

LECTURE 10: Standard Model (Part 2)

Overview:

- Construction of the Standard Model
- SM Higgs Mechanism

(I used Quigg and Novaes as references)

Standard Model Lagrangian

②

Last lecture, we ended with:

$$\mathcal{L}_{lep.} = \mathcal{L}_{lep} + \underbrace{\bar{L} i \gamma^m (i g_2 \tau^i W_m^i + \frac{g_1}{2} Y B_m) L}_{+ \bar{R} i \gamma^m (i \frac{g_2}{2} \tau^i B_m) R}$$

→ expanding: $-g \bar{L} \gamma^m \left(\frac{\tau^1 W_m^1 + \tau^2 W_m^2}{2} \right) L$ ①

$$-g \bar{L} \gamma^m \frac{\tau^3}{2} L W_m^3 - \frac{g'}{2} Y \bar{L} \gamma^m L B_m \quad \text{②}$$

We saw that Term ① involves charged current. We can write it as:

$$-\frac{g}{2} \bar{L} \gamma^m \begin{pmatrix} 0 & W_m^+ - i W_m^2 \\ W_m^+ + i W_m^2 & 0 \end{pmatrix} L$$

We can define the charged bosons as: $W_m^\pm = \frac{1}{\sqrt{2}} (W_m^1 \mp W_m^2)$

Standard Model Lagrangian (cont.)

(3)

Term ① becomes:

$$-\frac{g}{2\sqrt{2}} \left[\bar{\nu} \gamma^\mu (1 - \gamma^5) \ell W_\mu^+ + \bar{\ell} \gamma^\mu (1 - \gamma^5) \nu W_\mu^- \right]$$

note that $\frac{g}{2\sqrt{2}} = \left(\frac{M_W^2 G_F}{\sqrt{2}} \right)^{1/2}$

We now concentrate on the neutral current Terms which involves both L and R components:

$$\begin{aligned} & -g \bar{L} \gamma^\mu \frac{\tau_3}{2} L W_\mu^3 - \frac{g'}{2} (\bar{L} \gamma^\mu Y_L + \bar{R} \gamma^\mu Y_R) B_\mu \\ & = -g J_3^\mu W_\mu^3 - \frac{g'}{2} J_Y^\mu B_\mu \end{aligned}$$

→ From last lecture: $J_3^\mu = \frac{1}{2} (\bar{\nu}_L \gamma^\mu \nu_L - \bar{\ell} \gamma^\mu \ell)$

$$J_Y^\mu = -(\bar{\nu}_L \gamma^\mu \nu_L + \bar{\ell} \gamma^\mu \ell_L + 2 \bar{\ell}_R \gamma^\mu \ell_R)$$

Standard Model Lagrangian (cont.)

⑦

Remember that $J_{em} = J_3 + \frac{1}{2} J_Y$

We want the right combination of fields that couple to J_{em} .

We can do this by rotating the fields:

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_w & \sin \theta_w \\ -\sin \theta_w & \cos \theta_w \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix}$$

$$W_\mu^3 = \sin \theta_w A_\mu + \cos \theta_w Z_\mu$$

$$B_\mu = \cos \theta_w A_\mu - \sin \theta_w Z_\mu$$

$$\text{with } \sin \theta_w = \frac{g'}{\sqrt{g^2 + g'^2}} \quad \cos \theta_w = \frac{g}{\sqrt{g^2 + g'^2}}$$

$$\frac{g'}{g} = \tan \theta_w, \quad g \sin \theta_w = g' \cos \theta_w$$

Standard Model Lagrangian (cont.)

(5)

With the rotated fields, we get:

$$\left[-g \sin \theta_w J_3^m + \frac{1}{2} g' \cos \theta_w J_Y^m \right] A_\mu \quad \left. \begin{array}{l} \text{correct form for em} \\ \text{interaction} \end{array} \right\}$$

$$+ \left[-g \cos \theta_w J_3^m + \frac{1}{2} g' \sin \theta_w J_Y^m \right] Z_\mu \quad \left. \begin{array}{l} \text{something new ...} \end{array} \right\}$$

First term = $-g \sin \theta_w (\bar{\ell} \gamma^\mu \ell) A_\mu$

$e = g \sin \theta_w$

Second term: $\frac{-g}{2 \cos \theta_w} (2 \cos^2 \theta_w J_3^m - \frac{g'}{g} \cos \theta_w \sin \theta_w J_Y^m) Z_\mu$

$$= \frac{-g}{2 \cos \theta_w} (2 \cos^2 \theta_w J_3^m - \frac{g'}{g} \sin^2 \theta_w J_Y^m) Z_\mu$$

$$= \frac{-g}{2 \cos \theta_w} (2(1 - \sin^2 \theta_w) J_3^m - \sin^2 \theta_w J_Y^m) Z_\mu$$

$$J_Y^m = -(\bar{\nu}_L \gamma_\mu \nu_L + \bar{\ell}_L \gamma_\mu \ell_L + 2 \bar{\ell}_R \gamma_\mu \ell_R), \quad J_3^m = \frac{1}{2} (\bar{\nu}_L \gamma_\mu \nu_L - \bar{\ell}_L \gamma_\mu \ell_L)$$

note that $\sin^2 \theta_w$ terms cancel for neutrinos

Standard Model Lagrangian (cont.)

(6)

For electrons we have for $\sin^2 \theta_w$ terms:

$$+ \bar{l}_L \gamma^\mu l_L + \bar{l}_L \gamma^\mu l_L + 2 \bar{l}_R \gamma^\mu l_R = 2 (\bar{l}_L \gamma^\mu l_L + \bar{l}_R \gamma^\mu l_R)$$

other terms for neutrinos: $\bar{\nu}_L \gamma^\mu \nu_L$
" " " " electrons: $-\bar{l}_L \gamma^\mu l_L$

which can be summarized as:

$$-\frac{g}{2 \cos \theta_w} \sum_{\psi_i = \nu, l} \bar{\psi}_i \gamma^\mu (g_V^i - g_A^i \gamma^5) \psi_i Z_\mu$$

T_3^i
 $T_3^i - 2Q_i \sin^2 \theta_w$

→ note that SM predicted neutral weak interactions which were observed ~ 5 years after model was proposed more or that later...

Fermions and bosons are massless

Standard Model Lagrangian (cont.)

⑦

Back To the Higgs mechanism. We introduce the scalar doublet:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad \text{with } Y=1$$

$T = 1/2, \quad T^3 = -1/2$

$$\mathcal{L}_{\text{scalar}}: \partial_\mu \Phi^\dagger \partial^\mu \Phi - V(\Phi^\dagger \Phi)$$

$$V(\Phi^\dagger \Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

We introduce the gauge-covariant derivative:

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + i g \frac{\tau^i}{2} W_i + i g' Y B_\mu$$

$$\text{choose VEV: } \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad v = \sqrt{\frac{-\mu^2}{\lambda}}$$

→ we need to keep U(1) a symmetry of the vacuum (keep photon massless)

$$Q < \Phi >_0 = (T_3 + \frac{1}{2} Y) < \Phi >_0 = \frac{1}{2} \left[\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = 0$$

Standard Model Lagrangian (cont.)

⑧

As you've done in problem set 1, we parametrize the Higgs doublet

$$\Phi = \exp\left(\frac{i\tau^i \alpha_i}{2v}\right) \begin{pmatrix} 0 \\ (v+\eta)/\sqrt{2} \end{pmatrix}$$

we make a gauge trans. $\Phi \rightarrow \Phi' = \exp\left(-i\frac{\tau^i \gamma_i}{2v}\right) \Phi$

$$= \frac{(v+\eta)}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

A scalar: $\left| (d_n + i\frac{\tau^i}{2} W_n^i + i\frac{g_2}{2} \gamma B_n) \frac{(v+\eta)}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right|^2$

$$= \frac{M^2}{2} (v+\eta)^2 - \frac{\lambda}{4} (v+\eta)^4$$

$$\left| \begin{pmatrix} 0 \\ (d_n + i\frac{\tau^i}{2} (v+\eta)) \end{pmatrix} \begin{pmatrix} W_n^+ \\ \frac{-1}{\sqrt{2}} Z_n \end{pmatrix} \right|^2$$

Standard Model Lagrangian (cont.)

(9)

note: $-g W_n^3 + g' B_n$

$$\begin{aligned}
 &= -g \sin \theta_w A_n - g \cos \theta_w Z_n + g' \cos \theta_w A_n - g' \sin \theta_w Z_n \\
 &= -g \sin \theta_w A_n - g \cos \theta_w Z_n + g' \sin \theta_w A_n - g' \sin \theta_w Z_n \\
 &= -\frac{g}{\cos \theta_w} \left(\cos \theta_w Z_n + \frac{g'}{g} \sin \theta_w \cos \theta_w Z_n \right) \\
 &= -\frac{g}{\cos \theta_w} \left(\cos^2 \theta_w Z_n + g' \sin \theta_w \frac{1}{g} \sin \theta_w Z_n \right) \\
 &= -\frac{g}{\cos \theta_w} Z_n
 \end{aligned}$$

so:

$$\left(\begin{array}{c} 0 \\ \frac{1}{\sqrt{2}} \eta \end{array} \right) + \frac{i g}{2} (V + \eta) \left(\begin{array}{c} W_n^+ \\ -\frac{1}{\sqrt{2}} Z_n \end{array} \right) \left| \right|^2$$

becomes:

$$\frac{1}{2} \eta^2 \eta + \frac{g^2}{4} (V + \eta)^2 (W_n^+ W_n^- + \frac{1}{2 \cos^2 \theta_w} Z_n Z_n)$$

Standard Model Lagrangian (cont.)

(10)

The quadratic terms give:

$$\frac{g^2 v^2}{4} W_\mu^\dagger W^\mu + \frac{g^2 v^2}{4} \cdot \frac{1}{2 \cos^2 \theta_w} Z_\mu Z^\mu$$

$$\text{with } W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp i W_\mu^2)$$

Note that boson mass terms in Lagrangians have the form

$$\frac{1}{2} m^2 B_\mu B^\mu. \text{ So } \Rightarrow \boxed{M_W = \frac{g v}{2}, \quad M_Z = \frac{g v}{2 \cos \theta_w}}$$

with $v = (\sqrt{2} G_F)^{1/2}$ measured to be $\approx 246 \text{ GeV}$

and $\sin^2 \theta_w \approx 0.22$, we get

$$M_W^2 = \frac{e^2 v^2}{4 \sin^2 \theta_w} = \frac{\pi \alpha}{\sin^2 \theta_w} v^2 \approx \left(\frac{37.2 \text{ GeV}}{\sin \theta_w} \right)^2 \approx (80 \text{ GeV})^2$$
$$M_Z^2 = \left(\frac{37.2}{\sin \theta_w \cos \theta_w} \text{ GeV} \right)^2 \approx (98 \text{ GeV})^2$$

Standard Model Lagrangian (cont.)

(11)

The Terms involving the scalar field:

$$-\frac{1}{2}(-2\mu^2\eta^2 + \frac{1}{4}M^2v^2) \left(\frac{2}{v^3} |H^3 + \frac{1}{v^4} |H^4 - 1 \right)$$

$$\rightarrow M_H = \sqrt{-2\mu^2}$$

→ no prediction for Higgs mass

Another way To get the mass Terms For Z^0 and A^1 :

$$\mathcal{L}_{\text{scalar}} : \frac{v^2}{2} \left| \left(g \frac{\tau^3}{2} W_\mu^3 + \frac{g'}{2} Y B_\mu \right) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right|^2$$

$$\frac{v^2}{8} \left[\begin{pmatrix} B_\mu & W_\mu^3 \end{pmatrix} \begin{pmatrix} g'^2 & -g g' \\ -g g' & g^2 \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix} \right]$$

Two eigenvalues for the mass matrix: $0, \frac{1}{2} (g^2 + g'^2) v^2$

$$\hookrightarrow = M_A = \frac{1}{2} M_Z^2$$

Standard Model Lagrangian (cont.)

(12)

The ρ parameter: $\rho = \frac{M_W^2}{\cos^2 \theta_W M_Z^2}$

represents the relative strengths of the neutral and charged effective currents:

$$j_{0n} j_n / j_{+1} j_n^-$$

$\rho = 1$ in the SM (at Tree level)

In model with arbitrary number of Higgs multiplets ϕ_i with isospin T_i and 3rd component t_i , Higgs and VEV v_i ,

$$\rho \text{ is given by } : \frac{\sum_i [T_i(T_i + 1) - (T_{3i})^2] v_i^2}{2 \sum_i (T_{3i})^2 v_i^2}$$

will give $\rho = 1$ for doublets

→ Tests isospin structure of Higgs sector

Standard Model Lagrangian (cont.)

(13)

Lepton masses:

We saw that adding explicit mass terms in the Lagrangian breaks gauge invariance. We therefore add another Term:

$$\mathcal{L}_{\text{Yukawa}} = -G_e [\bar{R} (\ell + L) + (\bar{L} e) R]$$

$$= -G_e \begin{pmatrix} \nu + \eta \\ \nu_L \end{pmatrix} \begin{pmatrix} \bar{R} & (0 \ 1) \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} + (\bar{\nu}_L \bar{e}_L) \begin{pmatrix} 0 \\ 1 \end{pmatrix} R_e$$

$$= -\frac{G_e v}{\sqrt{2}} \bar{R} R - \frac{G_e}{\sqrt{2}} \bar{R} e \eta$$

$$\text{we get } M_e = \frac{G_e v}{\sqrt{2}}$$

→ no prediction for G_e

→ $G_{\text{Top}} = 1$ (within 1%)

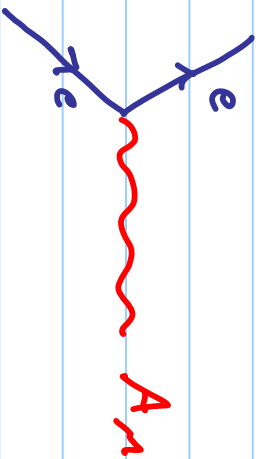
$$\text{coupling strength } \bar{Q} Q H = \frac{M_e}{v}$$

→ a Testable prediction

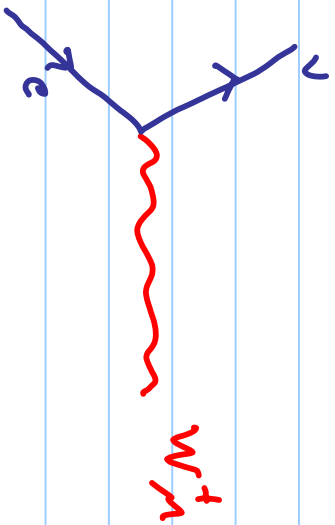
The Standard Model (cont.)

(14)

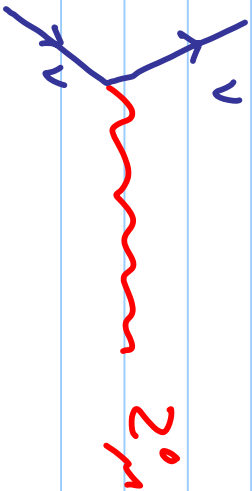
From the previous pieces of the SM Lagrangian we can deduce the following Feynman rules:



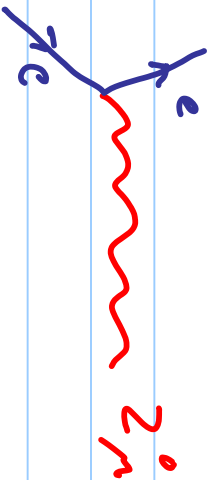
$$= ie \bar{\gamma}_\mu e$$



$$= i \left(\frac{GF M_W^2}{\sqrt{2}} \right)^{1/2} \bar{\nu} \gamma_\mu (1 - \gamma_5) \nu$$



$$= \frac{i}{\sqrt{2}} \left(\frac{GF M_Z^2}{\sqrt{2}} \right)^{1/2} \bar{\nu} \gamma_\mu (1 - \gamma_5) \nu$$

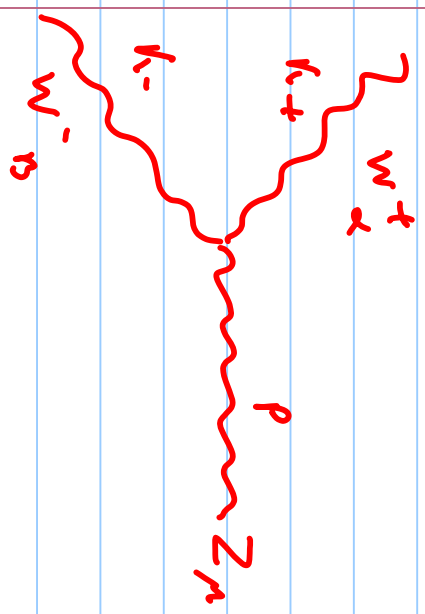


$$= \frac{i}{\sqrt{2}} \left(\frac{GF M_Z^2}{\sqrt{2}} \right)^{1/2} \bar{e} \gamma_\mu \left[2s_w^2 (1 + \gamma_5) + (2s_w^2 - 1) (1 - \gamma_5) \right] e$$

$$s_w = \sin \theta_w$$

The Standard Model (cont.)

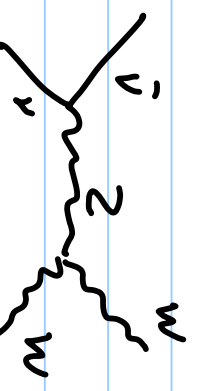
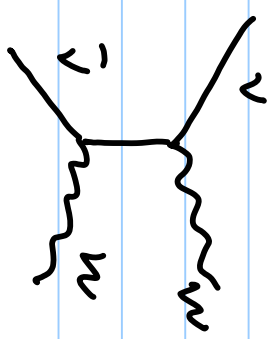
(15)



$$i e \cot \theta_w [g_{2B} (K_T - K_-)_\mu - g_{2W} (p + K_+)_\mu + g_{2W} (p + K_-)_\mu]$$

In lecture 9 we saw that the $\bar{\nu} \rightarrow W W$ cross section was not well behaved at high energies:

$$\sigma \sim \frac{G_F^2 s}{3\pi}$$



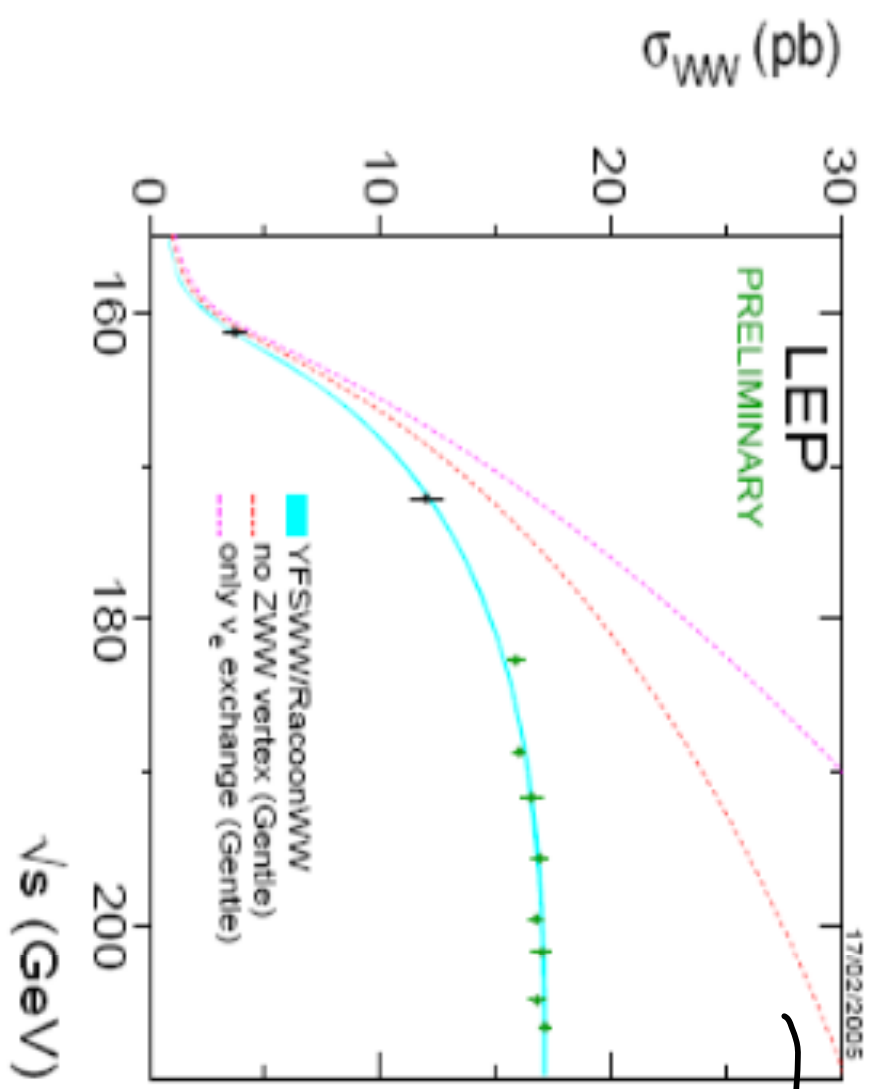
We know we were missing one diagram:

$$M_2 = \frac{-ig^2}{4(s-M_Z^2)} \bar{\nu} \gamma_\mu (1-\gamma_5) \nu (g_{2W} - \frac{g_{2B} g_{2W}}{M_Z^2}) \times \epsilon_+^{\mu\alpha} \epsilon_-^{\alpha\beta}$$

$$[g_{2B} (K_- - K_+)_\nu + g_{2W} (K_+ + p)_\nu - g_{2W} (K_- + p)_\nu]$$

PROBLEM SET 3, PROBLEM #1:
 Show that extra diagram cures our problem.

$e^+e^- \rightarrow WW$ cross sections from LEP II:



→ adding photon

