LECTURE 14: Z Boson Physics and Neutral Currents (Part 3)

Overview:

-Production asymmteries

-Neutral current and quarks

(I used Quigg and Halzen-Martin, K. Graham's thesis as references)

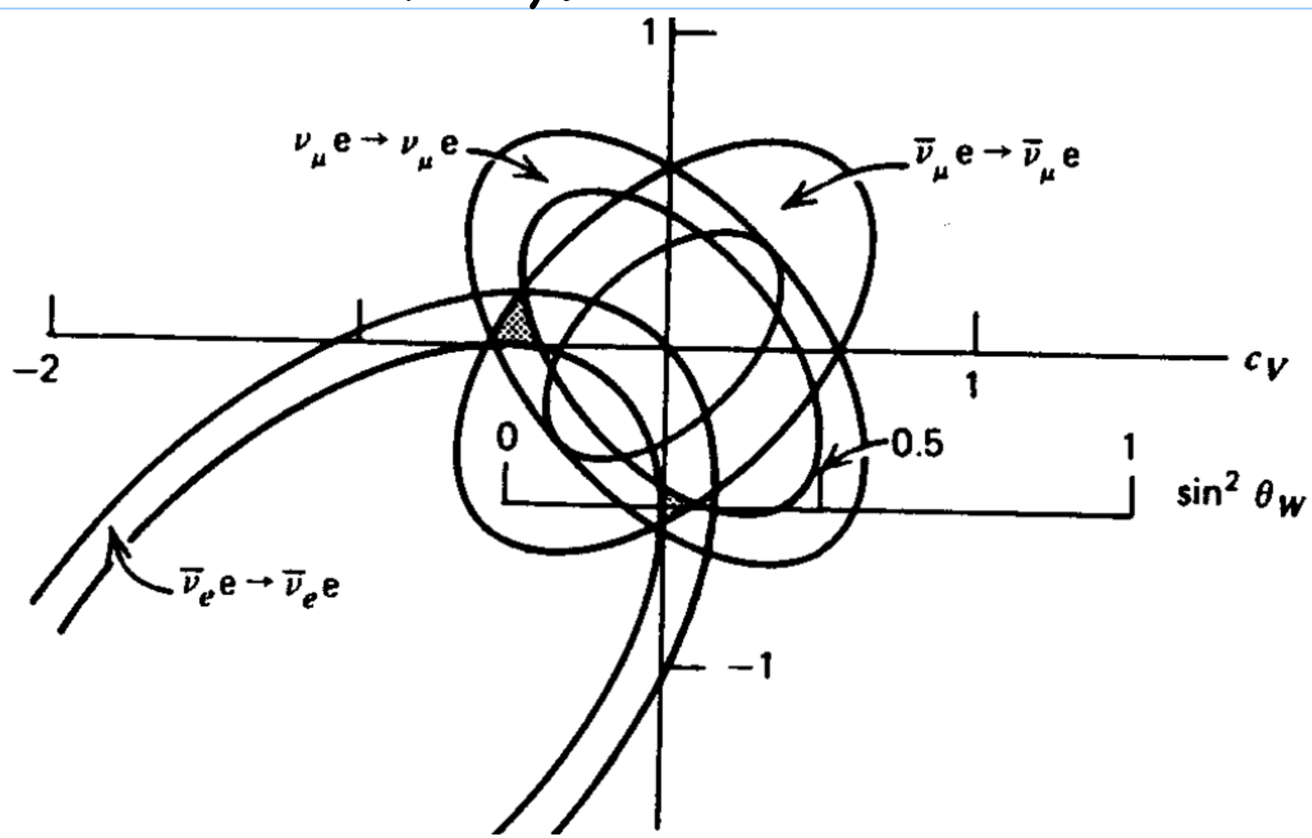
NEUTRAL CURRENT AND ASYMMETRIES

Last lecture we calculated the cross sections for the processes: $(s \ll M_Z^2)$

$$e\bar{\nu}_e \to e\bar{\nu}_e \qquad e\nu_\mu \to e\nu_\mu \qquad e\bar{\nu}_\mu \to e\bar{\nu}_\mu$$

We saw that the measurements gave two values for c_V and c_A.

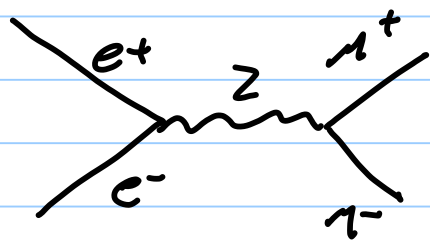
$$\frac{g}{2\cos\theta_W} \sum_{\psi_i = \nu, \ell} \bar{\psi}_i \gamma^\mu (c_V^i - c_A^i \gamma^5) \psi_i Z_\mu$$



NEUTRAL CURRENT AND ASYMMETRIES

We saw that this ambiguity could be removed with an asymmetry measurement. Let's look at this in more detail.

Two diagrams contribute to $e^+e^- \to \mu^+\mu^-$:



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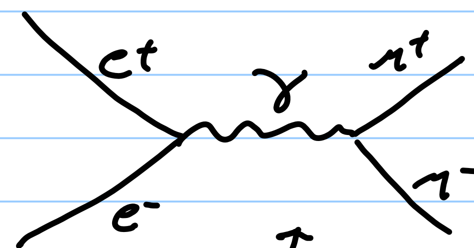


The QED part that we studied before gives us :

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}(1+\cos^2\theta) \quad , \quad \sigma = \frac{4\pi\alpha^2}{3s}$$ (full calculation in lecture 4)

You are calculating the pure Z contribution.

The full calculation involves an interference term.

NEUTRAL CURRENT AND ASYMMETRIES

$$M_\gamma = \frac{-e^2}{k^2}[\bar{u}\gamma^\nu u][\bar{e}\gamma_\nu e]$$

$$M_Z = -\frac{\sqrt{2}\,G M_Z^2}{s - M_Z^2}\left[R_\mu(\bar{\mu}_R\gamma^\nu \mu_R) + L_\mu(\bar{\mu}_L\gamma^\nu\mu_L)\right]\left[R_e(\bar{e}_R\gamma_\nu e_R) + L_e(\bar{e}_L\gamma_\nu e_L)\right]$$

$$L_e \equiv c_V + c_A = g_V + g_A \quad , \quad R_e = c_V - c_A = g_V - g_A$$

i.e. $\quad c_V - c_A\gamma^5 = (c_V - c_A)\frac{1}{2}(1+\gamma^5) + (c_V + c_A)\frac{1}{2}(1-\gamma^5)$

Doing the full calculation yields:

$$\frac{d\sigma}{d\Omega}(e_L^- e_R^+ \to \mu_L^- \mu_R^+) = \frac{\alpha^2}{4s}(1+\cos)^2 \left|1 + r L_e L_\mu\right|^2$$

$$\frac{d\sigma}{d\Omega}(e_L^- e_R^+ \to \mu_R^- \mu_L^+) = \frac{\alpha^2}{4s}(1-\cos)^2 \left|1 + r R_e R_\mu\right|^2$$

+ 2 other helicity contributions

$$r = \frac{\sqrt{2}\,G M_Z^2}{s - M_Z^2 + i M_Z \Gamma_Z}\left(\frac{s}{e^2}\right)$$

note that $R_e = R_\mu$, $L_e = L_\mu$

NEUTRAL CURRENT AND ASYMMETRIES

Results above averaged over all helicities give:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} \left[A_0 \underbrace{(1+\cos^2\theta)}_{\text{symmetric wrt } \cos\theta} + A_1 \underbrace{\cos\theta}_{\text{asymmetric wrt } \cos\theta} \right]$$

$$A_0 = 1 + \frac{1}{2} \operatorname{Real}(r) (L_e + R_e)^2 + \frac{1}{4} |r|^2 (L_e^2 + R_e^2)^2$$

$$= 1 + 2 \operatorname{Real}(r)\, c_V^2 + |r|^2 (c_V^2 + c_A^2)$$

$$A_1 = \operatorname{Real}(r) (L_e - R_e)^2 + \frac{1}{2} |r|^2 (L_e^2 - R_e^2)^2$$

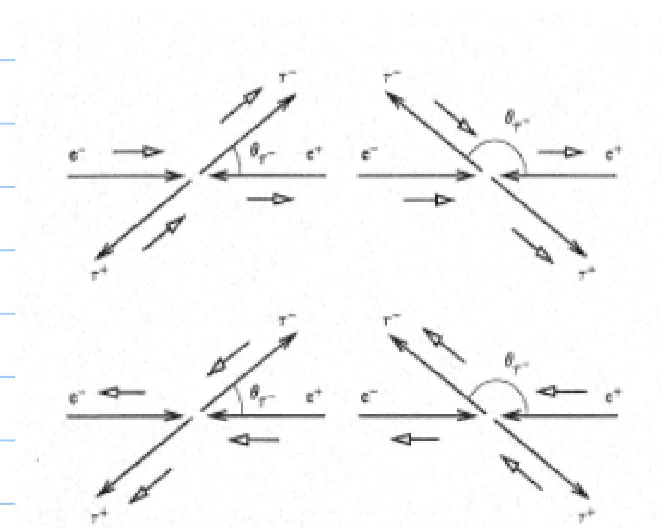
$$= 4 \operatorname{Real}(r)\, c_A^2 + 8 |r|^2 c_V^2 c_A^2$$

Note that with QED only we get: $A_0 = 1$, $A_1 = 0$ giving a symmetric distribution.

The weak interaction introduces an asymmetry in the $\cos\theta$ distribution $\longrightarrow$ a "forward-backward" asymmetry

NEUTRAL CURRENT AND ASYMMETRIES

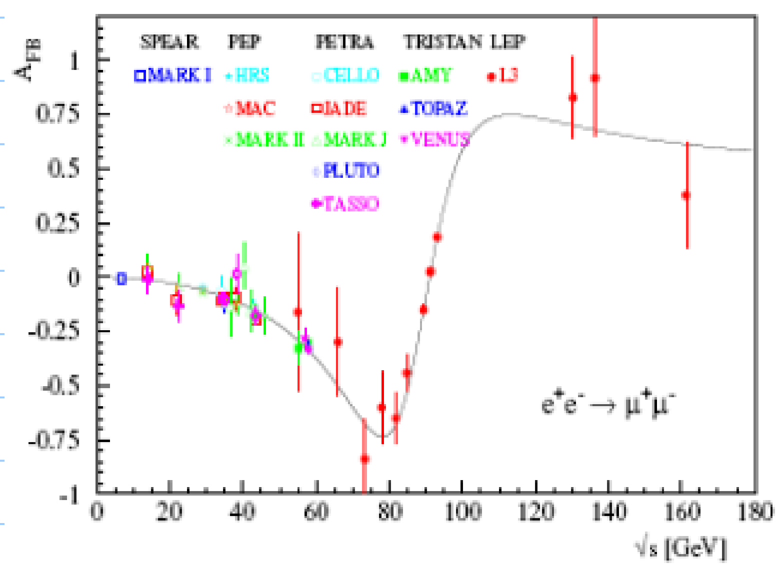
$$A_{FB} \equiv \frac{\int_0^1 dz \frac{d\sigma}{dz} - \int_1^0 dz \frac{d\sigma}{dz}}{\int_{-1}^1 dz \frac{d\sigma}{dz}}$$



$e^+e^- \to \tau^+\tau^-$

For $s \ll M_Z^2 \quad \to \quad |r| \ll 1$

$$A_{FB} = \frac{A_1}{8A_0/3} \approx \frac{3}{2} \text{Real } c_A^2 \approx -\frac{3 c_A^2}{\sqrt{2}} \left(\frac{G s}{e^2}\right)$$



J. Mnich *Phys. Rep.* **271**, 181-266 (1996)

$\to$ does not offer accurate determination of $\sin^2\theta_W$ (since $c_V \simeq 0$). We integrate $\frac{d\sigma}{d\Omega}$ and set $\sigma = \sigma_0 A_0$

where $\sigma_0 \equiv$ QED cross section

$$R_\mu \equiv \frac{\sigma(e^+e^- \to \mu^+\mu^-)}{\sigma_0} =$$

$$1 + 2\,\text{Real}(r)\, c_V^2 + |r|^2 (c_V^2 + c_A^2)^2$$

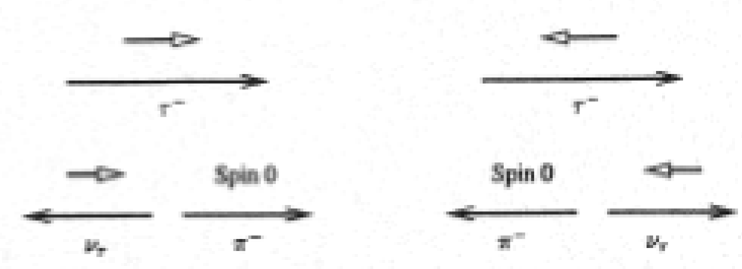
NEUTRAL CURRENT AND ASYMMETRIES

At the Z pole, we get very high values of R_μ (~ 150) since σ hits its maximum.

$$A_{FB}\Big|_{s \approx M_Z^2} = 3 \frac{c_V^2 c_A^2}{(c_V^2 + c_A^2)^2}$$

This can get more interesting if we can measure the polarization of the final state leptons. This can be done with $e^+e^- \to \tau^+\tau^-$

→ The Tau decays very quickly via the charged current interaction → ν_τ will be left-handed. Pion momentum depends on whether we started with τ_L or τ_R



$$P_\tau \equiv \frac{(\sigma_R - \sigma_L)}{(\sigma_R + \sigma_L)}$$

σ_R (σ_L) is σ for producing right (left)-handed τ^-

→ $P_\tau = 0$ if parity is conserved.

Neutral Current and Asymmetries

$$\frac{1}{\sigma_{TOT}}\frac{d\sigma_R}{d\cos\theta_{\tau^-}} = \frac{3}{16}\left[(1+\langle P_\tau\rangle)(1+\cos^2\theta_{\tau^-}) + \frac{8}{3}(A_{FB} + A^{FB}_{pol})\cos\theta_{\tau^-}\right.$$

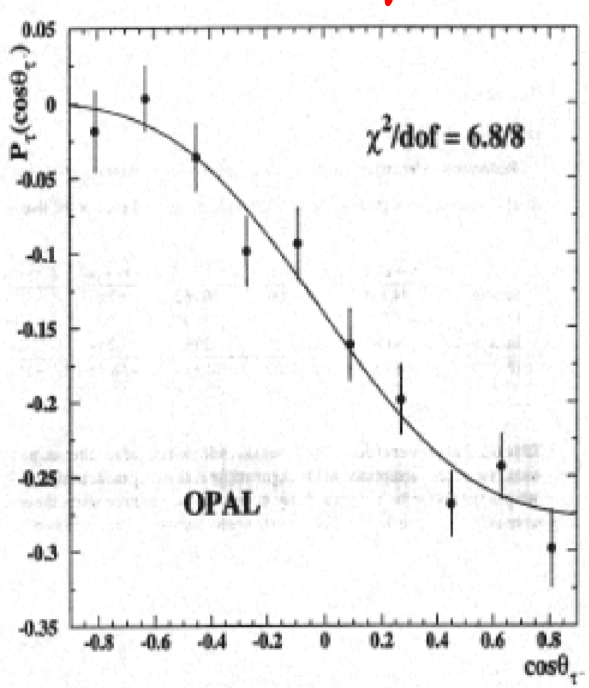
$$\frac{1}{\sigma_{TOT}}\frac{d\sigma_L}{d\cos\theta_{\tau^-}} = \frac{3}{16}\left[(1-\langle P_\tau\rangle)(1+\cos^2\theta_{\tau^-}) + \frac{8}{3}(A_{FB} - A^{FB}_{pol})\cos\theta_{\tau^-}\right.$$

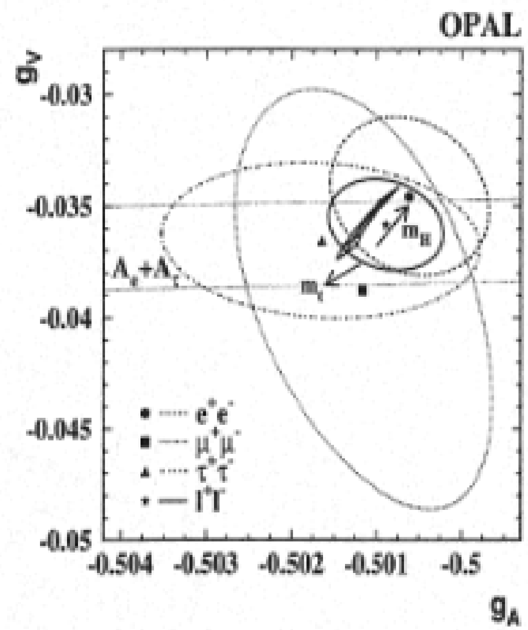
$$\langle P_\tau\rangle = \frac{\sigma_R - \sigma_L}{\sigma_{TOT}}, \quad A^{pol}_{FB} = \frac{(\sigma_R-\sigma_L)_{\cos\theta>0} - (\sigma_R-\sigma_L)_{\cos\theta<0}}{\sigma_{TOT}},$$ A_{FB} as prev. defined

$$\langle P_\tau\rangle = -A_\tau, \quad A^{FB}_{pol} = \frac{-3}{4}A_e, \quad A_{FB} = \frac{3}{4}A_e A_\tau$$

$$A_\ell = \frac{2g_V/g_A}{(1+g_V/g_A)^2}$$

note that g_V is very small. If $g_V = 0$ ($\sin^2\theta_W = 0.25$) $\Rightarrow A_\ell = 0$, no parity violation. Why?





NEUTRAL CURRENT FOR QUARKS

For charged leptons, we had the following vertex factor:

$$\frac{-i}{\sqrt{2}}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)^{1/2} \bar{e}\gamma_\lambda\left[R_e(1+\gamma_5)+L_e(1-\gamma_5)\right]e$$

$$R_e = 2\sin^2\theta_W \quad , \quad L_e = 2\sin^2\theta_W - 1$$

For quarks we have:

$$\frac{-i}{\sqrt{2}}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)^{1/2} \bar{q}\gamma_\lambda\left[R_q(1+\gamma_5)+L_q(1-\gamma_5)\right]q$$

$$R_q = -2Q_q\sin^2\theta_W$$
$$L_q = 1 - 2Q_q\sin^2\theta_W$$

Recall that $\Gamma_{e^+e^-} = (R_e^2 + L_e^2)\Gamma_{\nu\bar{\nu}}$ $\qquad R_e = \frac{1}{2}(c_V - c_A)$

Now we set: $\Gamma_{q\bar{q}} = 3(R_q^2 + L_q^2)\Gamma_{\nu\bar{\nu}}$ $\qquad L_e = \frac{1}{2}(c_V + c_A)$

For u quark : $Q_q = 2/3 \rightarrow R_q = -4/3\sin^2\theta_W,\ L_q = 1 - \frac{4}{3}\sin^2\theta_W$

d quark : $Q_q = -1/3 \rightarrow R_q = 2/3\sin^2\theta_W,\ L_q = -1 + \frac{2}{3}\sin^2\theta_W$

NEUTRAL CURRENT FOR QUARKS

$$R_u^2 = 4\cdot\left(\frac{2}{3}\right)^2 \sin^2\theta_W = \frac{16}{9}\sin^2\theta_W$$

$$L_u^2 = \left(1 - \frac{4}{3}\sin^2\theta_W\right) = 1 + \frac{16}{9}\sin^4\theta_W - \frac{8}{3}\sin^2\theta_W$$

$$R_d^2 = \frac{4}{9}\sin^2\theta_W$$

$$L_d^2 = -1 + \frac{2}{3}\sin^2\theta_W = 1 + \frac{4}{9}\sin^4\theta_W - \frac{4}{3}\sin^2\theta_W$$

$$\Gamma_{u\bar{u}} = 3\cdot\left(1 - \frac{8}{3}x_w + \frac{32}{9}x_w^2\right)\Gamma(Z\to\nu\bar{\nu}) \simeq 3\cdot 0.58\cdot\Gamma(Z\to\nu\bar{\nu})$$

$$\Gamma_{d\bar{d}} = 3\cdot\left(1 - \frac{4}{3}x_w + \frac{8}{9}x_w^2\right)\Gamma(Z\to\nu\bar{\nu}) \simeq 3\cdot 0.75\cdot\Gamma(Z\to\nu\bar{\nu})$$

$\rightarrow \sin^2\theta_W = x_w$

So what is approx. $\Gamma(Z\to q\bar{q})\,/\,\Gamma(Z\to\nu\nu)$?

Also $\quad R_{PEAK} \equiv \sigma(e^+e^-\to f\bar{f})\,/\,\sigma_{QED}(e^+e^-\to\mu^+\mu^-)$

$\hookrightarrow$ Z peak

$$R_{PEAK}(e^+e^-\to\mu^+\mu^-) = (\underbrace{1 - 4x_w + 8x_w^2}_{\simeq 0.5})\,R_{PEAK}(e^+e^-\to\nu\bar{\nu})$$

$$R_{PEAK}(e^+e^-\to u\bar{u}) = 3\cdot(1 - 8x_w + 32x_w^2)\,R_{PEAK}(e^+e^-\to\nu\bar{\nu})$$

$$R_{PEAK}(e^+e^-\to d\bar{d}) = 3\cdot\left(1 - \frac{4}{3}x_w + \frac{8}{9}x_w^2\right)R_{PEAK}(e^+e^-\to\nu\bar{\nu})$$

NEUTRAL CURRENT FOR QUARKS

FOR ASYMMETRIES WE HAVE:

$$A(q\bar{q}) = \frac{3 G_F s}{16\pi\alpha Q_q \sqrt{2}} (R_e - L_e)(R_q - L_q) \quad \text{FOR } s \ll M_Z^2$$

$$R_e - L_e = R_d - L_d = -(R_u - L_u) = 1$$

$$\Rightarrow \quad A(u\bar{u}) = \frac{3}{2} A(\mu^+\mu^-)$$

$$A(d\bar{d}) = 3 A(\mu^+\mu^-)$$

$$A_{PEAK}(f\bar{f}) = \frac{3}{4} \frac{(L_e^2 - R_e^2)(L_f^2 - R_f^2)}{(L_e^2 + R_e^2)(L_f^2 + R_f^2)} = 3 \frac{C_A^e C_V^e \, C_A^f C_V^f}{(C_A^{e2} + C_V^{e2})(C_A^{f2} + C_V^{f2})}$$

	ν	e	u	d
C_V	0.5	-0.05	0.2	-0.35

Early asymmetry measurement (DESY $\sqrt{s} = 30$ GeV)