Lecture 23: CP Violation (Part II)

- Direct CP violation
- B meson system

(I used mostly Thomson, Burgess, Cheng Li, Halzen)

CP violation in Kaon decays

- CP violation can occur through mixing:

$$|K_-\rangle = \frac{1}{\sqrt{1+|\tilde{\varepsilon}|^2}} \left[|\varphi_-\rangle + i\tilde{\varepsilon}\,|\varphi_+\rangle \right]$$

With $\mathcal{L}_W$ denoting weak interaction Lagrangian ($\Delta S = \pm 1$) we write $\langle \pi\pi | \mathcal{L}_W | K_+ \rangle$

- CP violation can also occur at the <u>decay</u>:

$\langle \pi\pi | \mathcal{L}_W | K_- \rangle$ i.e. $\mathcal{L}_W$ itself breaks CP (Known as "direct" CP violation.

To determine relative size of these contributions we can use the observables:

$$\eta_{00} = \frac{\langle \pi^0\pi^0 | \mathcal{L}_W | K_L \rangle}{\langle \pi^0\pi^0 | \mathcal{L}_W | K_S \rangle} \qquad \eta_{+-} = \frac{\langle \pi^+\pi^- | \mathcal{L}_W | K_L \rangle}{\langle \pi^+\pi^- | \mathcal{L}_W | K_S \rangle}$$

CP violation in Kaon decays

If mixing is sole source of CP violation: $\eta_{00} = \eta_{+-}$

Note that the Kaons decaying to two pions have either isospin 0 or 2. Amplitudes are:

$$\langle \pi^+ \pi^- | \mathcal{L}_w | K^0 \rangle = A_0 e^{i\delta_0} + A_2 e^{i\delta_2}$$

$$\langle \pi^+ \pi^- | \mathcal{L}_w | \bar{K}_0 \rangle = A_0 e^{-i\delta_0} + A_2 e^{-i\delta_2}$$

A_0, A_2 are CP-conserving strong interaction matrix elements for pion isospin channel, δ_0, δ_2 are the CP-violating phases due to $\mathcal{L}_w$ (assuming CP-violating $\mathcal{L}_w$)

$\rightarrow$ Physical decay rates prop. to $|A_0 e^{i\delta_0} + A_2 e^{i\delta_2}|^2$

$\Rightarrow$ relative phase is relevant

CP violation in Kaon decays (cont.)

with $\varepsilon = -\hat{\varepsilon} + \xi_0$

$$\varepsilon' = \left(\frac{A_2}{A_0 + A_2}\right)(\xi_2 - \xi_0)$$

we get : $\eta_{+-} = \varepsilon + \varepsilon'$, $\eta_{00} = \varepsilon - 2\varepsilon'$

Experiments (KTEV, NA48) have confirmed that

$\frac{\varepsilon'}{\varepsilon} \neq 0$, $\mathrm{Re}\left|\frac{\varepsilon'}{\varepsilon}\right| = 1.7 \times 10^{-3}$

$\longrightarrow$ ε' small relative to ε

Measurements with B mesons also have observed direct CP violation

$B-\bar{B}$ Mixing

Similar To $K-\bar{K}$ mixing buT b quarK mass $\gg$ s quarK mass and well above QCD scale:

- theoreTical uncertainTies are reduced.
- much larger phase space eliminates (essentially) The lifeTime difference. Simplifies expressions ...
- B decays are more CKM suppressed

With $\Gamma_- \simeq \Gamma_+$, we'll denote the Two states by "H" for heavy, and "L" for lighT. Previous oscillaTion probabiliTies become (non-relaTivisTic B's):

$$P_T[B^0 \to B^0] = e^{-\Gamma T} \cos^2\left(\frac{\Delta m T}{2}\right)$$

$$P_T[B^0 \to \bar{B}^0] = \left|\frac{q}{p}\right|^2 e^{-\Gamma T} \sin^2\left(\frac{\Delta m t}{2}\right)$$

$B-\bar{B}$ Mixing (cont.)

In our example (non-rel, e.g. CLEO), it is difficult to measure time t elapsed since the B was in a pure B^0 eigenstate. We start from:

$$e^+e^- \to \gamma^* \to B\bar{B}$$

→ relative angular momentum $\ell = 1$

$$CP|B\bar{B}\rangle = -|B\bar{B}\rangle$$

initial state $$|B\bar{B}\rangle = \frac{1}{\sqrt{2}}\left[|B(k)\,\bar{B}(-k)\rangle - |B(-k)\,\bar{B}(k)\rangle\right]$$

We can then "Tag" the flavour of the B using semi-leptonic decay of one B (there are other ways to Tag).

By reconstructing decay vertex, we can determine prob. of observing (for instance) same-sign leptons vs distance i.e. versus Time

→ asymmetric B factories make this easier

$B-\bar{B}$ Mixing (in more detail)

$$|\langle f|H|B^0\rangle|^2 = |\langle f|H|B^0(t)\rangle|^2$$

$$= \frac{1}{4|p|^2}\,|\langle f|B_L(t)\rangle + \langle f|B_H(t)\rangle|^2$$

$$= \frac{1}{4|p|^2}\,|pAe^{(-im_L-\Gamma_L/2)T} + pAe^{(-im_H-\Gamma_H/2)T}|^2$$

$$= \frac{1}{4}|A|^2\left(e^{-\Gamma_L T} + e^{-\Gamma_H T} + 2e^{-(\Gamma_H+\Gamma_L)T/2}\cos\Delta m t\right)$$

$$|\langle \bar{f}|H|B^0\rangle|^2 = |\langle \bar{f}|B^0(t)\rangle|^2$$

$$= \frac{1}{4|p|^2}\,|\langle \bar{f}|B_L(t)\rangle + \langle \bar{f}|B_H(t)\rangle|^2$$

$$= \frac{1}{4|p|^2}\,|q\bar{A}e^{(-im_L-\Gamma_L/2)t} - q\bar{A}e^{(-im_H-\Gamma_H/2)t}|^2$$

$$= \frac{1}{4}\left|\frac{p}{q}\right|^2|\bar{A}|^2\left(e^{-\Gamma_L T} + e^{-\Gamma_H T} - 2e^{-(\Gamma_H+\Gamma_L)t/2}\cos\Delta m T\right)$$

$B-\bar{B}$ Mixing (in more detail, cont.)

Remember we have:

$$|B_H\rangle = p|B^0\rangle + q|\bar{B}^0\rangle$$

$$|B_L\rangle = p|B^0\rangle - q|\bar{B}^0\rangle$$

$\rightarrow \langle F|B^0\rangle = A \quad , \quad \langle \bar{F}|B^0\rangle = 0$

$\langle \bar{F}|\bar{B}^0\rangle = \bar{A} \quad , \quad \langle F|\bar{B}^0\rangle = 0$

$B_{lt} \rightarrow F$ provides $p\,A$

$B_L \rightarrow \bar{F}$ provides $-q\,\bar{A}$

$\rightarrow$ if $A \neq \bar{A}$, CP violation in decay

$\rightarrow$ if $\left|\frac{q}{p}\right| \neq 1$, CP violation in mixing

Without CP violation we would have

$$|\langle F|H|B^0\rangle|^2 + |\langle \bar{F}|H|B^0\rangle|^2 = \frac{1}{2}|A|^2\left(e^{-\Gamma_L t} + e^{-\Gamma_H t}\right)$$

CKM Matrix

In SM, flavour changing and CP-violating physics is due to 4 parameters in unitary CKM matrix.

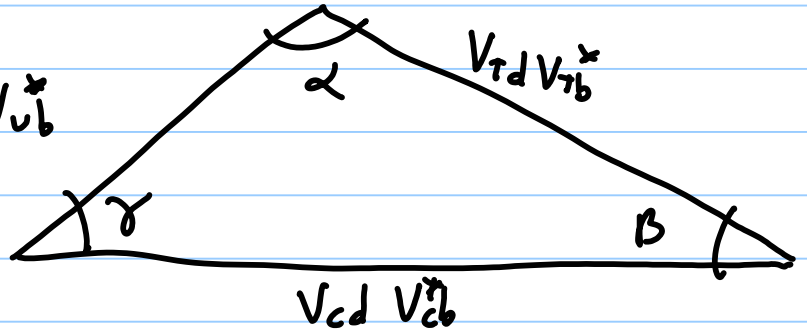
Physics beyond SM could provide new contributions.
B physics provides many opportunities of Testing SM.

Unitarity conditions $\rightarrow \sum_i V_{in} V_{im}^* = \delta_{mn}$

With Bd mesons, we have $m = b$, $n = d$, which implies:

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{Td} V_{Tb}^* = 0$$

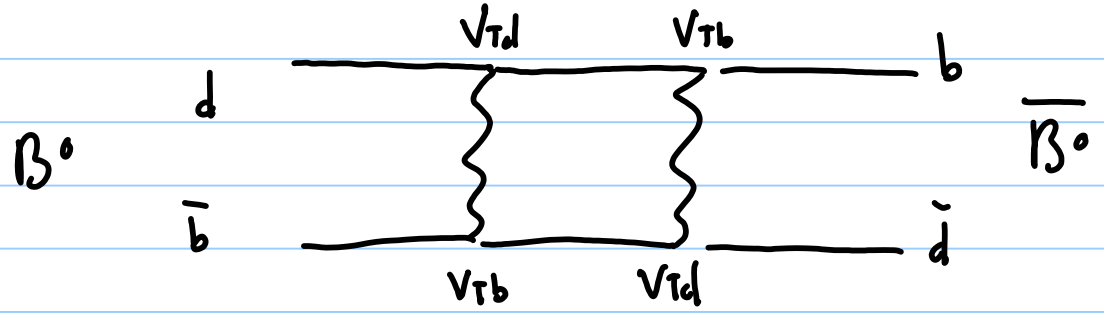
→ sum of 3 complex numbers vanish.
→ can be expressed 3 vectors in complex plane



$$\alpha = \arg\left(-\frac{V_{Td} V_{Tb}^*}{V_{ud} V_{ub}^*}\right)$$

$$\beta = \arg\left(\frac{V_{cd} V_{cb}^*}{V_{Td} V_{Tb}^*}\right)$$

CKM Matrix and B physics



$$\beta = \arg\left(\frac{V_{cd}\,V_{cb}^*}{V_{Td}\,V_{tb}^*}\right)$$

$B^0 \to 2K_S$: $V_{Td}V_{Tb}$ From mixing

V_{cb} From $b \to c$

V_{cd} From K^0 mixing

In asymmetric B Factories:

- reconstruct one B in CP eigenstate e.g. $2K_S$
- reconstruct decay of other B → determine flavour
- Measure distance between mesons and convert To proper Time

$$\varepsilon = \left(\frac{M_{12}^* - i/2\,\Gamma_{12}^*}{M_{12} - i/2\,\Gamma_{12}} \right)^{1/2} \simeq \frac{M_{12}^*}{|M_{12}|}$$

$$\rightarrow \quad M_{12}^* \propto (V_{Td} V_{Tb}^*)^2 \quad , \quad \varepsilon = \frac{M_{12}^*}{|M_{12}|} = \frac{(V_{Td} V_{Tb}^*)^2}{|(V_{Td} V_{Tb}^*)^2|} = \frac{V_{Td}^2}{|V_{Td}^2|}$$

$$\boxed{V_{Td} = |V_{Td}|\, e^{-i\beta}}$$

$$|B_L| = \frac{1}{\sqrt{2}} \left(|B^0\rangle + e^{-i2\beta} |\bar{B}^0\rangle \right) \quad , \quad |B_H\rangle = \ldots - \ldots$$

$$\Delta m_d = 2|M_{12}| \propto |(V_{Td} V_{Tb}^*)^2| \quad \rightarrow \quad \boxed{\Delta m_d \propto |V_{Td}|^2}$$

Now: $P(\bar{B}^0_{T=0} \rightarrow \bar{B}^0) = e^{-\Gamma T} \cos^2\left(\frac{\Delta m_d T}{2}\right)$

$$P(\bar{B}^0_{T=0} \rightarrow B^0) = \left|\frac{1}{\varepsilon}\right|^2 e^{-\Gamma T} \sin^2\left(\frac{\Delta m_d T}{2}\right)$$

For this system, very few decays are common, interference between decays is small

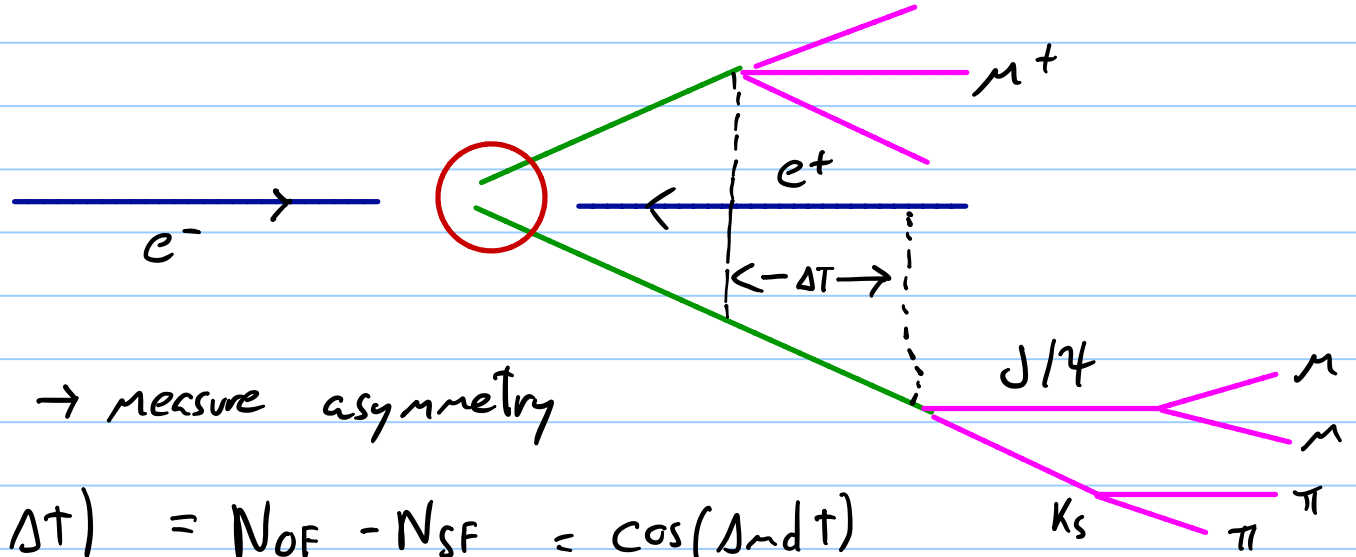
→ Hard To observe CP violation in mixing

What To do?

CP violation can be observed:

- directly : $\Gamma(A \to X) \neq \Gamma(\bar{A} \to \bar{X})$
- through mixing, as we saw for Kaon system
- CP violation in interference between decays

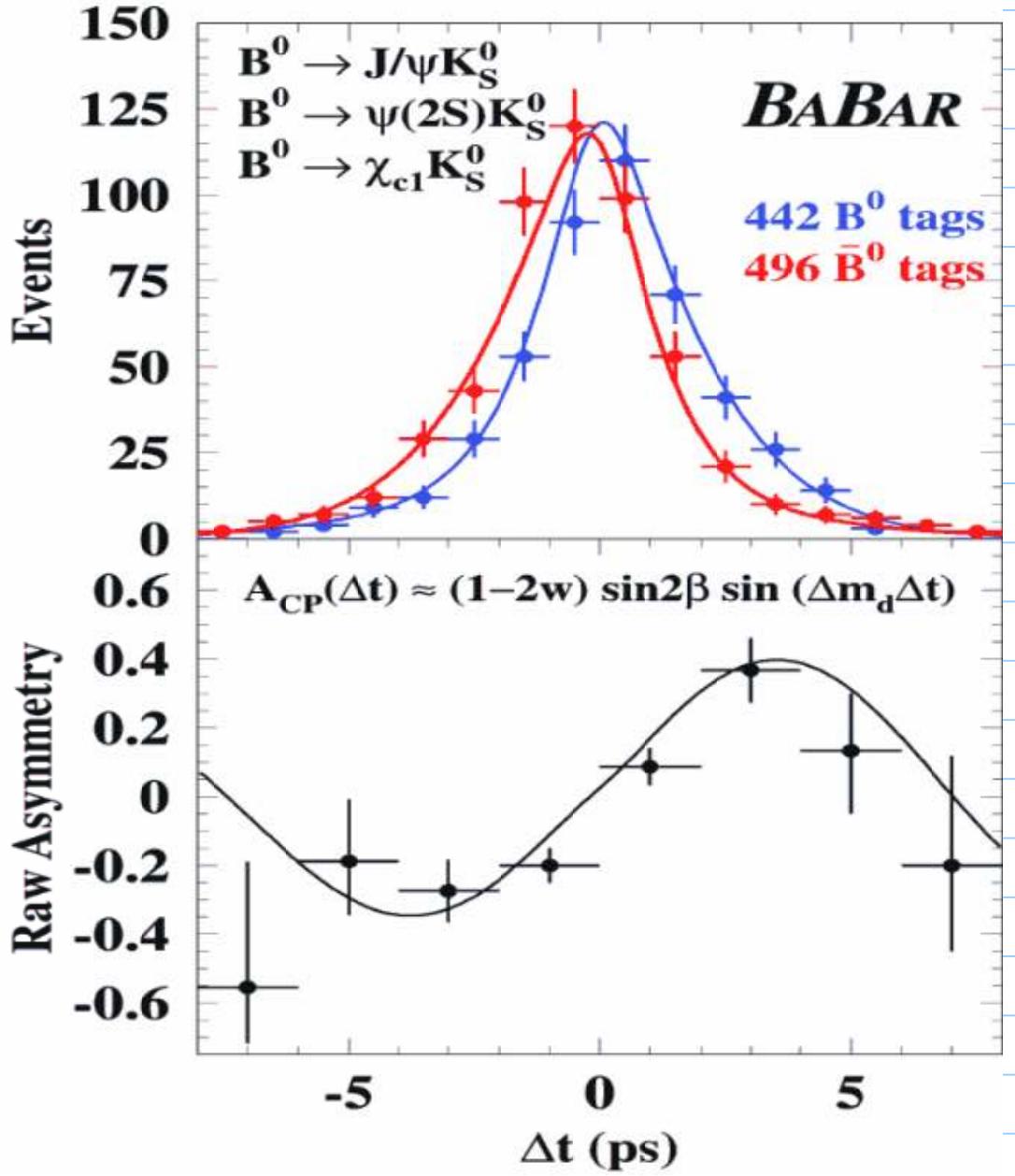
$$B^0 \to f \quad , \quad B^0 \to \bar{B}^0 \to f$$



$\Delta m_d \to$ measure asymmetry

$$A(\Delta t) = \frac{N_{OF} - N_{SF}}{N_{OF} + N_{SF}} = \cos(\Delta m_d t)$$

$$A_{CP}^{K_S} = \frac{\Gamma(\bar{B}^0_{t=0} \to J/\psi K_S) - \Gamma(B^0_{t=0} \to J/\psi K_S)}{\text{''} \qquad + \qquad \text{''}} = \sin 2\beta \sin \Delta m_d t$$


$B^0 \to J/\psi K^0_S$
$B^0 \to \psi(2S) K^0_S$
$B^0 \to \chi_{c1} K^0_S$
BABAR
442 B^0 tags
496 $\bar{B}^0$ tags
Events
150
125
100
75
50
25
0
$A_{CP}(\Delta t) \approx (1-2w) \sin 2\beta \sin(\Delta m_d \Delta t)$
Raw Asymmetry
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-5
0
5
Δt (ps)

Other sources of CP violation

CP violation due to CKM matrix is too small to explain observed matter anti-matter asymmetry in the Universe.

Are there other potential sources of CP violation in SM?

Yes:
- lepton sector (next lecture)
- Strong CP violation (very small if it exists)

Physics beyond SM could also contribute

STrong CP Problem

Most general renorm. Lagrangian involving SM Fields includes:

$$\mathcal{L}_{FS} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$- \frac{g_3^2 \Theta_3}{64\pi^2} \varepsilon_{\mu\nu\lambda\rho} G^{a\mu\nu} G^{a\lambda\rho} - \text{similar Terms for } W, B$$

→ This Term is effectively a Total derivative and would be expected To have no physical implications

$$\propto \partial^\mu K_\mu$$

Term determined by change in charge $\int d^3x K^0 \equiv N_{CS}$

Change in charge need not be zero in a vacuum To vacuum process → QCD vacuum Topologically non-trivial

Axions are a potential solution (need To add scalar field)