LECTURE 11: Standard Model (Part 3)

Overview:

-Quark Masses

-Higgs Physics

(I used Quigg and Novaes and my thesis as references)

The Standard Model (cont.)

We now need to generate masses for quarks.

Note that when we refer to doublets or singlets, we will assume 3 colours:

$$L_q = \begin{pmatrix} u \\ d' \end{pmatrix}_L \rightarrow \underset{\text{red}}{\begin{pmatrix} u \\ d' \end{pmatrix}_L} \quad \underset{\text{blue}}{\begin{pmatrix} u \\ d' \end{pmatrix}_L} \quad \underset{\text{green}}{\begin{pmatrix} u \\ d' \end{pmatrix}_L}$$

y for the doublets is $= 1/3$

For the singlets : $R_u = u_R = \frac{1}{2}(1+\gamma_5)\,u$

$R_d = d_R = \frac{1}{2}(1+\gamma_5)\,d$

$$y(u_R) = 4/3 \quad , \quad y(d_R) = -2/3$$

Refresher: $\Gamma(N_{udd} \to P_{uud}\, e\bar{\nu}) \gg \Gamma(\Lambda_{uds} \to P_{uud}\, e\bar{\nu})$

$\Delta s = 1$ Transition highly suppressed

The Standard Model (cont.)

We can write the hadronic current as:

$J_\mu^{(+)} = \bar{d}\gamma_\mu(1-\gamma_5)u + \bar{s}\gamma_\mu(1-\gamma_5)u$ but if we want the current to be universal, we can try this:

$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$ which gives:

$$\bar{d}'\gamma_\mu(1-\gamma_5)u = \cos\theta_c\,\bar{d}\gamma_\mu(1-\gamma_5)u + \sin\theta_c\,\bar{s}\gamma_\mu(1-\gamma_5)u$$

For neutral current:

$$J_\mu^{(+)}(0) = \bar{u}\gamma_\mu(1-\gamma_5)u + \bar{d}'(1-\gamma_5)d'$$

$$= \bar{u}\gamma_\mu(1-\gamma_5)u + \cos^2\theta_c\,\bar{d}\gamma_\mu(1-\gamma_5)d + \sin^2\theta_c\,\bar{s}(1-\gamma_5)s$$

$$+ \cos\theta_c\sin\theta_c\left[\bar{d}\gamma_\mu(1-\gamma_5)s + \bar{s}\gamma_\mu(1-\gamma_5)d\right]$$

last term generates FCNC (exper.: extremely small...)

The Standard Model (cont.)

→ GIM mechanism, add a quark (charm)

$$L_u \equiv \begin{pmatrix} u \\ d' \end{pmatrix} \qquad \begin{pmatrix} u \\ \cos\theta_c\, d + \sin\theta_c\, s \end{pmatrix}_L$$

$$L_c \equiv \begin{pmatrix} c \\ s' \end{pmatrix} \qquad \begin{pmatrix} c \\ -\sin\theta_c\, d + \cos\theta_c\, s \end{pmatrix}_L$$

We get a new contribution for the neutral current:

$$\bar{c}\gamma_\mu(1-\gamma_5)c + \bar{s}'\gamma_\mu(1-\gamma_5)s'$$

which will cancels the previous FCNC Term!

The neutral current can be written as:

$$\mathcal{L}^{(0)}_{\text{quarks}} = \frac{-g}{2\cos\theta_W} \sum_{\psi_q = u,d,s,c,b,T} \bar{\psi}_q \gamma^\mu \left(\boxed{g_V^q} - \boxed{g_A^q}\gamma_5\right)\psi_q Z_\mu$$

→ diff. from leptons

and

$$\mathcal{L}^{(\pm)}_{\text{quarks}} = \frac{g}{2\sqrt{2}} \left[\bar{u}\gamma_\mu(1-\gamma_5)d' + \bar{c}\gamma_\mu(1-\gamma_5)s' + \dots\right]$$

The Standard Model (cont.)

We had before $\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$, $Y_\varphi = 1$

we now need its cc to give mass to the top member of the doublet

$$\bar{\varphi} = -i\tau^2 \varphi^* = \begin{pmatrix} -\bar{\varphi}^0 \\ \varphi^- \end{pmatrix} \quad \text{with } Y_{\bar{\varphi}} = -1$$

We can obtain a gauge-invariant contribution to the Lagrangian:

$$-G_d (\bar{u}, \bar{d})_L \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} d_R - G_u (\bar{u}, \bar{d})_L \begin{pmatrix} -\bar{\varphi}^0 \\ \varphi^- \end{pmatrix} u_R + \dots$$

$$= -m_d \bar{d}d - m_u \bar{u}u - \frac{m_d}{v} \bar{d}d\eta - \frac{m_u}{v} \bar{u}u\eta + \dots$$

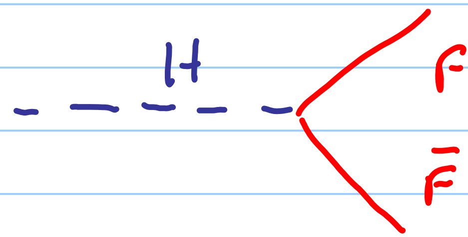
since weak interactions operate on $(u, d')_L$ etc., we write:

$$\mathcal{L}_{Y_q} - G_d^{ij} (\bar{u}_i, \bar{d}_i')_L \begin{pmatrix} \varphi^+ \\ \varphi_0 \end{pmatrix} d_{jR} - G_u^{ij} (\bar{u}_i, \bar{d}_i') \begin{pmatrix} -\bar{\varphi}^0 \\ \varphi^- \end{pmatrix} u_{jR} + \dots$$

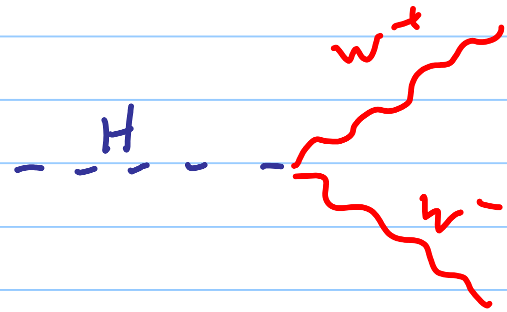
i, j = # of doublets

The Higgs Boson

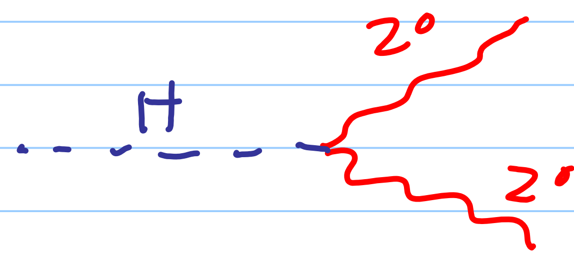
Some Feynman rules before we start:



$$-\frac{i m_f}{v} = -i m_f \left(G_F \sqrt{2}\right)^{1/2}$$



$$-i g M_W g_{\mu\nu} = -2 i M_W^2 \left(G_F \sqrt{2}\right)^{1/2} g_{\mu\nu}$$



$$\frac{-i g M_Z}{\cos\theta_W} g_{\mu\nu} = -2 i M_Z^2 \left(G_F \sqrt{2}\right)^{1/2} g_{\mu\nu}$$

Higgs Boson (cont.)

Let's look at some decays

$H \to f\bar{f}$: $\mathcal{M} = -i m_f (G_F \sqrt{2})^{1/2} \bar{u} v$

if $m_f \ll m_H$:

$$p_1 = \frac{m_H}{2}(1,0,0,1)$$

$$p_2 = \frac{m_H}{2}(1,0,0,-1)$$

$$|\mathcal{M}|^2 = G_F m_f^2 \sqrt{2}\, \mathrm{Tr}(\not{p}_2 \not{p}_1)$$

$$= 4 G_F m_f^2 \sqrt{2}\; p_1 \cdot p_2 \qquad = 2 G_F m_H^2 m_f^2 \sqrt{2}$$

$$\frac{d\Gamma}{d\Omega} = \frac{|\mathcal{M}|^2}{64\pi^2 m_H} = \frac{G_F m_H m_f^2}{16\pi^2 \sqrt{2}} .$$

$$\boxed{\Gamma(H \to f\bar{f}) = \frac{G_F m_H m_f^2}{4\pi \sqrt{2}}}$$

THE HIGGS BOSON (CONT.)

$$\boxed{\Gamma(H \to f\bar{f}) = \frac{G_F M_H m_f^2}{4\pi\sqrt{2}}}$$

note: width $\propto m_f^2$ $\Rightarrow$ will be dominated by heaviest fermion the Higgs can decay to

coupling to electrons very small ... cross section for $e^+e^- \to H$ at resonance:

$$\frac{4\pi}{M_H^2} \cdot \frac{\Gamma(H \to e^+e^-)}{\Gamma(H \to \text{all})} \longrightarrow \gtrsim 4 \text{ MeV}$$

$M_H > 114$ GeV

b quark $\sim$ 10 000 mass of e^- ...
$\sim$ 1000 mass of u, d

→ Dominant production mechanism at hadron colliders through gluon fusion (why?, how?)

THE HIGGS BOSON (cont)

$H \to W^+ W^-$

$$\mathcal{M} = \underbrace{\frac{ie}{\sin\theta_W}}_{ig} m_W \, g_{\mu\nu} \, \varepsilon_1^{\mu} \varepsilon_2^{\nu}$$

with: $$\varepsilon_\mu^0 = \frac{1}{m_W}(E, 0, 0, |p|)$$

$$\varepsilon_\mu^{\pm} = \frac{1}{\sqrt{2}}(0, 1, \pm i, 0)$$

$$\Gamma = \frac{p_F}{32\pi^2 m_H^2} \int_\Omega |\mathcal{M}|^2 \, d\Omega$$

$$p_F = \sqrt{\frac{m_H^2}{4} - m_W^2} = \frac{m_H}{2}\left(1 - \frac{4 m_W^2}{m_H^2}\right)^{1/2}$$

$$x = \frac{4 m_W^2}{m_H^2}$$

$$\Gamma = \frac{|\mathcal{M}|^2 (1-x)^{1/2}}{32\pi^2 m_H^2} \cdot \frac{m_H}{2} \cdot 4\pi$$

THE HIGGS BOSON (cont)

$$|M|^2 = g^2 m_W^2 (\varepsilon_1^\mu \varepsilon_{2\mu})^2 = G_F m_W^4 \frac{8}{\sqrt{2}} (\varepsilon_1^\mu \varepsilon_{2\mu})^2$$

Sum over helicities:

Long: $\frac{1}{m_W^2}\left(\frac{m_H^2}{4} + |p|^2\right) = \frac{1}{m_W^2}\left(\frac{m_H^2}{4} + \frac{m_H^2}{4} - m_W^2\right)$

$$= \frac{1}{m_W^2}\left(\frac{m_H^2}{2} - m_W^2\right) = \left(\frac{m_H^2}{2m_W^2} - 1\right)$$

$$= \frac{m_H^4}{4m_W^4} + 1 - \frac{m_H^2}{m_W^2} \qquad +2 \text{ for } \lambda \pm 1$$

$$|M|^2 = G_F m_W^4 \frac{8}{\sqrt{2}} \cdot \frac{1}{x^2} \cdot \left(4 - 4x + 3x^2\right)$$

$$\Gamma = \frac{(1-x)^{1/2}}{8\pi m_H^2} \frac{m_H}{2} \cdot G_F \cdot m_W^4 \cdot \frac{8}{\sqrt{2}} \cdot \frac{1}{x^2} \left(4 - 4x + 3x^2\right)$$

THE Higgs Boson (cont)

$$\Gamma = \frac{(1-x)^{1/2} G_F}{2\sqrt{2}\,\pi \cdot m_H} \frac{m_H^4}{16} \cdot \frac{x^2}{x^2} \left(4 - 4x + 3x^2\right)$$

$$= \frac{(1-x)^{1/2}}{2\sqrt{2}\,\pi} \frac{G_F}{16} \cdot m_H \cdot \frac{m_H^2}{4 m_W^2} m_W^2 \cdot 4 \left(4-4x+3x^2\right)$$

$$\boxed{= \frac{(1-x_W)^{1/2}}{8\sqrt{2}\,\pi} G_F \frac{m_W^2 m_H}{x_W} \left(4-4x_W+3x_W^2\right)}$$

$$x_W = \frac{4 m_W^2}{m_H^2}$$

For Z boson: $x_Z = \frac{4 m_Z^2}{m_H^2}$

Factor of 1/2 : $\boxed{\Gamma = \frac{(1-x_Z)^{1/2}}{16\sqrt{2}\,\pi} G_F \ldots}$