PHY 2408 : LONG PROBLEM SET 2

$\boxed{\text{Due: MARCH 8}}$

1- Calculate the cross section for the process $e^+e^- \to Z \to \mu^-\mu^+$ in terms of the vector and axial couplings c_V and c_A, and given that the e^+e^- are unpolarized.

note: we did this in class but try to do it by yourself and if you get stuck, consult the notes

2- the forward-backward asymmetry that can be obtained from the process above can be written as: (see notes)

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3}{4} A_e A_\mu ,$$

$$\boxed{\text{with} \quad A_f = \frac{(c_L^e)^2 - (c_R^e)^2}{(c_L^e)^2 + (c_R^e)^2} = \frac{2 c_V^f c_A^f}{(c_V^f)^2 + (c_A^f)^2}}$$

$\Rightarrow A_{FB} \propto c_V^2$ and

c_V is small for charged leptons

now $\frac{c_V}{c_A} \propto \sin^2\theta_W$ so given that $A_{Fb} \propto c_V^2$ and c_V is small, the measurement of $\sin^2\theta_W$ will not be as accurate. We can do better with either incoming polarized beams, or by measuring the polarization of the outgoing particles which is possible with τ leptons.

- Let's start with polarized incoming beams. Assume that the electron beam is fully polarized but that the positron beam is unpolarized.

Show that $$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = A_e$$

with σ_L as the cross section for a left-handed electron beam, and σ_R is the same for a right-handed beam. the cross section is at the Z^0 resonance.

PROBLEM 3: Now we consider the process $e^+e^- \to Z \to \tau^+\tau^-$

the polarization of the τ leptons can be inferred using the momentum of the τ decay products.

A - Show that the average Tau polarization: $\dfrac{N\uparrow - N\downarrow}{N\uparrow + N\downarrow} = -A_\tau$

where $N\uparrow$ and $N\downarrow$ are the number of Tau leptons produced in right-handed and left-handed helicity states.

→ this allows a measurement of $\sin^2\theta_W$ with a quantity that involves c_V (and not c_V^2)

B - Show that the Tau polarization where the τ^- is produced at an angle θ with respect to the initial e^- is:

$$P_{\tau^-}(\cos\theta) = \frac{N\uparrow(\cos\theta) - N\downarrow(\cos\theta)}{N\uparrow(\cos\theta + N\downarrow\cos\theta)}$$

$$= \frac{-A_\tau(1+\cos^2\theta) + 2A_e\cos\theta}{(1+\cos^2\theta) + 8/3\, A_{FB}\cos\theta}$$

PROBLEM 4 : Decay of the Top quark

We have studied the decay of the W boson to leptons and quarks. The W boson can't decay to a top quark because the top is heavier. However, the top quark can decay to a W boson and a b quark (this is the dominant decay).

- What are the fractions of the 3 polarizations of the W boson if the masses of the particles are $M_{Top} = 175$ GeV, $M_W = 80$ GeV, $M_b = 4$ GeV. How does your calculation compare to measurements?

- The W_L fraction increases as a function of the Top mass. Can you explain this (in words)?

PROBLEM J:

A- Calculate the octet $q\bar{q}$ colour factor using:

i - $b\bar{g}$

ii - $(r\bar{r} - b\bar{b})/\sqrt{2}$

iii - $(r\bar{r} + b\bar{b} - 2g\bar{g})/\sqrt{6}$

B - Find the overall colour factor for $qq \to qq$ if QCD corresponded to a SU(2) colour symmetry