LECTURE 22 : CP Violation

- Overview
- Meson mixing
- CP violation in Kaon system

(I used Thomson, Griffiths, Cheng Li)

RECAP

weak eigenstates $\neq$ mass eigenstates

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

W's couple to : $\begin{pmatrix} u \\ d' \end{pmatrix}$, $\begin{pmatrix} c \\ s' \end{pmatrix}$

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix} \qquad \begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ -d\sin\theta_c + s\cos\theta_c \end{pmatrix}$$

With 3 generations

$$\begin{pmatrix} u \\ d' \end{pmatrix} \quad \begin{pmatrix} c \\ s' \end{pmatrix} \quad \begin{pmatrix} T \\ b' \end{pmatrix} , \quad \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{Td} & V_{Ts} & V_{Tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

First doublet : $\begin{pmatrix} u \\ V_{ud}\, d + V_{us}\, s + V_{ub}\, b \end{pmatrix}$

RECAP (CONT.)

If CKM matrix was real, it would be orthogonal, couplings would be real.

A general 3x3 unitary matrix can be parameterized by 3 real angles and 6 complex phases.

If we start with:

$$q_1 = \begin{pmatrix} u \\ U_{11} d + U_{12} s + U_{13} b \end{pmatrix}$$

$U_{11} = R_{11} e^{i\delta}$, R_{11} real

we can redefine u-quark field:

$u \to u' = u e^{-i\delta}$ which gives:

$$q_1 = e^{i\delta} \begin{pmatrix} u' \\ R_{11} d + U'_{12} s + U'_{13} b \end{pmatrix}$$

Recap (cont.)

We can factor out complex phases for U_{21}, U_{31} too for c and t quarks.
We can also absorb two other phases U_{12} and U_{13} by redefining s, b fields

→ down to 13 parameters (9 R_{ij}, 4 phases)

→ normalization of each state reduces this by 3

→ orthogonality of 6 states reduces this by 6

→ need 3 parameters for 3x3 real orthogonal matrix

⟶ one independent phase

⟶ $n(n-1)/2$ angles

⟶ $(n-1)(n-2)/2$ indep. phases

C and P

$\theta - \tau$ puzzle :

$$\theta \to \pi^+ + \pi^0 \qquad P = +1$$

$$\tau \to \pi^+ + \pi^0 + \pi^0 \qquad P = -1$$
$$\pi^+ + \pi^+ + \pi^-$$

θ, τ same particle (K^+)

$\longrightarrow$ parity not conserved

$C \to$ "charge conjugation" changes all internal quantum numbers (charge, baryon and lepton number strangeness, etc.) but leaves mass, spin the same.

$\pi^+ \to \mu^+ + \nu_\mu \to$ left handed μ^+

$\pi^- \to \mu^- + \bar{\nu}_\mu \to$ no left-handed μ^-

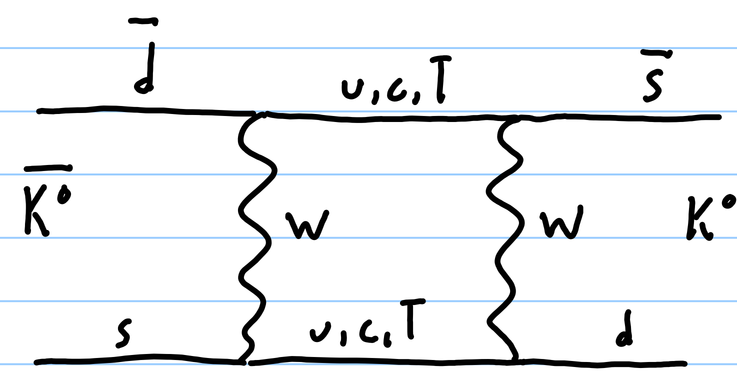
C and P

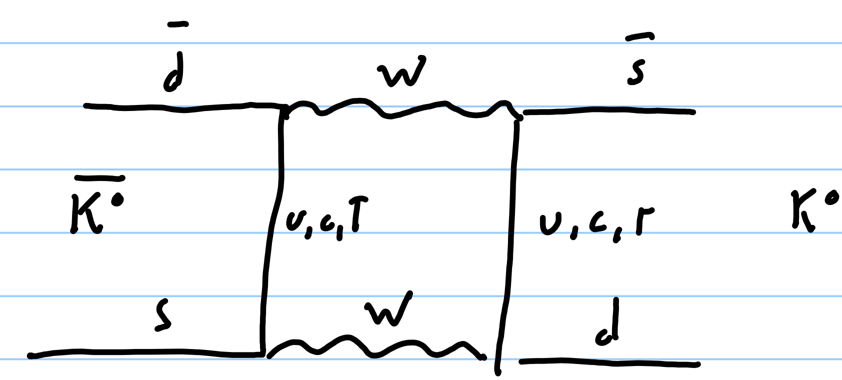
→ weak interactions not invariant under C either.

→ What about CP?

CP invariance has interesting implications on neutral mesons.

For Kaons:





→ When we turn weak interactions on,

$\overline{K^0}$ and K^0 will mix

We have to deal will superposition of K^0 and $\overline{K^0}$

CP Violation

$P|K^0\rangle = -|K^0\rangle$, $P|\bar{K}_0\rangle = -|\bar{K}^0\rangle$

$C|K^0\rangle = |\bar{K}^0\rangle$, $C|\bar{K}_0\rangle = |K^0\rangle$

$CP|K^0\rangle = -|\bar{K}^0\rangle$, $CP|\bar{K}^0\rangle = -|K^0\rangle$

→ normalized eigenstates of CP :

$$|K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right) , \quad K_2 = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)$$

$$CP|K_1\rangle = +|K_1\rangle , \quad CP|K_2\rangle = -|K_2\rangle$$

If CP is conserved in weak interactions:

K_1 will only decay into $CP = +1$ state

K_2 will only decay into $CP = -1$ state

CP Violation (cont.)

→ both states have $C = +1$

K_1 has $P = +1$
K_2 has $P = -1$

→ Decay to two pions gives state $P = +1$

→ Decay to three pions gives state $P = -1$

⇒ K_1 will decay to 2 pions and K_2 will decay to 3 pions

⇒ K_1 lifetime < K_2 lifetime

$$\tau_{K_1} = 0.9 \times 10^{-10}$$

$$\tau_{K_2} = 5 \times 10^{-8}$$

$$m_2 - m_1 = 3.5 \times 10^{-6}\ \text{eV} \ !$$

CP VIOLATION (cont.)

If we start with beam of K^0, weak interactions will turn it into linear combination K^0 and $\bar{K}^0$ $\rightarrow$ K_1 and K_2

$\rightarrow$ K_1 component will decay in first few centimetres

$\rightarrow$ K_2 component will be the only component left after a few metres

$\Longrightarrow$ Decays well away from initial K^0 production should all go to 3 pions (if CP conserved)

Observation: $\sim \frac{1}{500}$ K_2 decays go to $\pi^+ \pi^-$

$\Rightarrow$ CP not conserved

CP Violation (cont.)

So the long-lived Kaon is not a perfect CP eigenstate. We should write

$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} \left(|K_2\rangle + \varepsilon |K_1\rangle \right)$$

→ Note that the short component can be regenerated if K_L been interacts with matter

$\bar{K}^0$ and K^0 components interact differently in material e.g. $\bar{K}^0 p \to \Sigma^0 \pi^+$ $s \leftrightarrow u$ exchange

similar process doesn't exist for K^0

⇒ $\bar{K}^0$ component can be absorbed

⇒ regenerates K_1 !

Time evolution of K^0 - $\overline{K^0}$ system

Without K^0 - $\overline{K^0}$ mixing we would write

$$|K^0(t)\rangle = |K_0\rangle e^{-\Gamma t/2} e^{-imt}$$

$$\rightarrow \quad i\frac{\partial}{\partial t}|K^0(t)\rangle = (m - \frac{i}{2}\Gamma)\,|K^0(t)\rangle$$

$\rightarrow$ effective Hamiltonian

$$\mathcal{H}\,|K^0(t)\rangle = (m - \frac{i}{2}\Gamma)\,|K^0(t)\rangle$$

$\rightarrow$ eff. Hamiltonian not Hermitian due to exponential decay term

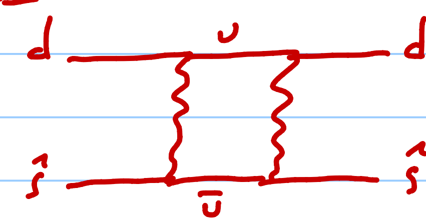
$$m = m_d + m_{\bar{s}} + \langle K^0|\hat{H}_{QCD} + \hat{H}_{em} + \hat{H}_w|K^0\rangle + \sum_j \frac{\langle K^0|\hat{H}_w|j\rangle\langle j|\hat{H}_w|K_0\rangle}{E_j - m_K}$$

$\hookrightarrow$ M_{11} later

$$\Gamma = 2\pi \sum_f |\langle f|\hat{H}_w|K^0\rangle|^2 \rho_f$$

$f \rightarrow$ final states

e.g.



with mixing: $|K(t)\rangle = a(t)\,|K^0\rangle + b(t)\,|\bar{K}^0\rangle$

$a(t)$, $b(t)$ are amplitudes + phases

we set

$$\begin{pmatrix} M_{11} - \tfrac{i}{2}\Gamma_{11} & M_{12} - \tfrac{i}{2}\Gamma_{12} \\ M_{21} - \tfrac{i}{2}\Gamma_{21} & M_{22} - \tfrac{i}{2}\Gamma_{22} \end{pmatrix} \begin{pmatrix} a(t)\,|K^0\rangle \\ b(t)\,|\bar{K}^0\rangle \end{pmatrix} = i\frac{\partial}{\partial t}\begin{pmatrix} |K^0\rangle \\ |\bar{K}^0\rangle \end{pmatrix}$$

effective Hamiltonian: $H = M - \tfrac{i}{2}\Gamma = \begin{pmatrix} M_{11} & \dots \\ \dots & M_{22} \end{pmatrix} - \tfrac{i}{2}\begin{pmatrix} \Gamma_{11} & \dots \\ \Gamma_{21} & \dots \end{pmatrix}$

$$\Gamma = 2\pi \sum_f |\langle f|\hat{H}_w|K(t)\rangle|^2 \rho_f$$

$$|\langle f|\hat{H}_w|K(t)\rangle|^2 = |a(t)|^2 |\langle f|\hat{H}_w|K_0\rangle|^2 + |b(t)|^2 |\langle f|\hat{H}_w|\bar{K}_0\rangle|^2$$
$$\left[\begin{array}{l} + a(t)\,b(t)^* \langle \bar{K}_0|\hat{H}_w|f\rangle\langle f|\hat{H}_w|K_0\rangle \\ + a(t)^* b(t) \langle K_0|\hat{H}_w|f\rangle\langle f|\hat{H}_w|\bar{K}_0\rangle \end{array}\right.$$

$$\Gamma_{11} = 2\pi \sum_f |\langle f|\hat{H}_w|K_0\rangle|^2 \rho_f$$

$$\Gamma_{22} = 2\pi \sum_f |\langle f|\hat{H}_w|\bar{K}_0\rangle|^2 \rho_f$$

interference $\rightarrow$ $\Gamma_{12} = \Gamma_{21}^*$

Mass Term M_{11} given before, M_{22} obtained by swapping $\bar{s} \to s$, $d \to \bar{d}$

$$M_{12} = M_{21}^* = \sum_j \frac{\langle \bar{K}^0 | \hat{H}_w | F \rangle \langle f | \hat{H}_w | K_0 \rangle}{E_j - M_K}$$

Note: no $\langle \bar{K}_0 | \hat{H}_w | K_0 \rangle$ Term

With CPT we have $M_{11} = M_{22}$, $\Gamma_{11} = \Gamma_{22}$

So we get: $\begin{pmatrix} M - i/_2 \Gamma & M_{12} - i/_2 \Gamma_{12} \\ M_{21} - i/_2 \Gamma_{21} & M - i/_2 \Gamma \end{pmatrix} \begin{pmatrix} a(t) \, |K^0\rangle \\ b(t) \, |\bar{K}_0\rangle \end{pmatrix} = \frac{i\partial}{\partial t} \begin{pmatrix} a(t) \, |K^0\rangle \\ b(t) \, |\bar{K}^0\rangle \end{pmatrix}$

Eigenstates and Eigenvalues:

$$\begin{pmatrix} |K_+\rangle \\ |K_-\rangle \end{pmatrix} = \frac{1}{\sqrt{1+|\xi|^2}} \begin{pmatrix} 1 & \xi \\ 1 & -\xi \end{pmatrix} \begin{pmatrix} |K^0\rangle \\ |\bar{K}_0\rangle \end{pmatrix} = \frac{1}{\sqrt{1+|\xi|^2}} \begin{pmatrix} |K^0\rangle + \xi |\bar{K}^0\rangle \\ |K^0\rangle - \xi |\bar{K}^0\rangle \end{pmatrix}$$

$$|K_+(t)\rangle = \frac{1}{\sqrt{1+|\xi|^2}} \left(|K^0\rangle + \xi |\bar{K}^0\rangle \right) e^{-i\lambda_+ T}$$
$$|K_-(t)\rangle = \frac{1}{\sqrt{1+|\xi|^2}} \left(|K^0\rangle - \xi |\bar{K}^0\rangle \right) e^{-i\lambda_- T}$$

$\lambda_\pm$ eigenvalues of matrix

Strangeness oscillations

We will neglect CP violation for now

We start with a K^0

$$|K(t)\rangle = \frac{1}{\sqrt{2}}\left[\Theta_s(t)|K_S\rangle + \Theta_L(t)|K_L\rangle\right]$$

with $\Theta_s(t) = \exp\left[(-im_s + \Gamma_s/2)t\right]$, $\Theta_L(t) = \exp\left[(-im_L + \Gamma_L/2)t\right]$

$$\simeq \frac{1}{2}\left[\Theta_s(|K^0\rangle + |\bar{K}^0\rangle) + \Theta_L(|K^0\rangle - |\bar{K}^0\rangle)\right]$$

$$= \frac{1}{2}(\Theta_s + \Theta_L)|K^0\rangle + \frac{1}{2}(\Theta_s - \Theta_L)|\bar{K}^0\rangle$$

$$P(K^0_{T=0} \to K^0) = |\langle K^0|K(t)\rangle|^2 = \frac{1}{4}|\Theta_s + \Theta_L|^2$$

$$P(K^0_{T=0} \to \bar{K}^0) = \frac{1}{4}|\Theta_s - \Theta_L|^2$$

we use: $|\Theta_s \pm \Theta_L|^2 = |\Theta_s|^2 + |\Theta_L|^2 \pm 2\,\mathrm{Re}(\Theta_s\Theta_L^*)$

$$= e^{-\Gamma_s T} + e^{-\Gamma_L T} \pm 2\,\mathrm{Re}\left(e^{-im_s T}\cdot e^{-1/2\Gamma_s T}\cdot e^{+im_L T}\cdot e^{-1/2\Gamma_L T}\right)$$

$$= e^{-\Gamma_s T} + e^{-\Gamma_L T} \pm 2e^{-1/2(\Gamma_s + \Gamma_L)T}\cos(\Delta m t),$$ with: $\Delta m = m_L - m_s$

We get

$$P(K_{T=0} \to K^0) = 1/4\; e^{-\Gamma_S T} + e^{-\Gamma_L T} + 2 e^{-1/2(\Gamma_S + \Gamma_L)T} \cos(\Delta m t)$$

$$P(K^0_{T=0} \to \bar{K}^0) = \;\; '' \;\; '' \; - \;\; '' \;\; ''$$

the period of the oscilliation is $\sim 10^{-9}$ s and is about 10x greater than K_S lifetime

CPLEAR Experiment

beam: $\bar{p}p \to K^- \pi^+ K^0$ and $\bar{p}p \to K^+ \pi^- \bar{K}^0$

use decays: $K^0 \to \pi^+ \ell^- \bar{\nu}$, $\bar{K}^0 \to \pi^- \ell^+ \nu$

Tag flavour at production and decay
↳ particle id using Cerenkov detector

Decay rates given by R_+ (for positive charge lepton) and
R_- (" negative charge lepton)

$R_+ \propto P(K^0_{T=0} \to K^0)$, $\bar{R}_+ \propto P(\bar{K}^0_{T=0} \to K_0)$

Measure Asymmetry: $A_{\Delta m}(t) = \dfrac{(R_+ + \bar{R}_-) - (R_- + \bar{R}_+)}{(R_+ + \bar{R}_-) + (R_- + \bar{R}_+)}$

$$A_{\Delta m}(t) = \frac{2e^{-(\Gamma_S + \Gamma_L)t/2} \cos(\Delta m T)}{e^{-\Gamma_S T} + e^{-\Gamma_L T}}$$

$\rightarrow$ extract $\Delta m = 3.5 \times 10^{-15}$ GeV

or 3.5×10^{-6} eV

CP violation in Kaon system

We have: $|K_S\rangle = \dfrac{1}{\sqrt{1+|\varepsilon|^2}} (|K_1\rangle + \varepsilon |K_2\rangle)$

$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} (|K_2\rangle + \varepsilon |K_1\rangle)$$

In Terms of flavour eigenstates:

$$|K_S\rangle = \frac{1}{\sqrt{2(1+|\varepsilon|^2}} \left[(1+\varepsilon)|K^0\rangle + (1-\varepsilon)|\bar{K}_0\rangle \right]$$

$$|K_L\rangle = \frac{1}{\sqrt{2(1+|\varepsilon|^2}} \left[(1+\varepsilon)|K^0\rangle - (1-\varepsilon)|\bar{K}_0\rangle \right]$$

In Terms of K^0 and $\overline{K^0}$:

$$|K^0\rangle = \frac{1}{1+\varepsilon}\sqrt{\frac{1+|\varepsilon|^2}{2}}\left(|K_S\rangle + |K_L\rangle\right)$$

$$|\overline{K^0}\rangle = \frac{1}{1+\varepsilon}\sqrt{\frac{1+|\varepsilon|^2}{2}}\left(|K_S\rangle - |K_L\rangle\right)$$

$$|K(t)\rangle = \frac{1}{1+\varepsilon}\sqrt{\frac{1+|\varepsilon|^2}{2}}\left(\Theta_S(t)|K_S\rangle - \Theta_L(t)|K_L\rangle\right)$$

In Terms of K_1 and K_2 :

$$|K(t)\rangle = \frac{1}{\sqrt{2}}\frac{1}{(1+\varepsilon)}\left[\Theta_S(|K_1\rangle + \varepsilon|K_2\rangle) + \Theta_L(|K_2\rangle + \varepsilon|K_1\rangle)\right]$$

$$= \frac{1}{\sqrt{2}}\frac{1}{1+\varepsilon}\left[(\Theta_S + \varepsilon\Theta_L)|K_1\rangle + (\Theta_L + \varepsilon\Theta_S)|K_2\rangle\right]$$

Decay rate To two pions:

$$\Gamma(K^0_{T=0} \to \pi\pi) \propto |\langle K_1|K(t)\rangle|^2 = \frac{1}{2}\left|\frac{1}{1+\varepsilon}\right|^2 |\Theta_S + \varepsilon\Theta_L|^2$$

$$\left|\frac{1}{1+\varepsilon}\right|^2 \approx 1 - 2\mathrm{Re}(\varepsilon) \quad \rightarrow \text{ with } \varepsilon \ll 1$$

$$|\theta_S + \varepsilon\theta_L|^2 = \left| e^{-im_S T - \Gamma_S t/2} + |\varepsilon| e^{i\varphi} e^{-im_L T - \Gamma_L t/2} \right|^2 \rightarrow \text{ with } \varepsilon = |\varepsilon| e^{i\varphi}$$

$$\rightarrow \Gamma(K^0_{T=0} \rightarrow \pi\pi) = \frac{N}{2}\left(1 - 2\mathrm{Re}(\varepsilon)\right)\left[e^{-\Gamma_S T} + |\varepsilon|^2 e^{-\Gamma_L T} + 2|\varepsilon| e^{-(\Gamma_L+\Gamma_S)t/2}\cos(\Delta m T - \varphi)\right]$$

and:

$$\Gamma(\bar{K}^0_{T=0} \rightarrow \pi\pi) = \frac{N}{2}\left(1 + 2\mathrm{Re}(\varepsilon)\right)\left[e^{-\Gamma_S T} + |\varepsilon|^2 e^{-\Gamma_L T} - 2|\varepsilon| e^{-(\Gamma_L+\Gamma_S)t/2}\cos(\Delta m T - \varphi)\right]$$

the measurement of ε was obtained from the asymmetry:

$$A_{+-} = \frac{\Gamma(\bar{K}^0_{T=0} \rightarrow \pi^+\pi^-) - \Gamma(K^0_{T=0} \rightarrow \pi^+\pi^-)}{\Gamma(\bar{K}_{0\,T=0} \rightarrow \pi^+\pi^-) + \Gamma(K_{T=0} \rightarrow \pi^+\pi^-)}$$

with ε small we ultimately get:

$$A_{+-} \simeq \frac{2\mathrm{Re}|\varepsilon| - 2|\varepsilon| e^{(\Gamma_S-\Gamma_L)T/2}\cos(\Delta m T - \varphi)}{1 + |\varepsilon|^2 e^{(\Gamma_S-\Gamma_L)T}}$$

$\longrightarrow$ extract $|\varepsilon|$ and φ

Another strategy to measure CP violation is to isolate the K_L component:

$$|K_L\rangle = \frac{1}{\sqrt{2(1+|\varepsilon|^2)}}\left[(1+\varepsilon)|K^0\rangle - (1-\varepsilon)|\bar{K}^0\rangle\right]$$

with decays: $\pi^- e^+ \nu$ and $\pi^+ e^- \bar{\nu}$

Decay rates:

$$\Gamma(\Gamma_L \to \pi^+ e^- \bar{\nu}) \propto |\langle \bar{K}_0 | K_L \rangle|^2 \propto |1-\varepsilon|^2 \approx 1 - 2\,\mathrm{Re}(\varepsilon)$$
$$\Gamma(\Gamma_L \to \pi^- e^+ \nu) \propto |\langle K_0 | K_L \rangle|^2 \propto |1+\varepsilon|^2 \approx 1 + 2\,\mathrm{Re}(\varepsilon)$$

Experimental measurement again in terms of asymmetry:

$$\delta = \frac{\Gamma(K_L \to \pi^- e^+ \nu_e) - \Gamma(K_L \to \pi^+ e^- \bar{\nu}_e)}{\Gamma(K_L \to \pi^- e^+ \nu_e) + \Gamma(K_L \to \pi^+ e^- \bar{\nu}_e)} = 2\,\mathrm{Re}(\varepsilon) = 2|\varepsilon|\cos\varphi$$

$K_L \to \pi^+ e^- \bar{\nu}_e$ 0.7% larger than $K_L \to \pi^- e^+ \nu_e$

Assignment (due: April 5th)

In the neutral Kaon system, we can express time-reversal violation in terms of the asymmetry:

$$A_T = \frac{\Gamma(\bar{K}^0 \to K^0) - \Gamma(K^0 \to \bar{K}^0)}{\Gamma(\bar{K}^0 \to K^0) + \Gamma(K^0 \to \bar{K}^0)}$$

Show that this is equivalent to:

$$A_T = \frac{\Gamma(\bar{K}^0_{T=0} \to \pi^- e^+ \nu_e) - \Gamma(K^0_{T=0} \to \pi^+ e^- \bar{\nu}_e)}{\Gamma(\bar{K}^0_{T=0} \to \pi^- e^+ \nu_e) + \Gamma(K^0_{T=0} \to \pi^+ e^- \bar{\nu}_e)}$$

and therefore:

$$\boxed{A_T \approx 4|\varepsilon| \cos \varphi}$$