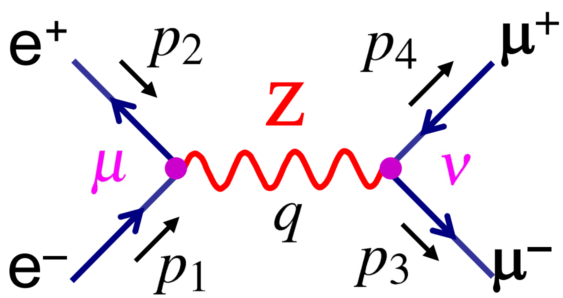
Slides from Mark Thomson

(See also content in textbook)

The Z Resonance

★ Want to calculate the cross-section for $e^+e^- \to Z \to \mu^+\mu^-$

- Feynman rules for the diagram below give:



e⁺e⁻ vertex: $\overline{v}(p_2) \cdot -ig_Z\gamma^\mu \frac{1}{2}(c_V^e - c_A^e\gamma^5) \cdot u(p_1)$

Z propagator: $\dfrac{-ig_{\mu\nu}}{q^2 - m_Z^2}$

μ⁺μ⁻ vertex: $\overline{u}(p_3) \cdot -ig_Z\gamma^\nu \frac{1}{2}(c_V^\mu - c_A^\mu\gamma^5) \cdot v(p_4)$

➡ $$-iM_{fi} = [\overline{v}(p_2) \cdot -ig_Z\gamma^\mu \tfrac{1}{2}(c_V^e - c_A^e\gamma^5) \cdot u(p_1)].\frac{-ig_{\mu\nu}}{q^2 - m_Z^2}.[\overline{u}(p_3) \cdot -ig_Z\gamma^\nu \tfrac{1}{2}(c_V^\mu - c_A^\mu\gamma^5) \cdot v(p_4)]$$

➡ $$M_{fi} = -\frac{g_Z^2}{q^2 - m_Z^2} g_{\mu\nu}[\overline{v}(p_2)\gamma^\mu \tfrac{1}{2}(c_V^e - c_A^e\gamma^5) \cdot u(p_1)].[\overline{u}(p_3)\gamma^\nu \tfrac{1}{2}(c_V^\mu - c_A^\mu\gamma^5) \cdot v(p_4)]$$

★ Convenient to work in terms of helicity states by explicitly using the Z coupling to **LH** and **RH** chiral states (ultra-relativistic limit so helicity = chirality)

$$\tfrac{1}{2}(c_V - c_A\gamma^5) = c_L\tfrac{1}{2}(1 - \gamma^5) + c_R\tfrac{1}{2}(1 + \gamma^5)$$

LH and RH projections operators

hence $c_V = (c_L + c_R), \;\; c_A = (c_L - c_R)$

and $$\frac{1}{2}(c_V - c_A\gamma^5) = \frac{1}{2}(c_L + c_R - (c_L - c_R)\gamma^5) = c_L\frac{1}{2}(1-\gamma^5) + c_R\frac{1}{2}(1+\gamma^5)$$

with $c_L = \frac{1}{2}(c_V + c_A), \;\; c_R = \frac{1}{2}(c_V - c_A)$

★ Rewriting the matrix element in terms of LH and RH couplings:

$$M_{fi} = -\frac{g_Z^2}{q^2 - m_Z^2} g_{\mu\nu}[c_L^e \overline{v}(p_2)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)u(p_1) + c_R^e \overline{v}(p_2)\gamma^\mu \tfrac{1}{2}(1+\gamma^5)u(p_1)]$$
$$\times[c_L^\mu \overline{u}(p_3)\gamma^\nu \tfrac{1}{2}(1-\gamma^5)v(p_4) + c_R^\mu \overline{u}(p_3)\gamma^\nu \tfrac{1}{2}(1+\gamma^5)v(p_4)]$$

★ Apply projection operators remembering that in the ultra-relativistic limit

$$\tfrac{1}{2}(1-\gamma^5)u = u_\downarrow; \quad \tfrac{1}{2}(1+\gamma^5)u = u_\uparrow, \quad \tfrac{1}{2}(1-\gamma^5)v = v_\uparrow, \quad \tfrac{1}{2}(1+\gamma^5)v = v_\downarrow$$

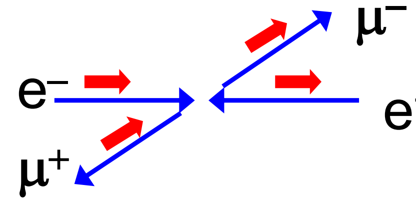
⇨ $$M_{fi} = -\frac{g_Z}{q^2 - m_Z^2} g_{\mu\nu}[c_L^e \overline{v}(p_2)\gamma^\mu u_\downarrow(p_1) + c_R^e \overline{v}(p_2)\gamma^\mu u_\uparrow(p_1)]$$
$$\times[c_L^\mu \overline{u}(p_3)\gamma^\nu v_\uparrow(p_4) + c_R^\mu \overline{u}(p_3)\gamma^\nu v_\downarrow(p_4)]$$

★ For a combination of **V** and **A** currents, $\overline{u}_\uparrow\gamma^\mu v_\uparrow = 0$ etc, gives four orthogonal contributions

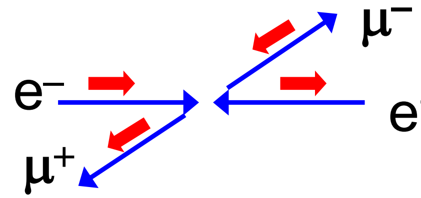
⇨ $$-\frac{g_Z^2}{q^2 - m_Z^2} g_{\mu\nu}[c_L^e \overline{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1) + c_R^e \overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)]$$
$$\times[c_L^\mu \overline{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4) + c_R^\mu \overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$

★ **Sum of 4 terms**

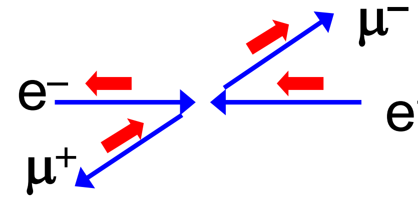
$$M_{RR} = -\frac{g_Z^2}{q^2 - m_Z^2} c_R^e c_R^\mu g_{\mu\nu} [\overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$



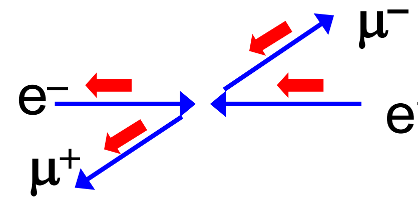
$$M_{RL} = -\frac{g_Z^2}{q^2 - m_Z^2} c_R^e c_L^\mu g_{\mu\nu} [\overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\overline{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)]$$



$$M_{LR} = -\frac{g_Z^2}{q^2 - m_Z^2} c_L^e c_R^\mu g_{\mu\nu} [\overline{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)][\overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$



$$M_{LL} = -\frac{g_Z^2}{q^2 - m_Z^2} c_L^e c_L^\mu g_{\mu\nu} [\overline{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)][\overline{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)]$$



Remember: the L/R refer to the helicities of the initial/final state <u>particles</u>

★ **Fortunately we have calculated these terms before when considering**

$e^+e^- \to \gamma \to \mu^+\mu^-$ **giving:** **(pages 137-138)**

$$[\overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)] = s(1+\cos\theta)$$ **etc.**

★ **Applying the QED results to the Z exchange with gives:**

$$\frac{e^2}{q^2} \rightarrow \frac{g_Z^2}{q^2 - m_Z^2} c^e c^\mu$$

where $q^2 = s = 4E_e^2$

$$|M_{RR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_R^e)^2 (c_R^\mu)^2 (1 + \cos\theta)^2$$

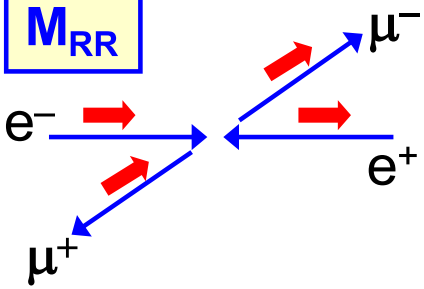
$$|M_{RL}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_R^e)^2 (c_L^\mu)^2 (1 - \cos\theta)^2$$

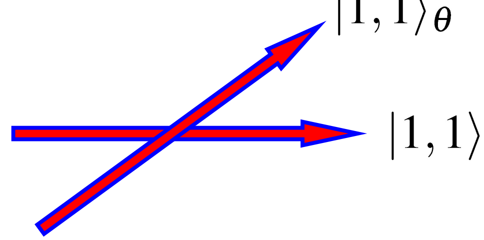
$$|M_{LR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_L^e)^2 (c_R^\mu)^2 (1 - \cos\theta)^2$$

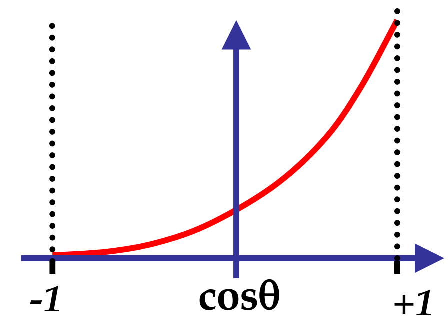
$$|M_{LR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_L^e)^2 (c_L^\mu)^2 (1 + \cos\theta)^2$$

★ **As before, the angular dependence of the matrix elements can be understood in terms of the spins of the incoming and outgoing particles e.g.**

M_{RR}







The Breit-Wigner Resonance

★ Need to consider carefully the propagator term $1/(s-m_Z^2)$ which diverges when the C.o.M. energy is equal to the rest mass of the Z boson

★ To do this need to account for the fact that the Z boson is an unstable particle

•For a stable particle at rest the time development of the wave-function is:

$$\psi \sim e^{-imt}$$

•For an unstable particle this must be modified to

$$\psi \sim e^{-imt} e^{-\Gamma t/2}$$

so that the particle probability decays away exponentially

$$\psi^* \psi \sim e^{-\Gamma t} = e^{-t/\tau} \quad \text{with} \quad \tau = \frac{1}{\Gamma_Z}$$

•Equivalent to making the replacement

$$m \rightarrow m - i\Gamma/2$$

★In the Z boson propagator make the substitution:

$$m_Z \rightarrow m_Z - i\Gamma_Z/2$$

★ Which gives:

$$(s-m_Z^2) \longrightarrow [s-(m_Z - i\Gamma_Z/2)] = s - m_Z^2 + im_Z\Gamma_Z + \tfrac{1}{4}\Gamma_Z^2 \approx s - m_Z^2 + im_Z\Gamma_Z$$

where it has been assumed that $\Gamma_Z \ll m_Z$

★ Which gives

$$\left|\frac{1}{s-m_Z^2}\right|^2 \rightarrow \left|\frac{1}{s-m_Z^2+im_Z\Gamma_Z}\right|^2 = \frac{1}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}$$

★ **And the Matrix elements become**

$$|M_{RR}|^2 = \frac{g_Z^4 s^2}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_R^e)^2(c_R^\mu)^2(1+\cos\theta)^2 \qquad \textbf{etc.}$$

★ **In the limit where initial and final state particle mass can be neglected:**

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|M_{fi}|^2$$

(page 31)

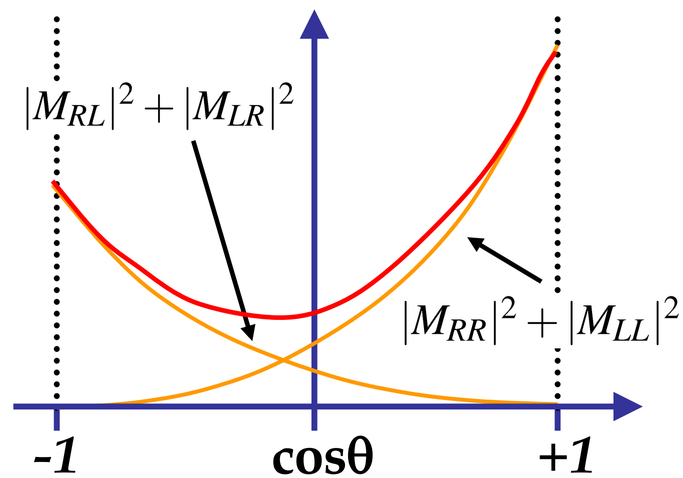
★ **Giving:**

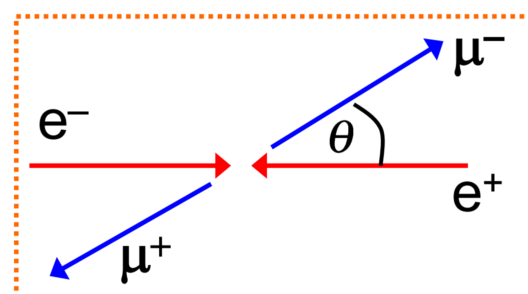
$$\frac{d\sigma_{RR}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_R^e)^2(c_R^\mu)^2(1+\cos\theta)^2$$

$$\frac{d\sigma_{LL}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_L^e)^2(c_L^\mu)^2(1+\cos\theta)^2$$

$$\frac{d\sigma_{LR}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_L^e)^2(c_R^\mu)^2(1-\cos\theta)^2$$

$$\frac{d\sigma_{RL}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_L^e)^2(c_R^\mu)^2(1-\cos\theta)^2$$





★ **Because $|M_{LL}|^2 + |M_{RR}|^2 \neq |M_{LR}|^2 + |M_{RL}|^2$, the differential cross section is asymmetric, i.e. parity violation (although not maximal as was the case for the W boson).**

Cross section with unpolarized beams

★ To calculate the total cross section need to sum over all matrix elements and **average** over the **initial spin states**. Here, assuming unpolarized beams (i.e. both e^+ and both e^- spin states equally likely) there a four combinations of initial electron/positron spins, so

$$\langle |M_{fi}|^2 \rangle = \frac{1}{2}.\frac{1}{2}.(|M_{RR}|^2 + |M_{LL}|^2 + |M_{LR}|^2 + |M_{RL}|^2)$$

$$= \frac{1}{2}.\frac{1}{2}\frac{g_Z^4 s^2}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2} \times \{[(c_R^e)^2(c_R^\mu)^2 + (c_L^e)^2(c_L^2)^2](1+\cos\theta)^2 + [(c_L^e)^2(c_R^\mu)^2 + (c_R^e)^2(c_L^2)^2](1-\cos\theta)^2\}$$

★ The part of the expression {...} can be rearranged:

$$\{\ldots\} = [(c_R^e)^2 + (c_L^e)^2][(c_R^\mu)^2 + (c_L^\mu)^2](1+\cos^2\theta) + 2[(c_R^e)^2 - (c_L^e)^2][(c_R^\mu)^2 - (c_L^\mu)^2]\cos\theta \qquad (1)$$

and using $c_V^2 + c_A^2 = 2(c_L^2 + c_R^2)$ and $c_V c_A = c_L^2 + c_R^2$

$$\{\ldots\} = \frac{1}{4}[(c_V^e)^2 + (c_A^e)^2][(c_V^\mu)^2 + (c_A^\mu)^2](1+\cos^2\theta) + 2c_V^e c_A^e c_V^\mu c_A^\mu \cos\theta$$

★ **Hence the complete expression for the unpolarized differential cross section is:**

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\langle|M_{fi}|^2\rangle$$

$$= \frac{1}{64\pi^2}\cdot\frac{1}{4}\cdot\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\times$$

$$\left\{\tfrac{1}{4}[(c_V^e)^2+(c_A^e)^2][(c_V^\mu)^2+(c_A^\mu)^2](1+\cos^2\theta)+2c_V^e c_A^e c_V^\mu c_A^\mu \cos\theta\right\}$$

★ **Integrating over solid angle** $d\Omega = d\phi\, d(\cos\theta) = 2\pi\, d(\cos\theta)$

$$\int_{-1}^{+1}(1+\cos^2\theta)d(\cos\theta) = \int_{-1}^{+1}(1+x^2)dx = \tfrac{8}{3} \quad \textbf{and} \quad \int_{-1}^{+1}\cos\theta\, d(\cos\theta) = 0$$

$$\boxed{\sigma_{e^+e^-\to Z\to\mu^+\mu^-} = \frac{1}{192\pi}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\left[(c_V^e)^2+(c_A^e)^2\right]\left[(c_V^\mu)^2+(c_A^\mu)^2\right]}$$

★ **Note: the total cross section is proportional to the sums of the squares of the vector- and axial-vector couplings of the initial and final state fermions**

$$(c_V^f)^2+(c_A^f)^2$$

Connection to the Breit-Wigner Formula

★ **Can write the total cross section**

$$\sigma_{e^+e^-\to Z\to\mu^+\mu^-} = \frac{1}{192\pi}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\left[(c_V^e)^2+(c_A^e)^2\right]\left[(c_V^\mu)^2+(c_A^\mu)^2\right]$$

in terms of the Z boson decay rates (partial widths) from page 473 (question 26)

$$\Gamma(Z\to e^+e^-) = \frac{g_Z^2 m_Z}{48\pi}\left[(c_V^e)^2+(c_A^e)^2\right] \quad \textbf{and} \quad \Gamma(Z\to\mu^+\mu^-) = \frac{g_Z^2 m_Z}{48\pi}\left[(c_V^\mu)^2+(c_A^\mu)^2\right]$$

$$\Rightarrow \quad \sigma = \frac{12\pi}{m_Z^2}\frac{s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\Gamma(Z\to e^+e^-)\Gamma(Z\to\mu^+\mu^-)$$

★ **Writing the partial widths as** $\Gamma_{ee}=\Gamma(Z\to e^+e^-)$ **etc., the total cross section can be written**

$$\sigma(e^+e^-\to Z\to f\overline{f}) = \frac{12\pi}{m_Z^2}\frac{s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\Gamma_{ee}\Gamma_{ff} \qquad \textbf{(2)}$$

where f is the final state fermion flavour:

(The relation to the non-relativistic form of the part II course is given in the appendix)

Forward-Backward Asymmetry

★ On page 495 we obtained the expression for the differential cross section:

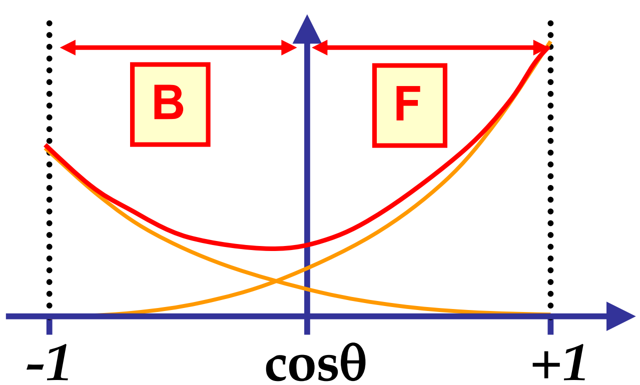
$$\langle|M_{fi}|\rangle^2 \propto [(c_L^e)^2+(c_R^e)^2][(c_L^\mu)^2+(c_R^\mu)^2](1+\cos^2\theta)+[(c_L^e)^2-(c_R^e)^2][(c_L^\mu)^2-(c_R^\mu)^2]\cos\theta$$

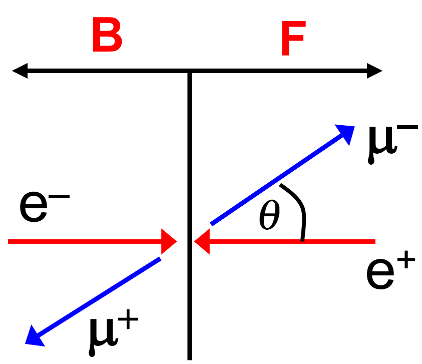
★ The differential cross sections is therefore of the form:

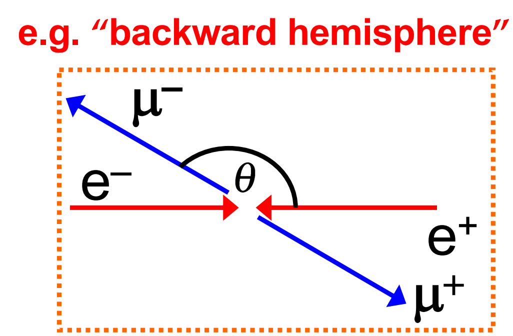
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \kappa\times[A(1+\cos^2\theta)+B\cos\theta] \qquad \begin{cases} A=[(c_L^e)^2+(c_R^e)^2][(c_L^\mu)^2+(c_R^\mu)^2] \\ B=[(c_L^e)^2-(c_R^e)^2][(c_L^\mu)^2-(c_R^\mu)^2] \end{cases}$$

★ Define the **FORWARD** and **BACKWARD** cross sections in terms of angle incoming electron and out-going particle

$$\sigma_F \equiv \int_0^1 \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\mathrm{d}\cos\theta \qquad \sigma_B \equiv \int_{-1}^0 \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\mathrm{d}\cos\theta$$

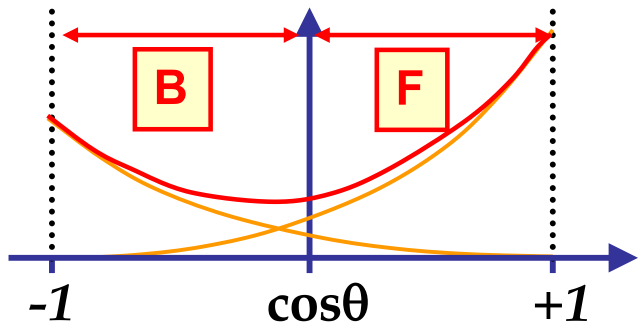






★ The level of asymmetry about **cosθ=0** is expressed in terms of the Forward-Backward Asymmetry

$$A_{\rm FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$



• Integrating equation (1):

$$\sigma_F = \kappa \int_0^1 [A(1+\cos^2\theta) + B\cos\theta]\,{\rm d}\cos\theta = \kappa \int_0^1 [A(1+x^2)+Bx]{\rm d}x = \kappa\left(\frac{4}{3}A + \frac{1}{2}B\right)$$

$$\sigma_B = \kappa \int_{-1}^0 [A(1+\cos^2\theta) + B\cos\theta]\,{\rm d}\cos\theta = \kappa \int_{-1}^0 [A(1+x^2)+Bx]{\rm d}x = \kappa\left(\frac{4}{3}A - \frac{1}{2}B\right)$$

★ Which gives:

$$A_{\rm FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{B}{(8/3)A} = \frac{3}{4}\left[\frac{(c_L^e)^2 - (c_R^e)^2}{(c_L^e)^2 + (c_R^e)^2}\right]\cdot\left[\frac{(c_L^\mu)^2 - (c_R^\mu)^2}{(c_L^\mu)^2 + (c_R^\mu)^2}\right]$$

★ This can be written as

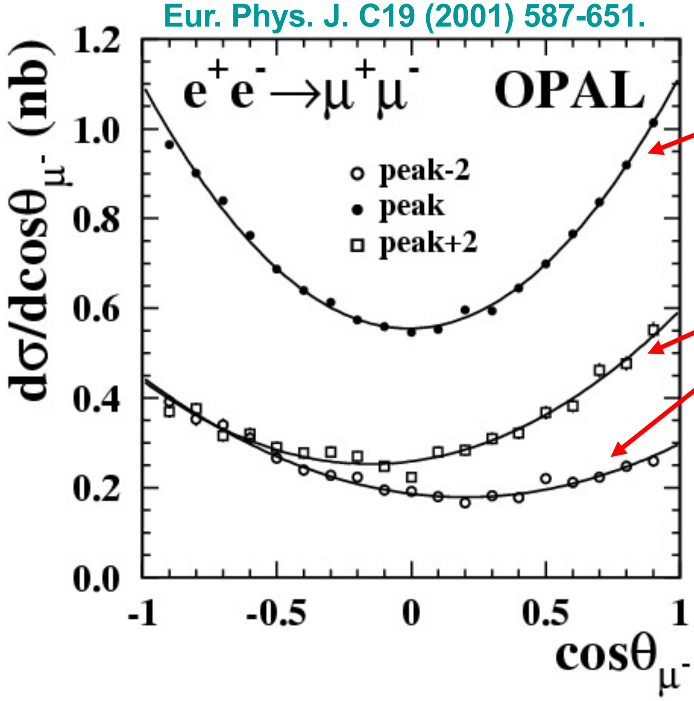
$$A_{\rm FB} = \frac{3}{4}A_e A_\mu$$ with $$A_f \equiv \frac{(c_L^e)^2 - (c_R^e)^2}{(c_L^e)^2 + (c_R^e)^2} = \frac{2c_V^f c_A^f}{(c_V^f)^2 + (c_A^f)^2} \qquad (4)$$

★ Observe a non-zero asymmetry because the couplings of the Z to LH and RH particles are different. Contrast with QED where the couplings to LH and RH particles are the same (parity is conserved) and the interaction is FB symmetric

Measured Forward-Backward Asymmetries

★ Forward-backward asymmetries can only be measured for final states where the charge of the fermion can be determined, e.g. $e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-$

OPAL Collaboration,
Eur. Phys. J. C19 (2001) 587-651.



Because $\sin^2\theta_W \approx 0.25$, the value of A_{FB} for leptons is almost zero

For data above and below the peak of the Z resonance interference with $e^+e^- \rightarrow \gamma \rightarrow \mu^+\mu^-$ leads to a larger asymmetry

★LEP data combined:

$$A_{FB}^{0,e} = 0.0145 \pm 0.0025$$
$$A_{FB}^{0,\mu} = 0.0169 \pm 0.0013$$
$$A_{FB}^{0,\tau} = 0.0188 \pm 0.0017$$

★To relate these measurements to the couplings uses $A_{\mathrm{FB}} = \frac{3}{4}A_e A_\mu$

★ In all cases asymmetries depend on A_e

★ To obtain A_e could use $A_{FB}^{0,e} = \frac{3}{4}A_e^2$ (also see Appendix II for A_{LR})

Determination of the Weak Mixing Angle

★ **From LEP :** $A_{FB}^{0,f} = \frac{3}{4} A_e A_f$

★ **From SLC :** $A_{LR} = A_e$

$\Rightarrow A_e, A_\mu, A_\tau, \ldots$

Putting everything together ⇨

$$A_e = 0.1514 \pm 0.0019$$
$$A_\mu = 0.1456 \pm 0.0091$$
$$A_\tau = 0.1449 \pm 0.0040$$

includes results from other measurements

with $$A_f \equiv \frac{2 c_V^f c_A^f}{(c_V^f)^2 + (c_A^f)^2} = 2 \frac{c_V / c_A}{1 + (c_V / c_A)^2}$$

★ **Measured asymmetries give ratio of vector to axial-vector Z coupings.**

★ **In SM these are related to the weak mixing angle**

$$\frac{c_V}{c_A} = \frac{I_W^3 - 2Q \sin^2\theta_W}{I_W^3} = 1 - \frac{2Q}{I_3} \sin^2\theta_W = 1 - 4|Q| \sin^2\theta_W$$

★ **Asymmetry measurements give precise determination of** $\sin^2\theta_W$

$$\sin^2\theta_W = 0.23154 \pm 0.00016$$