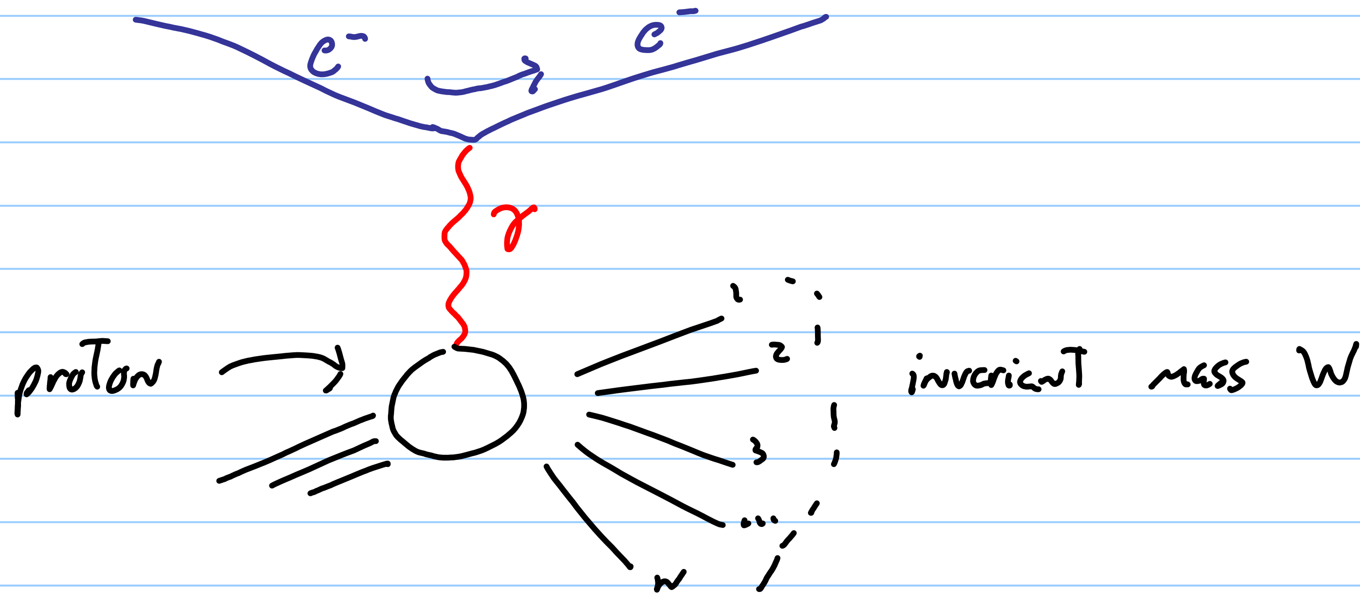
LECTURE 16: Hadron Structure (Part 2)

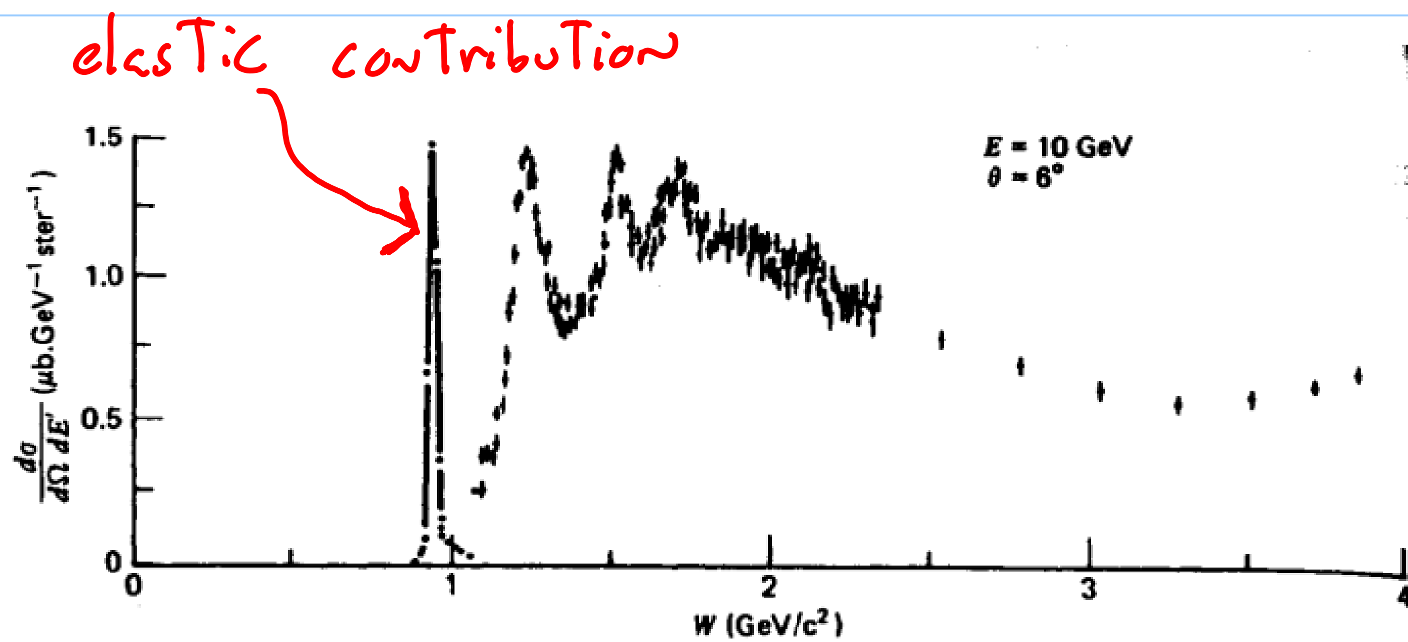
Overview:

- -Inelastic Scattering of protons and electrons
- -Contents of the proton and neutron
- -Parton Distribution Functions

(I used Quigg and mostly Halzen-Martin as references)

INELASTIC ELECTRON - PROTON SCATTERING





INELASTIC ELECTRON-PROTON SCATTERING

We had before for $e\mu$ scattering : $d\sigma \sim L^{e}_{\mu\nu} (L^{\mu})^{\mu\nu}$

We'll Try : $L^{e}_{\mu\nu} W^{\mu\nu}$

$$W^{\mu\nu} = -W_1 g^{\mu\nu} + \frac{W_2}{M^2} p^\mu p^\nu + \frac{W_4}{M^2} q^\mu q^\nu + \frac{W_5}{M^2} (p^\mu q^\nu + p^\nu q^\mu)$$

$L_{\mu\nu}$ is symmetric $\rightarrow$ antisymmetric Terms will vanish

W_3 will be parity violating Term

We use $q^\mu L^{e}_{\mu\nu} = q^\nu L^{e}_{\mu\nu} = 0$ (check this)

We can also show that $q_\mu W^{\mu\nu} = 0$ which follows from $\partial_\mu J^\mu = 0$

$$\Rightarrow \quad W_5 = -\frac{p\cdot q}{q^2} W_2 \quad , \quad W_4 = \left(\frac{p\cdot q}{q^2}\right)^2 W_2 + \frac{M^2}{q^2} W_1$$

$\Rightarrow$ only Two structure functions are indep.

Inelastic Electron-Proton Scattering

$$W^{\mu\nu} = W_1\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) + W_2\left(p^\mu - \frac{p\cdot q}{q^2}q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}q^\nu\right)\cdot\frac{1}{M^2}$$

We can chose two indep. variables : q^2 , $\frac{p\cdot q}{M} \equiv \nu$

or dimensionless : $x = \frac{-q^2}{2p\cdot q} = \frac{-q^2}{2M\nu}$, $y = \frac{p\cdot q}{p\cdot K}$

Invariant mass : $W^2 = (p+q)^2 = M^2 + 2M\nu + q^2$

In rest frame of proton : $\nu = E - E'$

$$y = \frac{E-E'}{E}$$

We now have:

$$(L^e)^{\mu\nu}W_{\mu\nu} = 4W_1(K\cdot K') + \frac{2W_2}{M^2}\left[2(p\cdot K)(p\cdot K') - M^2 K\cdot K'\right]$$

Inelastic Electron - Proton Scattering

In lab frame:

$$L^{e\,\mu\nu} W_{\mu\nu} = 4EE' \left[\cos^2\frac{\theta}{2}\, W_2(\nu, q^2) + \sin^2\frac{\theta}{2}\, 2W_1(\nu, q^2) \right]$$

$$d\sigma = \frac{1}{4\left((k\cdot p)^2 - m^2 M^2\right)^{1/2}} \underbrace{\left[\frac{e^4}{q^4} L^{\mu\nu} W_{\mu\nu} \underbrace{4\pi M}_{\text{normalisation of } W^{\mu\nu}} \right]}_{|\mathcal{M}|^2} \frac{d^3k'}{2E'(2\pi)^3}$$

$$\left.\frac{d\sigma}{dE'\,d\Omega}\right|_{lab} = \frac{\alpha^2}{4E^2 \sin^4\frac{\theta}{2}} \left[W_2(\nu, q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu, q^2)\sin^2\frac{\theta}{2} \right]$$

→ neglected electron mass.

INELASTIC ELECTRON-PROTON SCATTERING

Summary of recent results:

For all reactions, the diff. cross section can be written as

$$\boxed{\frac{d\sigma}{dE'\,d\Omega} = \frac{4\alpha^2 E'^2}{q^4}\left[\quad\right]}$$

Point particle (we did muon but we'll deal with quarks...)

1
$$\boxed{[\ ] = \left(\cos^2\frac{\theta}{2} - \frac{q^2}{2m^2}\sin^2\frac{\theta}{2}\right)\delta\left(\nu + \frac{q^2}{2m}\right)}$$

elastic proton-electron:

2
$$\boxed{[\ ] = \left(\frac{G_E^2 + \tau G_M^2}{1+\tau}\cos^2\frac{\theta}{2} + 2\tau G_M^2 \sin^2\frac{\theta}{2}\right)\delta\left(\nu + \frac{q^2}{2m}\right), \quad \tau = \frac{-q^2}{4m^2}}$$

inelastic proton-electron:

3
$$\boxed{[\ ] = W_2(\nu, q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu, q^2)\sin^2\frac{\theta}{2}}$$

Inelastic Electron-Proton Scattering

If point-like spin $\frac{1}{2}$ quarks live inside the proton, we should be able to resolve them with photons that have small enough wavelength.

→ structure functions would become: $Q^2 \equiv -q^2$

$$2W_1 \rightarrow \frac{Q^2}{2m^2}\,\delta\left(\nu - \frac{Q^2}{2m}\right), \qquad W_2 = \delta\left(\nu - \frac{Q^2}{2m}\right)$$

inelastic proton-electron scattering → elastic electron-quark scattering

Using $\delta\left(\frac{x}{a}\right) = a\,\delta(x)$:

$$2m\,W_1(\nu, Q^2) = \frac{Q^2}{2m\nu}\,\delta\left(1 - \frac{Q^2}{2m\nu}\right)$$

$$\nu W_2(\nu, Q^2) = \delta\left(1 - \frac{Q^2}{2m\nu}\right)$$

→ W_1, W_2 now functions of $\frac{Q^2}{2m\nu}$ and not Q^2 and ν independently.

INELASTIC ELECTRON-PROTON SCATTERING

For elastic scattering of ep with $\kappa = 0 \Rightarrow G_E = G_M = G$

$$W_1 = \frac{Q^2}{4M^2} G^2(Q^2)\, \delta\left(\nu - \frac{Q^2}{2m}\right)$$

$$W_2 = G^2(Q^2)\, \delta\left(\nu - \frac{Q^2}{2m}\right)$$

$\hookrightarrow$ reflects size of proton

So for point constituents probed by large Q^2 photons:

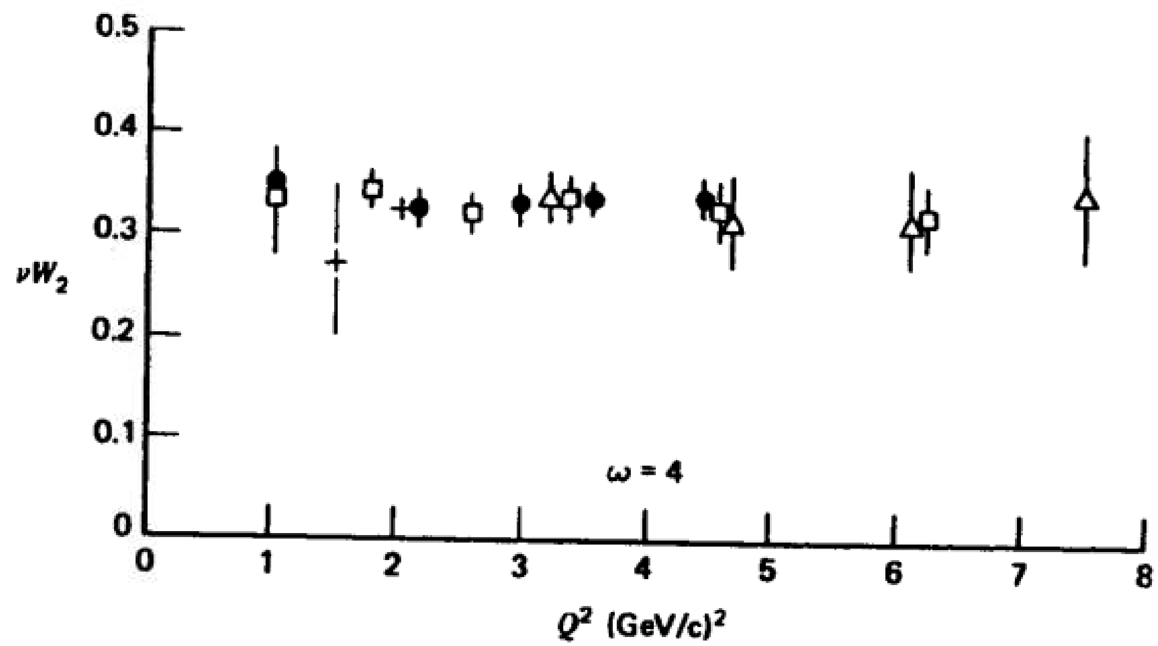
$$M W_1(\nu, Q^2) \to F_1(\omega), \qquad \omega = \frac{2q \cdot p}{Q^2} = \frac{2M\nu}{Q^2}$$
$$\nu W_2(\nu, Q^2) \to F_2(\omega)$$

$\to$ we should see that νW_2 is independent of Q^2 for a given ω

$\to$ evidence for "partons"

$\to$ Bjorken Scaling

Inelastic Electron-Proton Scattering



"PDFs" : parton distribution functions

$f_i(x) = \frac{dP_i}{dx}$ describes probability that a parton will carry a fraction x of the proton's momentum

$$\sum_i \int dx \, x f_i(x) = 1$$

Inelastic Electron-Proton Scattering

In terms of x and ω, structure functions given by:

$$F_1(\omega) = \frac{Q^2}{2M\nu x}\,\delta\left(1 - \frac{Q^2}{2M\nu}\right) = \frac{1}{2x^2\omega}\,\delta\left(1 - \frac{1}{x\omega}\right)$$

$$F_2(\omega) = \delta\left(1 - \frac{Q^2}{2M\nu}\right) = \delta\left(1 - \frac{1}{x\omega}\right)$$

$$\Rightarrow \quad F_2(\omega) = \sum_i \int dx\, e_i^2 f_i(x)\, x\, \delta\left(x - \frac{1}{\omega}\right)$$

$$F_1(\omega) = \frac{\omega}{2} F_2(\omega)$$

In terms of x:

$$\left.\begin{aligned} \nu W_2(\nu, Q^2) &\to F_2(x) = \sum_i e_i^2 x f_i(x) \\ M W_1(\nu, Q^2) &\to F_1(x) = \frac{1}{2x} F_2(x) \end{aligned}\right]$$

satisfies Bjorken scaling

$$\boxed{x = \frac{1}{\omega} = \frac{Q^2}{2M\nu}}$$

INELASTIC ELECTRON-PROTON SCATTERING

We can reexpress the results we obtained in terms of x and $y = \frac{\nu}{E} = \frac{p \cdot q}{p \cdot k}$, $1-y \simeq \frac{1}{2}(1+\cos\theta)$

$$dE'\,d\Omega = \frac{\pi}{EE'}\,dQ^2\,d\nu = \frac{2ME}{E'}\,\pi y\,dx\,dy$$

$$M\nu_{max}\frac{d\sigma}{dx\,dy} = \frac{2\pi\alpha^2}{x^2y^2}\left\{ x y^2 F_1 + \left[(1-y) - \frac{Mxy}{2\nu_{max}}\right] F_2 \right\}$$

$\nu_{max} = E$ in the lab frame

Parton model predicts:

$$\frac{d\sigma}{dx\,dy} = \frac{2\pi\alpha^2}{Q^4}\, s\,\left[1 + (1-y)^2\right] \sum_i e_i^2\, x f_i(x)$$

Proton QUARK CONTENT

We had: $F_2(x) = \sum_i e_i^2 \, x f_i(x)$

$$F_1(x) = \frac{1}{2x} F_2(x)$$

$$\rightarrow \frac{1}{x} F_2^{ep} = \left(\frac{2}{3}\right)^2 \left[u^p(x) + \bar{u}^p(x)\right] + \left(\frac{1}{3}\right)^2 \left[d^p(x) + \bar{d}^p(x)\right]$$
$$+ \left(\frac{1}{3}\right)^2 \left[s^p(x) + \bar{s}^p(x)\right]$$

$\rightarrow$ neglect other heavy quarks

We have a similar expression for neutrons.

$$\frac{1}{x} F_2^{en} = \left(\frac{2}{3}\right)^2 \left[u^n(x) + \bar{u}^n(x)\right] + \ldots$$

$$u^p(x) = d^n(x) \equiv u(x)$$
$$d^p(x) = u^n(x) \equiv d(x)$$
$$s^p(x) = s^n(x) \equiv s(x)$$

proton $u_v \; u_v \; d_v$ (valence)
$u_s\bar{u}_s$, $d_s\bar{d}_s$, $s_s\bar{s}_s$ (sea)

Proton Quark Content

$$u_s(x) = \bar{u}_s(x) = d_s(x) = \ldots = s(x)$$

$$u(x) = u_v(x) + u_s(x)$$

$$d(x) = d_v(x) + d_s(x)$$

$$\int_0^1 [u(x) - \bar{u}(x)]\, dx = 2$$

$$\int_0^1 [d(x) - \bar{d}(x)]\, dx = 1$$

$$\int_0^1 [s(x) - \bar{s}(x)]\, dx = 0$$

$$\Rightarrow \quad \frac{1}{x} F_2^{ep} = \frac{1}{9}[4u_v + d_v] + \frac{4}{3} s$$

$$\frac{1}{x} F_2^{en} = \frac{1}{9}[u_v + 4d_v] + \frac{4}{3} s$$

AT low x we expect:

$$\frac{F_2^{en}(x)}{F_2^{ep}(x)} \xrightarrow[x \to 0]{} 1$$

AT high x we expect

$$\frac{F_2^{en}(x)}{F_2^{ep}(x)} \to \frac{u_v + 4d_v}{4u_v + d_v}$$

$\rightarrow$ proton $u_v \gg d_v$: ratio Tends Towards 0.25

Proton Quark Content

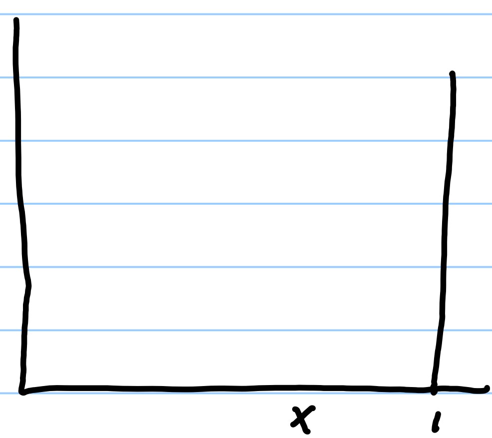
$$\frac{1}{x} F_2^{ep} = \frac{1}{9}\left[4u_v + d_v\right] + \frac{4}{3} S$$

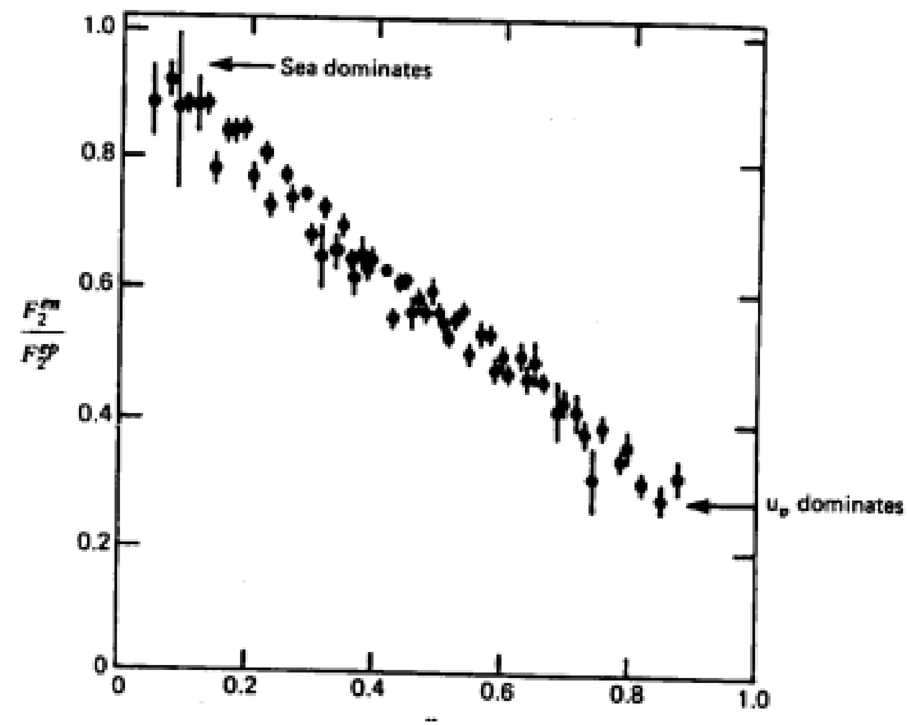
$$\frac{1}{x} F_2^{en} = \frac{1}{9}\left[u_v + 4d_v\right] + \frac{4}{3} S$$

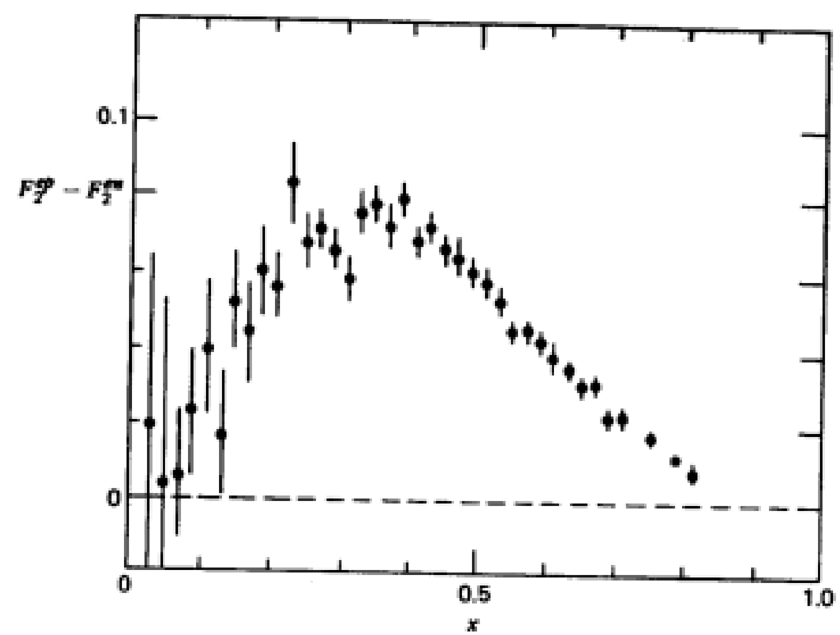
If we subtract the two equations above, we get:

$$\frac{1}{3}\left[u_v(x) - d_v(x)\right]$$

For 1 quark, we should get:



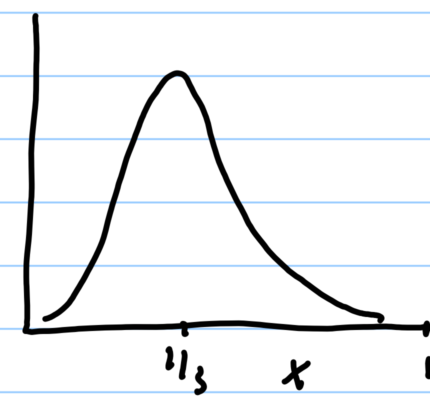


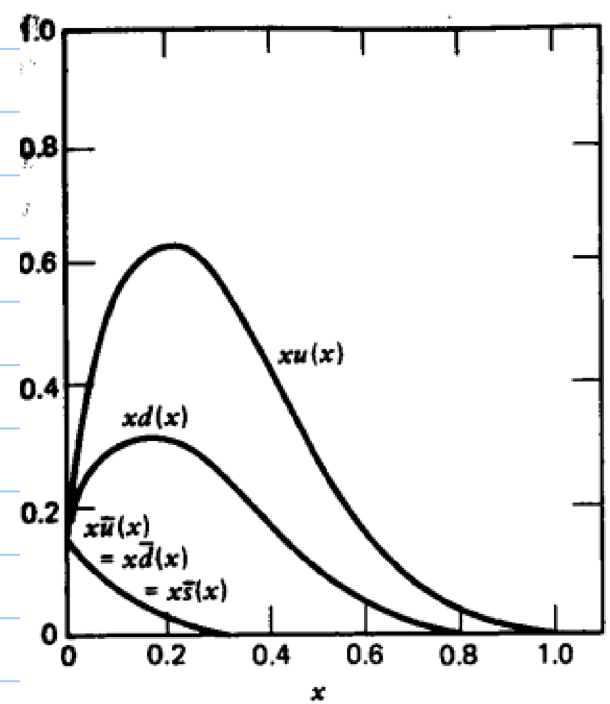


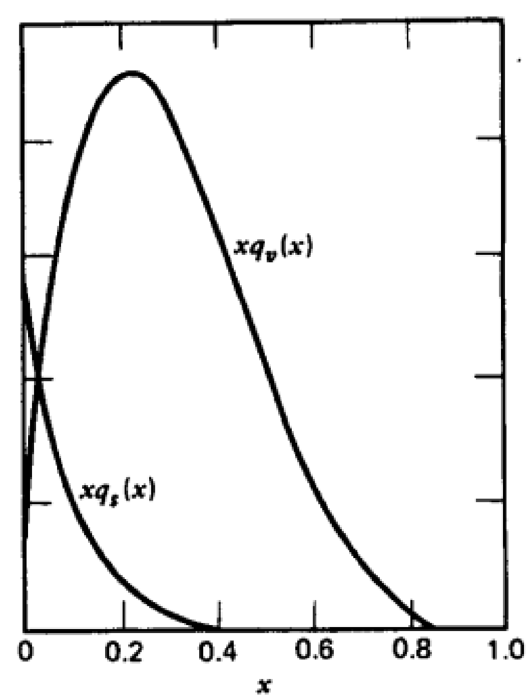
PROTON QUARK CONTENT

FOR 3 QUARKS:

3 QUARKS + INTERACTIONS:





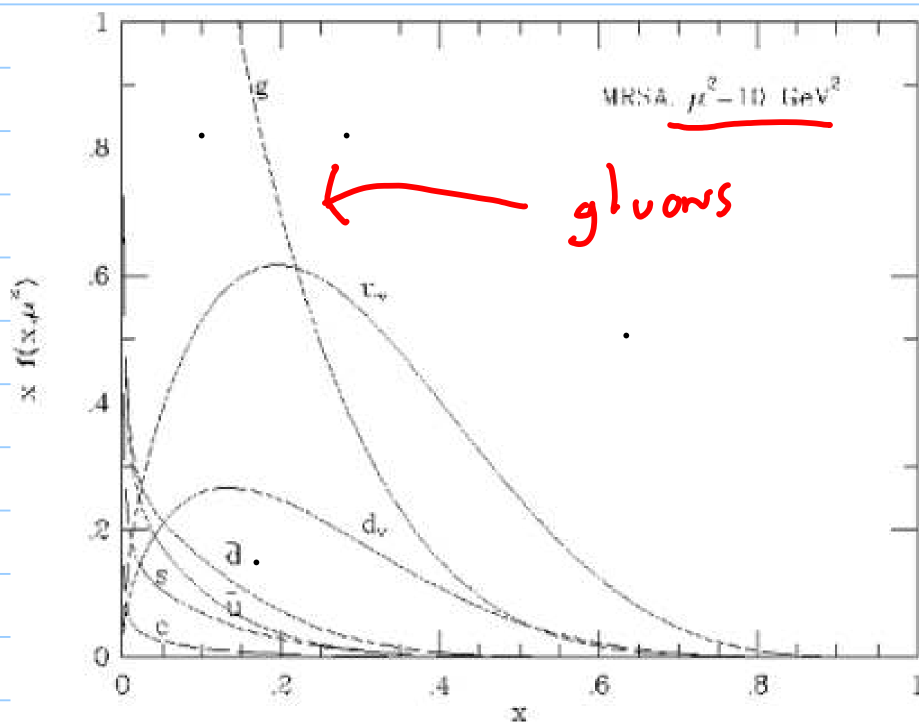


Proton Gluon Content

If we integrate $\int_0^1 dx\, x\left[u+\bar{u}+d+\bar{d}+s+\bar{s}\right] = 1 - F_{gluon}$

The photon does not interact with the gluon ...

We get $F_{gluon} \sim 0.45$!



note that these depend on Q^2 ...

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