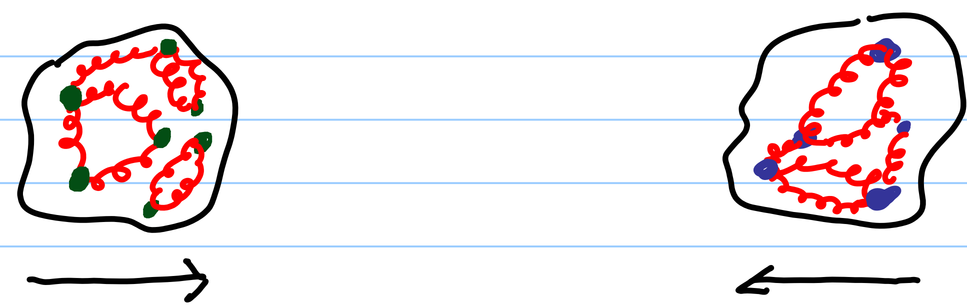
# LECTURE 15: Hadron Structure

## Overview:

- Formalism for the kinematics
- Elastic Scattering of protons and neutrons
- Deep Inelastic scattering

(I used Quigg and Halzen-Martin as references)

# HADRON STRUCTURE



We would like to understand the composition of hadrons:

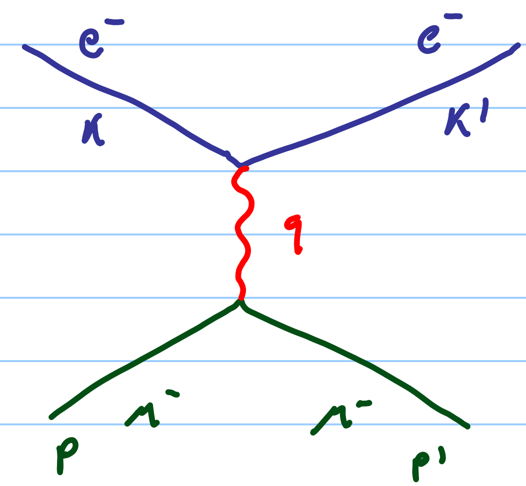
→ determine parton flux in hadron collisions

→ learn more about QCD

We'll start by doing a cross section calculation in the lab frame. This will prove convenient later on.

We go back to $e$-$\mu$ scattering. We calculated the cross section in the CM frame. Here we switch to the lab frame.

# Electron-muon scattering




we neglected $m_e$ and obtained:

$$\overline{|M|^2} = \frac{8e^4}{q^4}\left[(k'\cdot p)(k\cdot p) + (k'\cdot p)(k\cdot p') - m^2 k'\cdot k\right]$$

↪ muon mass

We have:

$$q = k - k'$$
$$p' = (k - k' + p) \quad , \quad k^2 \simeq 0$$
$$q^2 \simeq -2k\cdot k'$$

Eliminate $p'$

$$\rightarrow \overline{|M^2|} \quad \frac{8e^4}{q^4}\left[-\frac{1}{2}q^2(k\cdot p - k'\cdot p) + 2(k'\cdot p)(k\cdot p) + \frac{1}{2}m^2 q^2\right]$$

# Electron-muon scattering

lab frame:

$$p = (M, 0)$$
$$k = (E, \vec{k})$$
$$k' = (E', \vec{k}')$$
$$q = (\nu, \vec{q})$$

We get $$\overline{|M^2|} = \frac{8e^4}{q^4}\left[-\frac{1}{2}q^2 \underbrace{M}_{\text{muon mass}}(E-E') + 2EE'M^2 + \frac{1}{2}M^2 q^2\right]$$

$$= \frac{8e^4}{q^4} 2M^2 E'E \left[1 + \frac{q^2}{4EE'} - \frac{q^2}{2M^2}\frac{M(E-E')}{2EE'}\right]$$

we have $$q^2 \simeq -2k\cdot k' \simeq -2EE'(1-\cos\theta)$$
$$= -4EE'\sin^2\frac{\theta}{2}$$

$$p+q = p' \rightarrow (p+q)^2 = p'^2 \rightarrow p^2 + q^2 + 2(q\cdot p) = p'^2 \rightarrow q^2 = -2(q\cdot p)$$

$$q^2 = -2\nu M, \quad \nu = E-E' \Rightarrow E-E' = \frac{-q^2}{2M}$$

$$\Rightarrow \overline{|M^2|} = \frac{8e^4}{q^4} 2M^2 EE' \left\{\underbrace{1 - \sin^2\frac{\theta}{2}}_{\cos^2\theta/2} - \frac{q^2}{2M^2}\sin^2\frac{\theta}{2}\right)$$

# Electron-muon scattering

We get: $\frac{8e^4}{q^4} 2M^2 E' E \left[ \cos^2\frac{\theta}{2} - \frac{q^2}{2M^2} \sin^2\frac{\theta}{2} \right]$

Now, the cross section:

$$d\sigma = \frac{1}{(2E)(2M)} \frac{|\bar{M}|^2}{4\pi^2} \frac{d^3k'}{2E'} \frac{d^3p'}{2p_0'} \delta^4(p+k-p'-k')$$

$$= \frac{1}{4ME} \frac{|\bar{M}|^2}{4\pi^2} \frac{1}{2} E' dE' d\Omega \frac{d^3p'}{2p_0'} \delta^4(p+q-p')$$

For the $\int \frac{d^3p'}{2p_0'} \delta^4(p+q-p')$ integration we get

note that $\delta(p_0'^2 - a^2) = \frac{1}{2|a|} (\delta(p_0'-a) + \delta(p_0'+a))$

$$\Rightarrow \delta((p+q)^2 - M^2) = \delta(2p\cdot q + q^2) = \frac{1}{2M} \delta\left(\nu + \frac{q^2}{2M}\right)$$

Electron-muon scattering

$$\frac{1}{2M}\,\delta\left(\nu + \frac{q^2}{2M}\right) = \frac{1}{2M}\,\delta\left[E - E' - EE'(1-\cos\theta)/M\right]$$

$$= \frac{1}{2MA}\,\delta(E' - E/A)$$

$$A = 1 + \frac{2E}{M}\sin^2\frac{\theta}{2}$$

Inserting matrix element and using result above, we get:

$$\frac{d\sigma}{dE'\,d\Omega} = \left(\frac{2\alpha E'}{q^2}\right)^2\left[\cos^2\frac{\theta}{2} - \frac{q^2}{2M}\sin^2\frac{\theta}{2}\right]\delta\left(\nu + \frac{q^2}{2M}\right)$$

$$\frac{d\sigma}{d\Omega} = \left[\frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\right]\frac{E'}{E}\left[\cos^2\frac{\theta}{2} \underbrace{- \frac{q^2}{2M}\sin^2\frac{\theta}{2}}_{\text{due to magnetic moment}}\right]$$

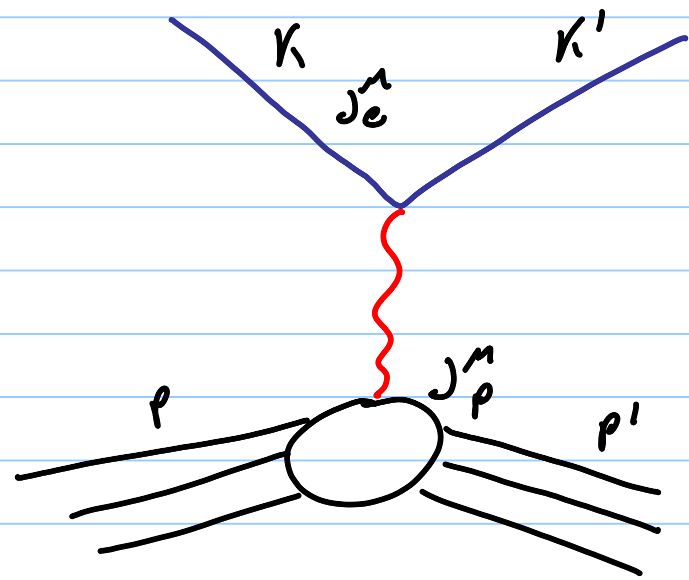
Note that for spinless particles we had vertex factors of the form: $ie(p_A + p_C)^\mu$ and

$$\bar{u}_f \gamma^\mu u = \frac{1}{2M}\,\bar{u}_f\left((p_f + p_i)^\mu + i\sigma^{\mu\nu}(p_f - p_i)_\nu\right)u_i$$

# Electron-Proton scattering

If the proton was a point charge with Dirac magnetic moment $\frac{e}{2M}$, we would just replace

$\mu_\mu \to \mu_p$. But it is not... so:




$$J_e^\mu = -e\,\bar{u}(k')\gamma^\mu u(k)\, e^{i(k'-k)\cdot x}$$
$$J_p^\mu = e\,\bar{u}(p')[\,?\,]u(p)\, e^{i(p'-p)\cdot x}$$

# Electron - Proton scattering

- $J_p^\mu$ must be a 4-vector
- Two indep. Terms : $\gamma^\mu$ , $i\sigma^{\mu\nu} q_\nu$
- no $\gamma^5$ $\rightarrow$ parity is conserved

$$[\,?\,] = \left[ F_1(q^2)\,\gamma^\mu + \frac{K}{2m} F_2(q^2)\, i\sigma^{\mu\nu} q_\nu \right]$$

For $q^2 \to 0$ proton contents not resolved. We have a particle of charge $e$ and magnetic moment $(1+K)\frac{e}{2m}$

$\hookrightarrow$ anomalous mag. moment : proton = 1.79, neutron = −1.91

with $F_1(0) = 1$ , $F_2(0) = 1$

The expression similar to $e\mu$ scattering is:

# ELECTRON - PROTON SCATTERING

$$\left.\frac{d\sigma}{d\Omega}\right|_{lab} = \left(\frac{\alpha^2}{4E^2 \sin^2\theta/2}\right) \frac{E'}{E} \left\{ \left( F_1^2 - \frac{\kappa^2 q^2}{4m^2} F_2^2 \right) \cos^2\frac{\theta}{2} - \frac{q^2}{2m} (F_1 + \kappa F_2)^2 \sin^2\frac{\theta}{2} \right]$$

$\rightarrow$ ROSENBLUTH FORMULA

$\kappa = 0 \ , \ F_1 = 1$ recovers muon result

In practice, different factors are used:

$$G_E \equiv F_1 + \frac{\kappa q^2}{4m^2} F_2 \quad , \quad G_m \equiv F_1 + \kappa F_2$$

we get:
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4\frac{\theta}{2}} \frac{E'}{E} \left( \frac{G_E^2 + \tau G_m^2}{1+\tau} \cos^2\frac{\theta}{2} + 2\tau G_m^2 \sin^2\frac{\theta}{2} \right)$$

$$\tau = \frac{-q^2}{4m^2}$$

$\longrightarrow$ no "interference" Term

$G_E$ AND $G_m$ RELATED TO PROTON CHARGE AND MAGNETIC MOMENT DIST. IN BREIT FRAME $p' = -p$

# Electron - Proton Scattering

$$G_E(q^2) \simeq \left(1 - \frac{q^2}{0.71}\right)^{-2} \quad \text{in } GeV^2$$

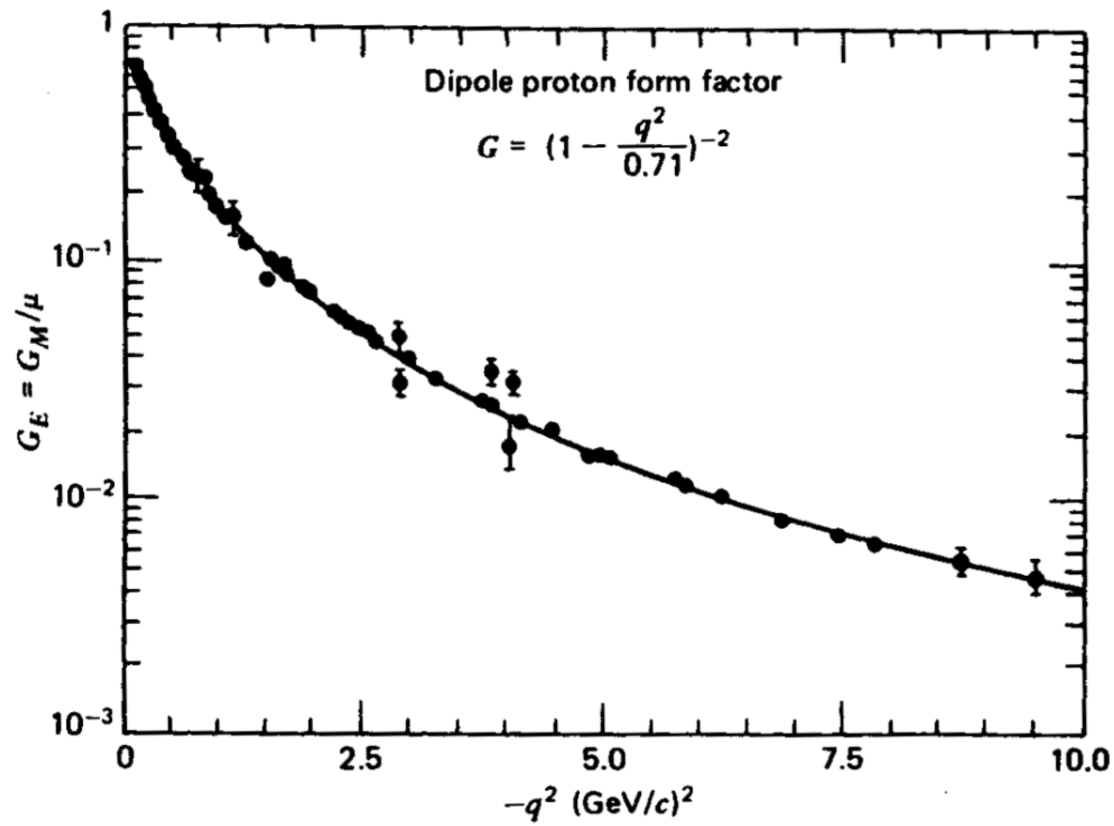
$$\langle r^2 \rangle = 6 \left( \frac{dG_E(q^2)}{dq^2} \right)_{q^2=0} = 0.8 \times 10^{-15}\ m$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{point} |F(q)|^2$$

$$F(q) = \int \rho(x) e^{iq\cdot x} d^3x$$

$$\int \rho(x)\, d^3x = 1$$

$$F(0) = 1$$




$$F(q) = \int \left(1 + iq\cdot x - \frac{(q\cdot x)^2}{2} + \dots\right) \rho(x)\, d^3x$$

$$= 1 - \frac{1}{6}|q|^2 \langle r^2 \rangle + \dots$$