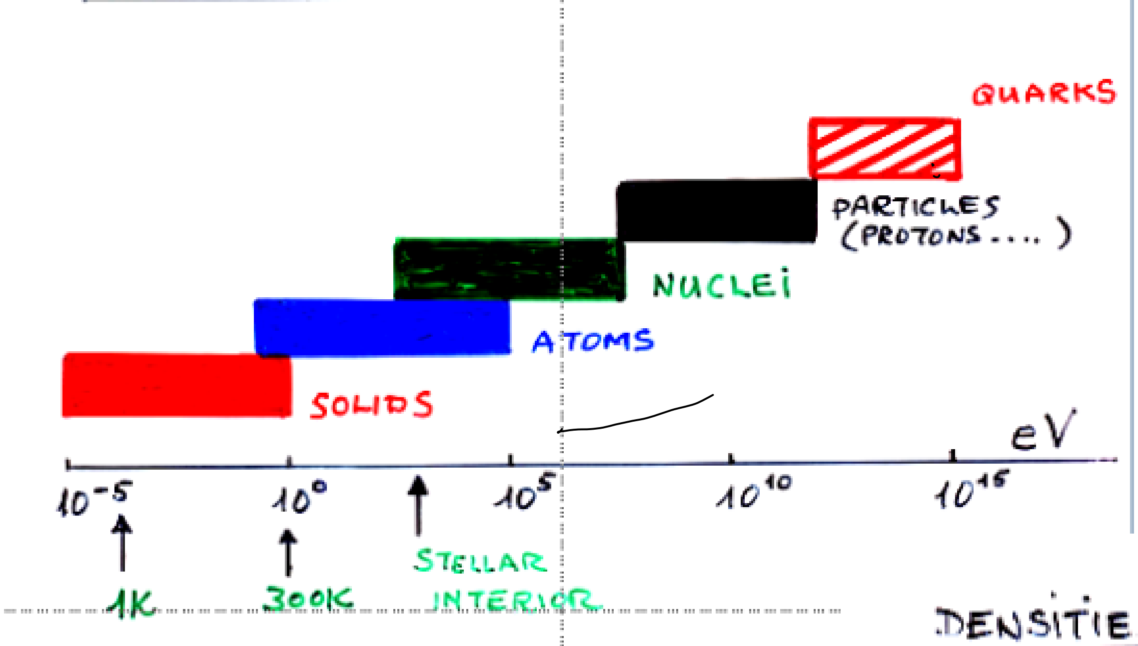
ENERGY DENSITIES



DENSITIES

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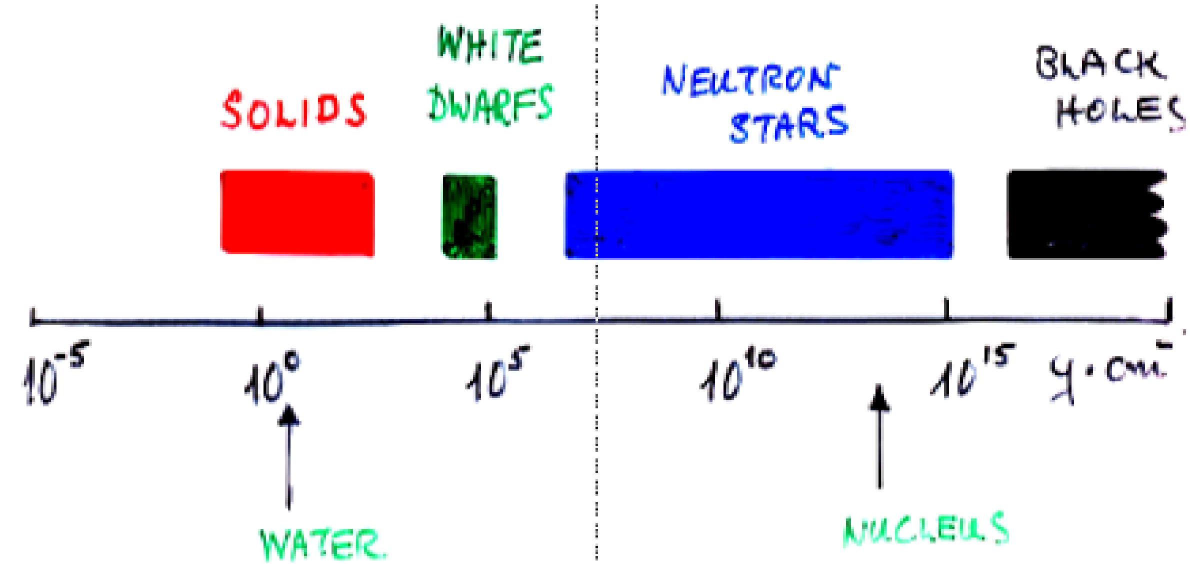


SPECIAL RELATIVITY

- TO REACH HIGH ENERGY DENSITIES AND SMALL DISTANCES USE PROBES $v \approx c$
- RESULTS OF MEASUREMENTS DEPEND ON:
  LORENTZ FRAME
- ANY SENSIBLE THEORY MUST BE:
  COVARIANT → LORENTZ INVARIANT
- BASED ON PROPERTIES / QUANTITIES THAT DO NOT DEPEND ON LORENTZ FRAME
  E.G. REST MASS
  PROPER LIFETIME
- FORMULATE IN TERMS OF 4-VECTORS

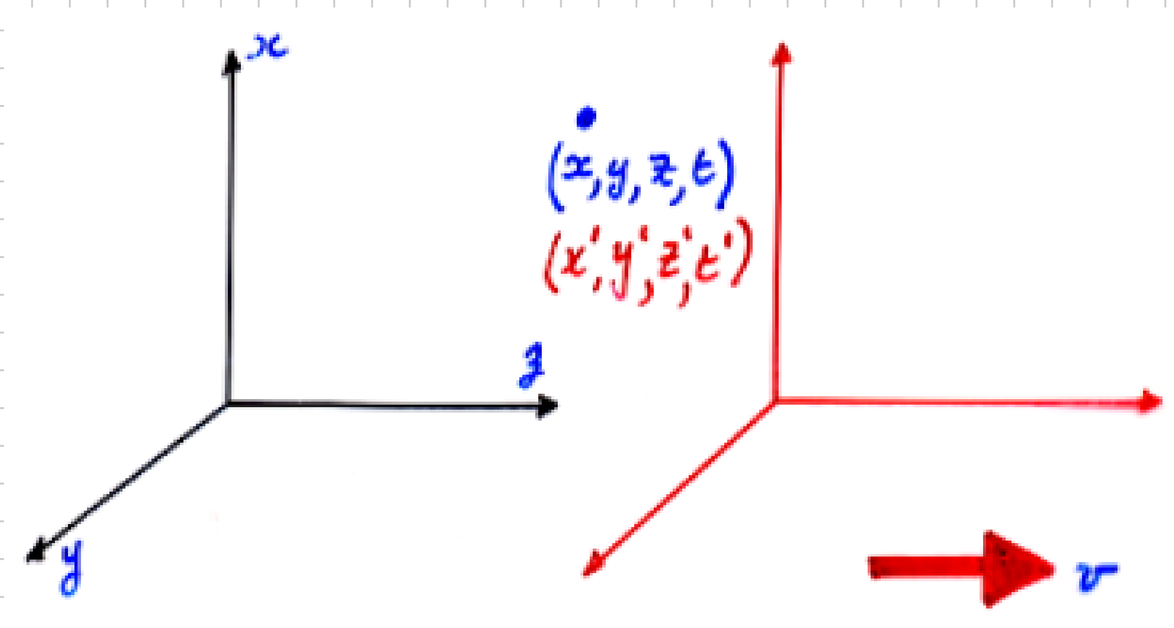
FIRST LET'S REMEMBER SIMPLE SPECIAL RELATIVITY

→ THEN MAKE IT A BIT MORE ELEGANT WITH 4-VECTORS.

SPECIAL RELATIVITY

ELECTRONS, PROTONS, NUCLEI HAVE ELECTRIC CHARGES.

MUCH OF WHAT WE TALK ABOUT WILL INVOLVE ELECTRO MAGNETIC INTERACTION OF PARTICLE AND IT WILL BE RELATIVISTIC.



$$x' = x \; ; \; y' = y$$

$$\left.\begin{aligned} z' &= \gamma(z - vt) \\ t' &= \gamma\left(t - \frac{\beta}{c} \cdot z\right) \end{aligned}\right\} \text{note mixing of } z \text{ and } t.$$

$$t' = \gamma (t - \beta/c \, z)$$

γ IS THE LORENTZ BOOST FACTOR

β IS VELOCITY IN UNITS WHERE $C=1$

$$\gamma = \frac{1}{(1-\beta^2)^{1/2}} \quad ; \quad \beta = v/c$$

$$\bar{p} = m \gamma \bar{v}$$

SO

$$v^2 = \frac{p^2}{m^2\gamma^2} = \frac{p^2}{m^2}(1-\beta^2) = \frac{p^2}{m^2}\left(1 - v^2/c^2\right)$$

$$v^2 m^2 c^4 = c^4 p^2 - p^2 v^2 c^2$$

$$v^2 \underbrace{(m^2c^4 + p^2c^2)}_{E^2} = c^2 \cdot c^2 p^2$$

$$\frac{v^2}{c^2} E^2 = c^2 p^2 \longrightarrow \beta = \frac{cp}{E}$$

$\beta \to 1$

$cp \to E$

$p \to E$ (UNITS $C=1$)

- 3 DIMENSIONS $\left.\begin{matrix}\bar{x}\\ \bar{p}\end{matrix}\right\}$ VECTORS

- 4 DIMENSIONS $(ct, \bar{x})$

$\left(\frac{E}{c}, \bar{p}\right)$ ENERGY MOMENTUM

$\left(\frac{E}{c}, \bar{p}\right) \cdot \left(\frac{E}{c}, \bar{p}\right) \rightarrow$ LORENTZ INVARIANT

$\rightarrow \; = \frac{E^2}{c^2} - p^2$ WHY?

KNOW $E^2 = p^2c^2 + m^2c^4 \rightarrow \frac{E^2}{c^2} - p^2 = m^2c^2$ → REST MASS

OBVIOUSLY UNITS WHERE $c = 1$ CONVENIENT

$(E, \bar{p}) \cdot (E, \bar{p}) = E^2 - p^2 = m^2$

LORENTZ INVARIANT → SCALAR PRODUCT

FOUR VECTORS

- 3 DIMENSIONAL VECTOR $\rightarrow$ (x, y, z), (p_x, p_y, p_z)
- VECTORS NOT FRAME INVARIANT
- SCALAR PRODUCTS ARE FRAME INVARIANT
  - ↳ DISTANCE $(\bar{s} \cdot \bar{s}) \rightarrow$ FRAME INVARIANT
- CAN DEFINE 4-DIMENSIONAL VECTOR

get dimensions correct

$$(ct, x, y, z) = (ct, \bar{x}) \rightarrow \text{SPACE POINT}$$

$$(ct, \bar{x}) \cdot (ct, \bar{x}) \rightarrow \text{INVARIANT INTERVAL}$$

ANYTHING WHICH TRANSFORMS LIKE

$$(ct, \bar{x})$$

UNDER A LORENTZ TRANSFORM IS A

4-VECTOR

4 - VECTOR DEFINITIONS

THE PHYSICS IS $E^2 = p^2c^2 + m^2c^4$ [C=1]

THIS IS WHAT DEFINES MULTIPLICATION RULE

$$(E, \bar{p}) \cdot (E, \bar{p}) = E^2 - p^2 = m^2$$

BETTER NOTATION MATHEMATICALLY

$$A \cdot B = g_{\mu\nu} A^\mu B^\nu = g^{\mu\nu} A_\mu B_\nu = A^\mu B_\mu$$

$\mu = 0, 1, 2, 3$

CONTRAVARIANT $A^\mu = (A^0, \bar{A})$

COVARIANT $A_\mu = (A^0, -\bar{A})$

METRIC

$$g_{\mu\nu} = \begin{pmatrix} +1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix}$$

LORENTZ INVARIANT → UPPER & LOWER INDICES BALANCE

$$(E, \bar{p}) = p^\mu = p$$

$$p \cdot p = p^\mu p_\mu = E^2 - p^2 = m^2$$

MINKOWSKI NOTATION

SOMETIMES SEE FOLLOWING NOTATION (e.g. PERKINS)

3 REAL "SPACE" COMPONENTS

1 IMAGINARY "TIME" COMPONENT

$$p_x \quad p_y \quad p_z \quad E$$

$$\mu = 1, 2, 3, 4$$

$$p_1 = p_x, \quad p_2 = p_y, \quad p_3 = p_z, \quad p_4 = i E$$

$$p = (\bar{p}, i E)$$

$$p^2 = \sum_\mu p_\mu^2 = p_1^2 + p_2^2 + p_3^2 + p_2^4$$

$$= \bar{p}^2 - E^2 = -m^2$$

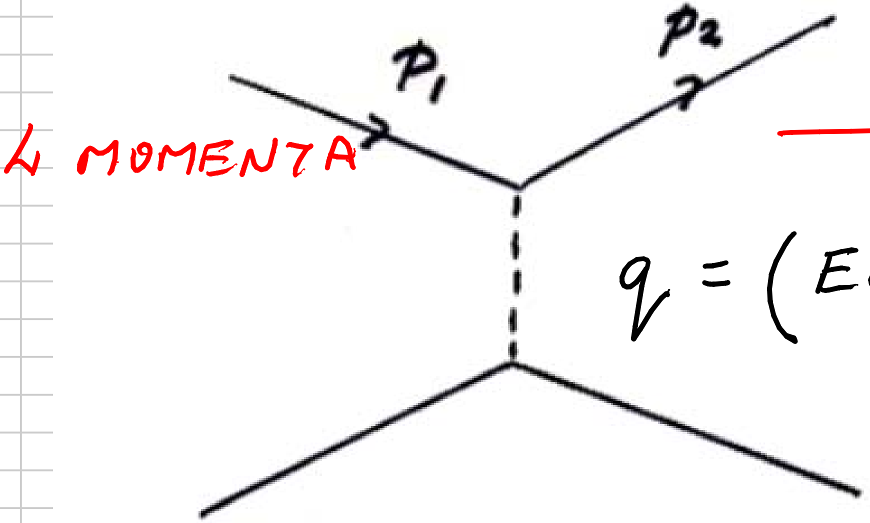
PHYSICS IS THE SAME

$E^2 = p^2 + m^2$

$$p^2 = -m^2$$

JUST DEFINITION OF p^2

FOR A SCATTERING PROCESS



$$q = (E_q, \bar{p}_q)$$

EXCERCISE

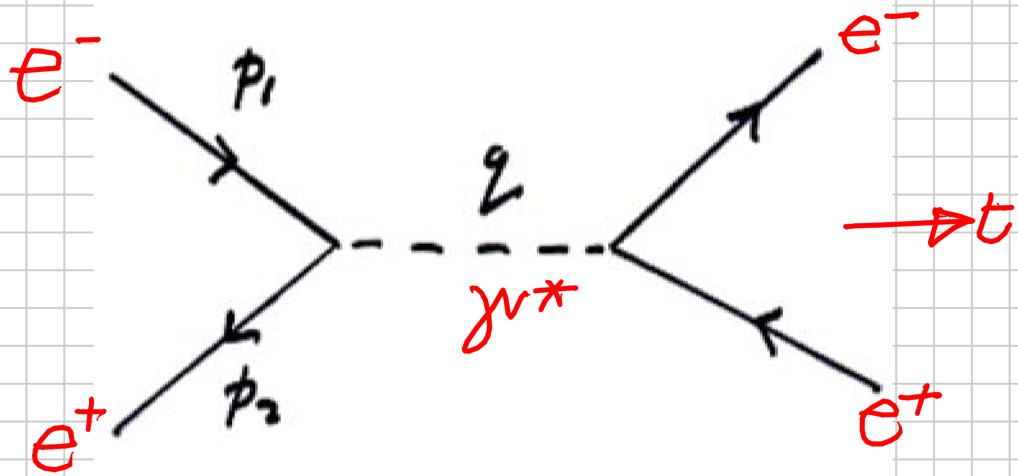
$$q^2 = (p_1 - p_2)^2 = -ve$$

$$= E_q^2 - \bar{p}_q^2$$

SPACE LIKE

SPACE COMPONENT

FOR AN ANNIHILATION PROCESS



$$q^2 = (p_1 + p_2)^2 = +ve$$

$$= E_q^2 - p_q^2 = m_{\gamma^*}^2$$

"TIME" COMPONENT

TIME LIKE

SIGNS REVERSED FOR MINKOWSKI

LORENTZ TRANSFORMATION

$$p'_\mu = \sum \alpha_{\mu\nu} p_\nu$$

FOR A BOOST ALONG x-AXIS OF β

MINKOWSKI

$$\begin{pmatrix} p'_x \\ p'_y \\ p'_z \\ iE' \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & i\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\beta\gamma & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} p_x \\ p_y \\ p_z \\ iE \end{pmatrix}$$

"MODERN" METRIC

$$\begin{pmatrix} E' \\ p'_x \\ p'_y \\ p'_z \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix}$$

$$E' = \gamma(E - \beta p_x)$$

$$p_x = \gamma(p_x - \beta E)$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$C = 1$ ON THIS PAGE

UNITS IN SUBATOMIC PHYSICS

- PHYSICS & TECHNOLOGY → SI UNITS USED
- RELATIVISTIC PHYSICS — CGS / GAUSSIAN
  NATURAL SYSTEM → NO "$4\pi\varepsilon_0$" IN EM

LENGTH	METER	m
TIME	SECOND	s
ENERGY	ELECTRON VOLT	eV
MASS	"	eV/c^2
MOMENTUM	"	eV/c

$10^6\,eV = MeV$ — BINDING ENERGY OF NUCLEI

$10^9\,eV = GeV$ — MASS ENERGY OF PROTON

$10^{12}\,eV = TeV$ — MASS ENERGY OF HIGGS BOSON

- SCALE OF COLOR FORCE
- SCALE OF ELECTROWEAK FORCE

- $10^{-15}\,m$ = FEMTOMETER = 1 FERMI = DIAMETER OF PROTON

- TIME FOR LIGHT SIGNAL TO CROSS PROTON

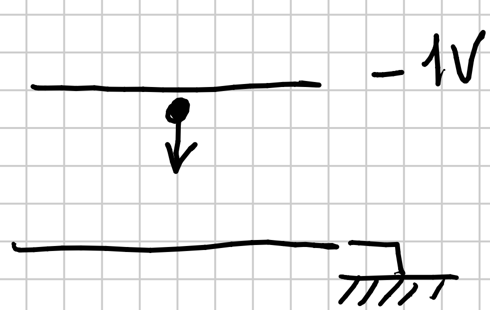
$$\approx \frac{10^{-15}\,[m]}{3\times10^8\,[m]}\,[s] \approx 10^{-23}\,sec$$

- TIME SCALE OF STRONG INTERACTION

WHY USE ELECTRON VOLTS?

- CONVENIENT ENERGY UNIT
- SUBATOMIC PHYSICS EXPERIMENTS DONE BY ACCELERATING BEAMS OF PARTICLES IN ELECTRIC FIELDS, AND SCATTERING OFF TARGET

- ONE ELECTRON VOLT (eV) IS THE KINETIC ENERGY AN ELECTRON GAINS BY ACCELERATING THRU A POTENTIAL OF ONE VOLT



$$1\,\text{eV} = e \times 1\,\text{VOLT}$$
$$= 1.60 \times 10^{-19}\,(\text{COULOMB}) \times 1\,\text{VOLT}$$
$$= 1.60 \times 10^{-19}\ \text{JOULES}$$
$$= 1.60 \times 10^{-12}\ \text{ERGS}$$

WHY ELECTRON VOLTS FOR MASSES?

WHEN A BEAM OF RELATIVISTIC PARTICLES SCATTERS FROM A TARGET, SOME OF THE KINETIC ENERGY CAN APPEAR AS MASS → NEW PARTICLES

- CONVENIENT TO MEASURE MASS IN SAME UNITS AS ENERGY

$$E^2 = p^2 c^2 + m^2 c^4$$

TOTAL RELATIVISTIC ENERGY

MOMENTUM

VELOCITY OF LIGHT

MASS
SOME TIMES CALLED
"REST MASS"

$$E^2 = p^2c^2 + m^2c^4$$

- FOR A PARTICLE WITH NO MASS

PHOTON — γ

NEUTRINO — ν (?)

$$E = p \cdot c \rightarrow p = \frac{E}{c} = \underbrace{\frac{[eV]}{c}}_{\text{UNIT OF MOMENTUM}}$$

MASSLESS PARTICLE $E = 1eV \rightarrow p = 1eV/c$

- FOR A MASSIVE PARTICLE AT REST — IN ITS REST FRAME

$$E = mc^2 \rightarrow m = \frac{E}{c^2} = \underbrace{\frac{[eV]}{c^2}}_{\text{UNIT OF MASS}}$$

PARTICLE OF MASS $1eV/c^2$ AT REST HAS A TOTAL ENERGY OF $1eV$

$$E^2 = p^2c^2 + m^2c^4 \quad \text{—— (1)}$$

$$E^2 = p^2 + m^2 \qquad [\text{WHERE } c = 1]$$

EACH TERM OF (1) HAS DIMENSIONS $[E]^2$

EXAMPLE – PARTICLE MASS = $1000 \frac{MeV}{c^2}$, $p = 1000 \frac{MeV}{c}$

$$E^2 = 10^6 \frac{MeV^2}{c^2} \cdot c^2 + 10^6 \frac{MeV^2}{c^4} \cdot c^4$$

$$E^2 = 2 \times 10^6 \ MeV^2$$

THE PARTICLE HAS A TOTAL RELATIVISTIC ENERGY

$$E = \sqrt{2} \times 10^3 \ MeV$$

Table 1.1. *Units in high energy physics*

(a)

Quantity	High energy unit	Value in SI units
length	1 fm	10^{-15} m
energy	1 GeV = 10^9 eV	1.602×10^{-10} J
mass, E/c^2	1 GeV/c^2	1.78×10^{-27} kg
$\hbar = h/(2\pi)$	6.588×10^{-25} GeV s	1.055×10^{-34} J s
c	2.998×10^{23} fm s^{-1}	2.998×10^{8} m s^{-1}
$\hbar c$	0.1975 GeV fm	3.162×10^{-26} J m

PROTON MASS

(b)

natural units, $\hbar = c = 1$	
~~mass, Mc^2/c^2~~	1 GeV
length, $\hbar c/(Mc^2)$	1 GeV^{-1} = 0.1975 fm
time, $\hbar c/(Mc^3)$	1 GeV^{-1} = 6.59×10^{-25} s
Heaviside–Lorentz units, $\epsilon_0 = \mu_0 = \hbar = c = 1$	
fine structure constant	$\alpha = e^2/(4\pi) = 1/137.06$

Relations between energy units

1 MeV = 10^6 eV   1 GeV = 10^3 MeV   1 TeV = 10^3 GeV

$\hbar c = 1 = 0.1975$ GeV. fm

$\hbar = 1 = 6.59 \times 10^{-25}$ GeV.s

CHOOSE $\hbar c = 1 \left[\begin{matrix}\text{NATURAL}\\ \text{UNITS}\end{matrix}\right] = 0.1975 \left[\mathrm{GeV \cdot fm}\right]$

CAN CONVERT BETWEEN UNITS

<u>LENGTH</u>

$[\mathrm{GeV \cdot fm}]$

$$\left[\frac{\mathrm{GeV}}{c^2} \cdot c^2\right] \quad \frac{\hbar c}{m c^2} = \frac{\hbar c}{\mathrm{GeV}} \; [L] = 0.1975 \left[\frac{\mathrm{GeV \cdot fm}}{\mathrm{GeV}}\right]$$

$$1\,\mathrm{GeV}^{-1} = 0.1975\ \mathrm{fm}$$

<u>TIME</u>

$$\frac{\hbar c}{m c^3} \; [T] = \frac{\hbar c}{\mathrm{GeV}} = 6.588 \times 10^{-25} \left[\frac{\mathrm{GeV \cdot s}}{\mathrm{GeV}}\right]$$

$$1\,\mathrm{GeV}^{-1} = 6.588 \times 10^{-25}\ \mathrm{s}$$

$\hbar = c = 1$

RANDOMNESS OF DECAYS

- QUANTUM MECHANICS → IN AN ENSEMBLE OF UNSTABLE PARTICLES, ANY ONE MAY DECAY AT RANDOM IN A SMALL TIME INTERVAL.
- EACH PARTICLE DECAYS AFTER A RANDOM TIME → ENSEMBLE CHARACTERIZED BY MEAN LIFETIME
- NUMBER DECAYING IN TIME INTERVAL dt:

$$dN = -\omega N(t)\, dt$$

TIME INTERVAL

DECAYING

PROBABILITY PER UNIT TIME FOR DECAY

UNDECAYED AT START OF TIME INTERVAL

$$\frac{dN}{N(t)} = -\omega\, dt$$

DECAY CONSTANT
TRANSITION RATE

$$dN = -\omega\, N(t)\, dt \rightarrow \frac{dN}{N} = -\omega\, dt$$

$$\int_{N(0)}^{N(t)} dN/N = -\omega \int_0^t dt \rightarrow \ln N(t) - \ln N(0) = -\omega t$$

$$N(t) = N(0)\, e^{-\omega t}$$

SURVIVAL EQUATION

INTENSITY OF RADIATION = ACTIVITY

$$\underline{I}(t) = \frac{-dN(t)}{dt} = N(0)\,\omega\, e^{-\omega t}$$

$$\underline{I}(t) = \underline{I}(0)\, e^{-\omega t}$$

INTENSITY OF EMITTED RADIATION

INITIAL ACTIVITY OR INTENSITY

UNITS OF RADIOACTIVITY

- CURIE (Ci) AMOUNT OF RADIOACTIVE MATERIAL IN WHICH NUMBER OF DISINTEGRATIONS PER SECOND = 1g OF RADIUM

  $3 \cdot 7 \times 10^{10}\ s^{-1}$

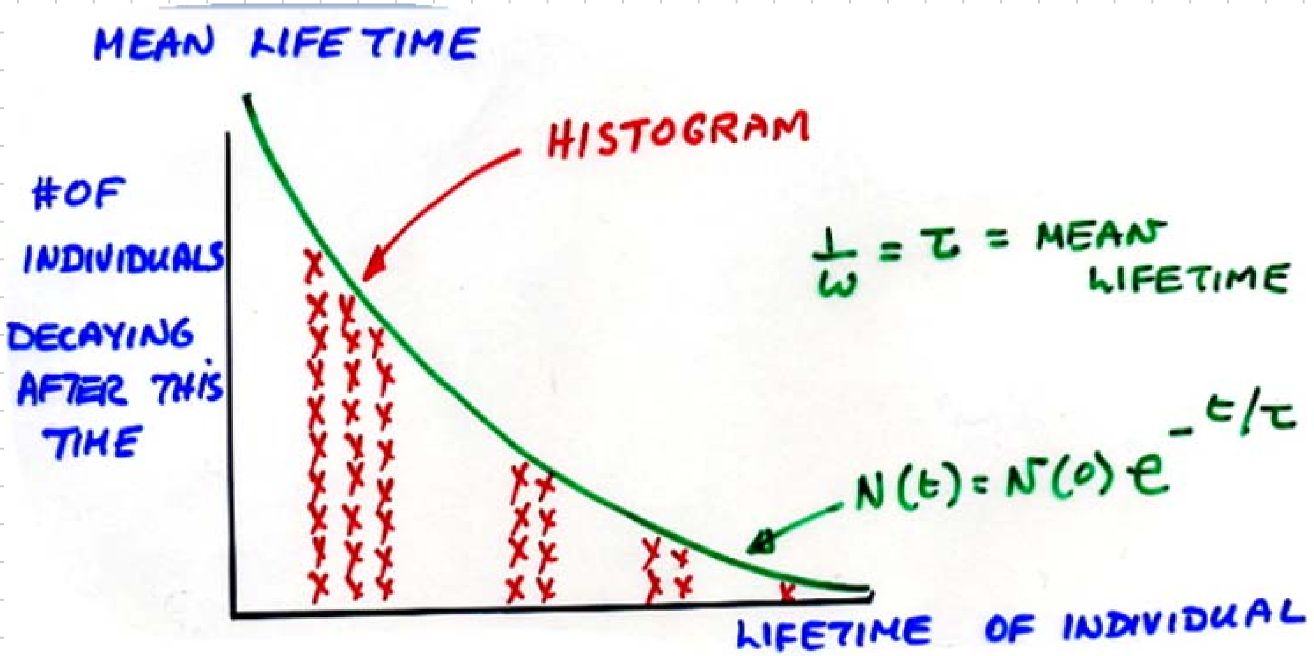
- BECQUEREL (Bq) ONE DISINTEGRATION PER SECOND

  STARTED WHOLE SUBJECT BY DISCOVERING RADIOACTIVITY IN 1896

? HOW MUCH RADIOACTIVITY DID CHERNOBYL RELEASE

1000'S OF CURIES !

MEAN LIFETIME



- INDIVIDUAL DECAY RANDOM
- POPULATION CHARACTERIZED BY MEAN LIFETIME

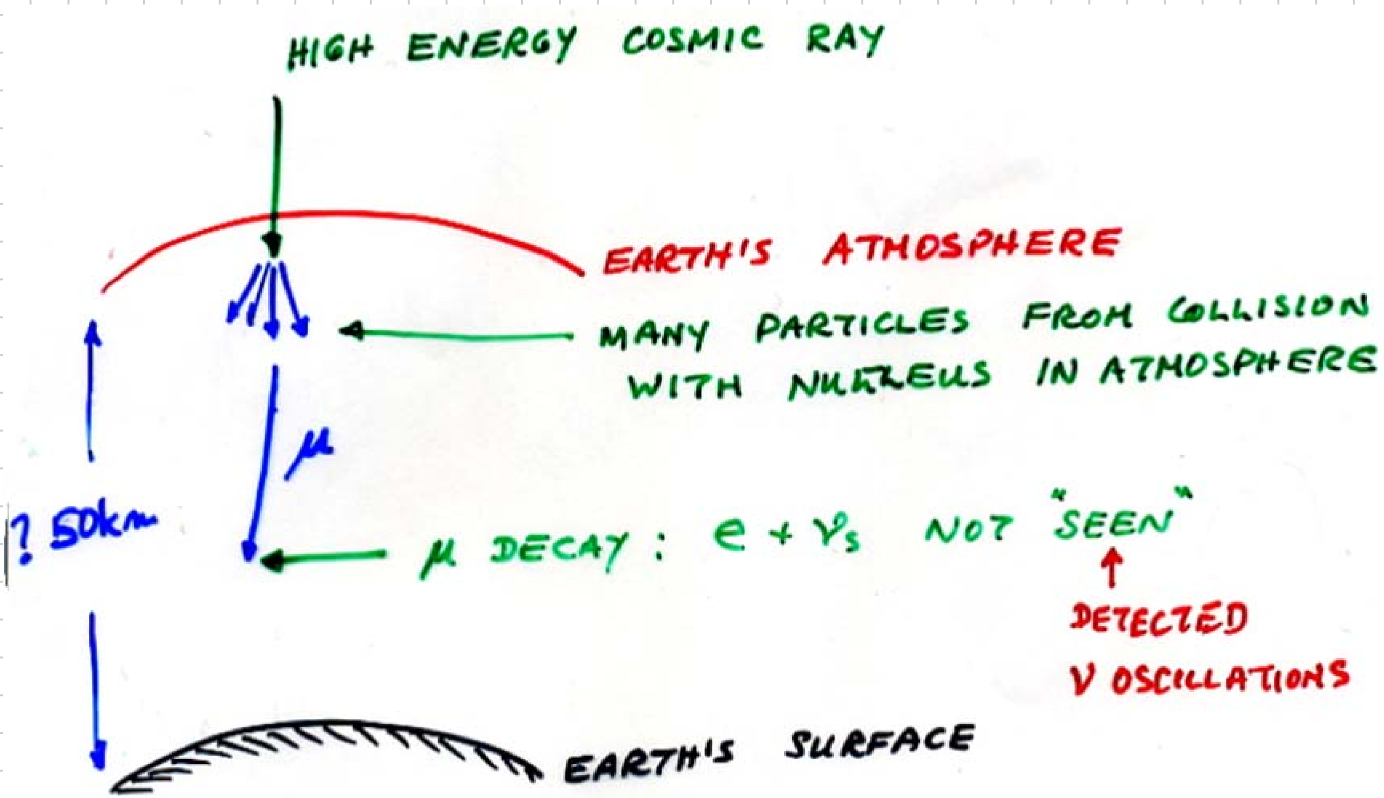
ENORMOUS RANGE OF LIFETIMES

PROTON DECAY	$> 10^{33}$ YEARS	DECAY RATE GOVERNED BY FORCE (INTERACTION) CAUSING THE DECAY PROCESS
$^{238}U \to {}^{234}Th + \alpha$	6.5×10^{9} YEARS	
$^{215}Po \to {}^{211}Pb + \alpha$	2.0×10^{3} s	
$\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$	2.2×10^{-6} s	
$\pi^0 \to \gamma\gamma$	8.3×10^{-17} s	
$\Delta^{++} \to p\pi^+$	6×10^{-24} s	

SPECIAL RELATIVITY & LIFETIME

LIFETIME ONLY MAKES SENSE IN SPECIAL FRAME

- REST FRAME OF PARTICLE



eg $\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$

$\tau = 2.2 \times 10^{-6}\,s$

- ASSUME μ TRAVELLING AT SPEED OF LIGHT

MEAN DISTANCE TO DECAY: $c\tau$

$$= 3\times10^8 \times 2\times10^{-6} \frac{[m][s]}{[s]}$$

$$\approx 600\ m.$$

- TIME TO REACH EARTH'S SURFACE FROM 50km

$$t_{50} = \frac{50 \times 10^3}{3 \times 10^8} = \frac{50}{3} \times 10^{-5} \simeq 2 \times 10^{-4}\,s$$

- HOW MANY UNDECAYED AFTER THIS TIME?

FRACTION SURVIVING $= \dfrac{N(t_{50})}{N(0)} = e^{-t_{50}/\tau_{LAB}}$

$$= \exp\left\{\frac{2 \times 10^{-4}}{2 \times 10^{-6}}\right\} \sim 10^{-44} \sim 0$$

- WHY DO WE SEE ANY MUONS AT EARTH'S SURFACE?

$\tau_{LAB} \neq \tau_\mu \Rightarrow \tau^{REST} = t_2^{REST} - t_1^{REST}$ DEFINES τ IN REST FRAME

SO! $\tau_{LAB} = \gamma t_2^{REST} - \gamma t_1^{REST}$

$$\tau_{LAB} = \gamma \tau_\mu^{REST}$$

THAT'S MORE SENSIBLE!

ASSUME μ HAVE ENERGY OF 100 GeV

$$E = pc/\beta \quad \text{AND} \quad \gamma^2 = \frac{1}{1-\beta^2}$$

$$\gamma^2 = \left(1 - \frac{p^2c^2}{E^2}\right) = E^2/(E^2 - p^2c^2)$$

$$\therefore \quad \gamma_\mu = E/mc^2$$

MASS OF μ IS $m_\mu = 106 \text{ MeV}/c^2$

$$\gamma_\mu = \frac{10^5 \,[\text{MeV}]}{10^2 \,\frac{[\text{MeV}]}{[c^2]} \cdot c^2} \approx 10^3 \Longrightarrow \tau_{LAB} = 10^3 \times 2.2 \times 10^{-6} = 2.2 \times 10^{-3}$$

- FRACTION LEFT AT SEA LEVEL

$$= \exp - \left\{ \frac{t_{50}}{\tau_{LAB}} \right\} = \exp - \left\{ \frac{2 \times 10^{-4}}{2 \times 10^{-3}} \right\} = \exp - \left\{ 10^{-1} \right\} = .905$$

- FRACTION LEFT AT SEA LEVEL $\sim 90\%$