

LECTURE 24: Neutrinos

Overview:

- 1- 2-Family Oscillations
- 2- 3-Family Oscillations
- 3- Experimental Results

→ Main text is Thomson

I used Burgess, and Akhmedov (mainly), C. Giunti as references

↳ MSW Effect

Neutrino Mixing

②

IF NEUTRINOS HAVE MASS, WE CAN HAVE WEAK EIGENSTATES THAT ARE DIFFERENT THAN MASS EIGENSTATES?

WEAK EIGENSTATES

$$\begin{pmatrix} e^- \\ \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

MASS EIGENSTATES

$$\begin{pmatrix} \bar{\nu}_1 \\ \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Mixing "PMNS" Matrix

PONTECORVO, MAKI
NAKAGAWA, SAKATA

$$(\nu_e, \nu_\mu, \nu_\tau) = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

WE'LL START WITH 2-FAMILY MIXING FIRST

2 - Family Mixing

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$$\begin{pmatrix} v_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

m_1, m_2 will be masses of two steps

θ mixing angle

- Note that different masses for ν_1 and ν_2 imply different velocities

$$\begin{aligned} v_e &= \cos\theta \nu_1 + \sin\theta \nu_2 \\ \nu_\mu &= -\sin\theta \nu_1 + \cos\theta \nu_2 \end{aligned}$$

For state of mass, energy, momentum given by m_i, E_i, p_i :

$$\begin{aligned} \nu_i(t, x) &= \nu_i(0, 0) e^{iQ_i(t, x)} \\ \rightarrow Q_i &= E_i t - p_i x, \quad i = 1, 2 \end{aligned}$$

with initial state given by: $t = x = 0$ $\nu_e(0) = 1, \nu_\mu(0) = 0$

$$\nu_1(0) = \nu_e(0) \cos\theta$$

$$\nu_2(0) = \nu_e(0) \sin\theta$$

2-Family Mixing (cont.)

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As a function of T and x :

$$v_e(T, x) = \cos \theta v_1(T, x) + \sin \theta v_2(T, x)$$

$$\begin{aligned} P_{e \rightarrow e} &= \left| \frac{v_e(T, x)}{v_e(0, 0)} \right|^2 = \left| \cos^2 \theta e^{i\varphi_1(T, x)} + \sin^2 \theta e^{i\varphi_2(T, x)} \right|^2 \\ &= 1 - \sin^2 2\theta \sin^2 \left(\frac{\varphi_1 - \varphi_2}{2} \right) \end{aligned}$$

$$\varphi_1 - \varphi_2 = (E_1 - E_2)T - (p_1 - p_2)x$$

with $T = x \left(\frac{E_1 + E_2}{p_1 + p_2} \right)$ we set:

$$\varphi_1 - \varphi_2 = (E_1^2 - E_2^2) - (p_1^2 - p_2^2) \cdot x = \frac{M_1^2 - M_2^2}{p_1 + p_2} x$$

$$= \frac{\Delta M^2}{2p} x \approx \frac{\Delta M^2}{2E} x$$

2-Family Mixing (cont.)

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$$P(\nu_e \rightarrow \nu_e) = 1 - \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 x}{4E\nu} \right)$$

$$P(\nu_e \rightarrow \nu_\mu) = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 x}{4E\nu} \right)$$

Max mixing at $\theta = \frac{\pi}{4}$

and $L = \frac{L_{osc}}{2}$

$$\left(\frac{1.27 \Delta m^2 L}{E\nu} \right)$$

L in km
 $E\nu$ in GeV
 m in eV

3-Family Mixing

With 3 generations we will have 3 Δm^2 values but 2 are independent

$$\Delta m_{12}^2 = m_1^2 - m_2^2, \quad \Delta m_{23}^2 = m_2^2 - m_3^2, \quad \Delta m_{31}^2 = m_3^2 - m_1^2$$

PMNS will have 4 indep. parameters, like CKM

Use $C_{ij} = \cos \theta_{ij}$, $S_{ij} = \sin \theta_{ij}$: $\theta_{12}, \theta_{23}, \theta_{13}$, $\underline{\varphi}$

★ Hence the two-flavour oscillation probability is:

$$P(\nu_e \rightarrow \nu_\mu) = \sin^2 2\theta \sin^2 \left(\frac{\Delta m_{21}^2 L}{4E} \right)$$

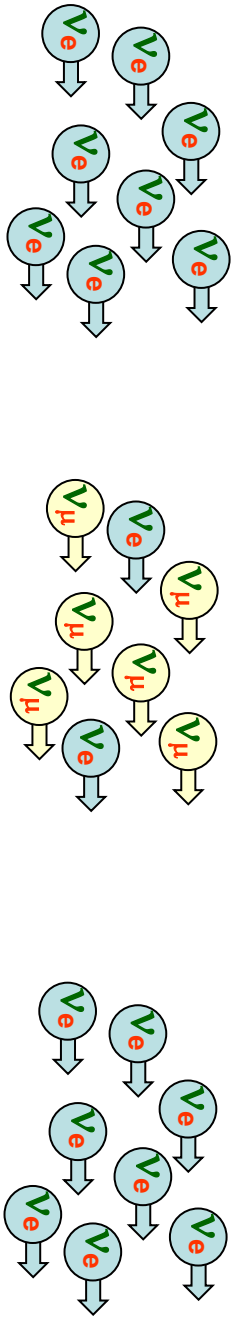
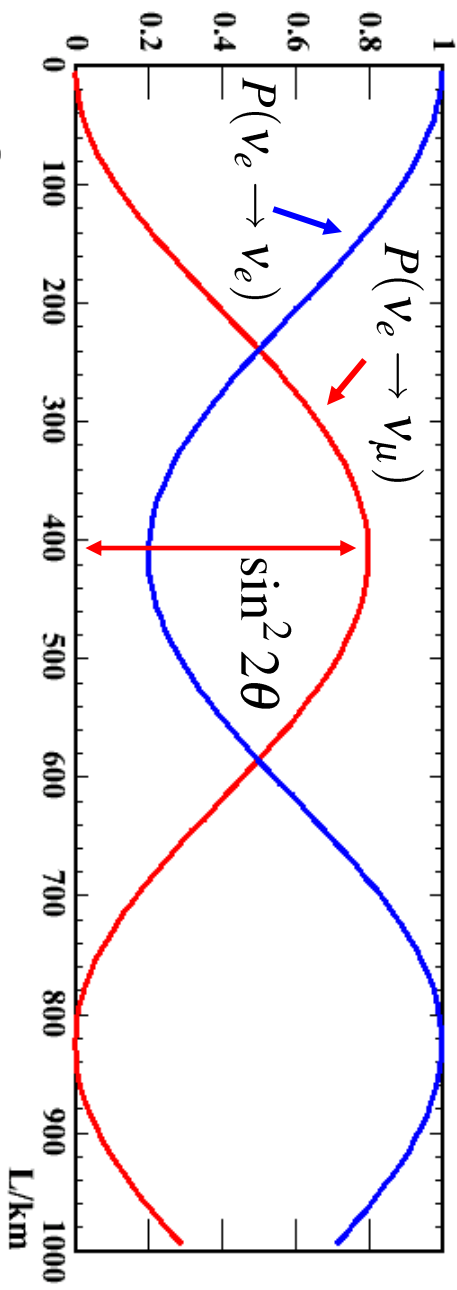
with

$$\Delta m_{21}^2 = m_2^2 - m_1^2$$

★ The corresponding two-flavour survival probability is:

$$P(\nu_e \rightarrow \nu_e) = 1 - \sin^2 2\theta \sin^2 \left(\frac{\Delta m_{21}^2 L}{4E} \right)$$

•e.g. $\Delta m^2 = 0.003 \text{ eV}^2$, $\sin^2 2\theta = 0.8$, $E_\nu = 1 \text{ GeV}$



•wavelength

$$\lambda_{\text{osc}} = \frac{4\pi E}{\Delta m^2}$$

3-Family Mixing (cont.)

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We will need a 3×3 matrix, the PMNS matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$\text{Unitary} \rightarrow \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} U_{e1}^* & U_{\mu 1}^* & U_{\tau 1}^* \\ U_{e2}^* & U_{\mu 2}^* & U_{\tau 2}^* \\ U_{e3}^* & U_{\mu 3}^* & U_{\tau 3}^* \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

At Time = 0, let's say we start with ν_e

$$|\psi(0)\rangle = |\nu_e\rangle \equiv U_{e1}^* |\nu_1\rangle + U_{e2}^* |\nu_2\rangle + U_{e3}^* |\nu_3\rangle$$

Time evolution:

$$\begin{aligned} |\psi(x,t)\rangle &= U_{e1}^* |\nu_1\rangle e^{-iQ_1} + U_{e2}^* |\nu_2\rangle e^{-iQ_2} + U_{e3}^* |\nu_3\rangle e^{-iQ_3} \\ &= U_{e1}^* (U_{e1} |\nu_e\rangle + U_{\mu 1} |\nu_\mu\rangle + U_{\tau 1} |\nu_\tau\rangle) e^{-iQ_1} \\ &\quad + U_{e2}^* (U_{e2} |\nu_e\rangle + U_{\mu 2} |\nu_\mu\rangle + U_{\tau 2} |\nu_\tau\rangle) e^{-iQ_2} \\ &\quad + U_{e3}^* (U_{e3} |\nu_e\rangle + U_{\mu 3} |\nu_\mu\rangle + U_{\tau 3} |\nu_\tau\rangle) e^{-iQ_3} \end{aligned}$$

Grouping Terms for all eigensates:

$$| \nu(x, t) \rangle = \begin{pmatrix} U_{e1}^* U_{e1} e^{-iQ_1} + U_{e2}^* U_{e2} e^{-iQ_2} + U_{e3}^* U_{e3} e^{-iQ_3} \\ U_{e1}^* U_{\nu 1} e^{-iQ_1} + U_{e2}^* U_{\nu 2} e^{-iQ_2} + U_{e3}^* U_{\nu 3} e^{-iQ_3} \\ U_{e1}^* U_{\nu 1} e^{-iQ_1} + U_{e2}^* U_{\nu 2} e^{-iQ_2} + U_{e3}^* U_{\nu 3} e^{-iQ_3} \end{pmatrix} | \nu \rangle$$

Oscillation probability of $\nu_e \rightarrow \nu_\mu$: $P(\nu_e \rightarrow \nu_\mu) = | \langle \nu_\mu | \nu(x, t) \rangle |^2$

$$= | U_{e1}^* U_{\nu 1} e^{-iQ_1} + U_{e2}^* U_{\nu 2} e^{-iQ_2} + U_{e3}^* U_{\nu 3} e^{-iQ_3} |^2$$

note: if phases the same, $P(\nu_e \rightarrow \nu_\mu) = 0$ because of
 Unitarity of PMNS matrix

$$P(\nu_e \rightarrow \nu_\mu) = | U_{e1}^* U_{\nu 1} |^2 + | U_{e2}^* U_{\nu 2} |^2 + | U_{e3}^* U_{\nu 3} |^2 + \\
 2 \operatorname{Re} (U_{e1}^* U_{\nu 1} U_{e2} U_{\nu 2}^* e^{-i(Q_1 - Q_2)}) + \\
 2 \operatorname{Re} (U_{e1}^* U_{\nu 1} U_{e3} U_{\nu 3}^* e^{-i(Q_1 - Q_3)}) + \\
 2 \operatorname{Re} (U_{e2}^* U_{\nu 2} U_{e3} U_{\nu 3}^* e^{-i(Q_2 - Q_3)})$$

We use: $| U_{e1}^* U_{\nu 1} |^2 + | U_{e2}^* U_{\nu 2} |^2 + | U_{e3}^* U_{\nu 3} |^2 + \\
 2 \operatorname{Re} (U_{e1}^* U_{\nu 1} U_{e2} U_{\nu 2}^* + U_{e1}^* U_{\nu 1} U_{e3} U_{\nu 3}^* + U_{e2}^* U_{\nu 2} U_{e3} U_{\nu 3}^*) = 0$

And obtain:

$$\begin{aligned}
 P(\nu_e \rightarrow \nu_\mu) &= 2 \operatorname{Re} \left(U_{e1}^x U_{\mu 1} U_{e2} U_{\mu 2}^* \left[e^{i(\theta_2 - \theta_1)} - 1 \right] \right) \\
 &+ 2 \operatorname{Re} \left(U_{e1}^x U_{\mu 1} U_{e3} U_{\mu 3}^* \left[e^{i(\theta_3 - \theta_1)} - 1 \right] \right) \\
 &+ 2 \operatorname{Re} \left(U_{e2}^x U_{\mu 2} U_{e3} U_{\mu 3}^* \left[e^{i(\theta_3 - \theta_2)} - 1 \right] \right)
 \end{aligned}$$

$$\begin{aligned}
 \text{For } P(\nu_e \rightarrow \nu_e) &= 1 + 2 |U_{e1}|^2 |U_{e2}|^2 \operatorname{Re} \left(\left[e^{i(\theta_2 - \theta_1)} - 1 \right] \right) \\
 &+ 2 |U_{e1}|^2 |U_{e3}|^2 \operatorname{Re} \left(\left[e^{i(\theta_3 - \theta_1)} - 1 \right] \right) \\
 &+ 2 |U_{e2}|^2 |U_{e3}|^2 \operatorname{Re} \left(\left[e^{i(\theta_3 - \theta_2)} - 1 \right] \right)
 \end{aligned}$$

$$\text{we have } \operatorname{Re} \left(e^{i(\theta_j - \theta_i)} - 1 \right) = \cos(\theta_j - \theta_i) - 1 = -2 \sin^2 \left(\frac{\theta_j - \theta_i}{2} \right) = -2 \sin^2 \Delta_{ji}$$

$$\Delta_{ji} = \frac{\theta_j - \theta_i}{2} = \frac{(m_j^2 - m_i^2) L}{4E\nu}$$

$$\begin{aligned}
 \rightarrow P(\nu_e \rightarrow \nu_e) &= 1 - 4 |U_{e1}|^2 |U_{e2}|^2 \sin^2 \Delta_{21} - 4 |U_{e1}|^2 |U_{e3}|^2 \sin^2 \Delta_{31} \\
 &- 4 |U_{e2}|^2 |U_{e3}|^2 \sin^2 \Delta_{32}
 \end{aligned}$$

Note That $\Delta_{31} = \Delta_{32} + \Delta_{21} \rightarrow$ 2 independent mass² differences

Experimentally, we know that $(m_2^2 - m_1^2) \ll (m_3^2 - m_1^2) \approx (m_3^2 - m_2^2)$

CP and T Violation

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$$\begin{aligned} \text{We had: } P(\nu_e \rightarrow \nu_\mu) &= 2 \operatorname{Re} \left(U_{e1}^* U_{\mu 1} U_{e2} U_{\mu 2}^* \left[e^{i(\theta_2 - \theta_1)} - 1 \right] \right) \\ &2 \operatorname{Re} \left(U_{e1}^* U_{\mu 1} U_{e3} U_{\mu 3}^* \left[e^{i(\theta_3 - \theta_1)} - 1 \right] \right) \\ &2 \operatorname{Re} \left(U_{e2}^* U_{\mu 2} U_{e3} U_{\mu 3}^* \left[e^{i(\theta_3 - \theta_2)} - 1 \right] \right) \end{aligned}$$

For $P(\nu_\mu \rightarrow \nu_e)$ we would change e and μ labels

T reversal symmetry implies $P(\nu_e \rightarrow \nu_\mu) = P(\nu_\mu \rightarrow \nu_e)$

this is true if matrix elements are real.

$$\text{For T reversal: } \nu_e \rightarrow \nu_\mu \rightarrow \nu_\mu \rightarrow \nu_e$$

$$\text{For CP Trans.: } \nu_e \rightarrow \nu_\mu \rightarrow \bar{\nu}_e \rightarrow \bar{\nu}_\mu$$

$$\text{For CPT Trans.: } \nu_e \rightarrow \nu_\mu \rightarrow \bar{\nu}_\mu \rightarrow \bar{\nu}_e$$

So if PMNS is not all real, then

$$\begin{aligned} P(\nu_e \rightarrow \nu_\mu) &\neq P(\nu_\mu \rightarrow \nu_e) \\ P(\nu_e \rightarrow \nu_\mu) &\neq P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu) \end{aligned}$$

(10)

Neglecting CP violation for now, we can simplify some more:

$$P(\nu_e \rightarrow \nu_\mu) = -4 U_{e1} U_{\mu 1} U_{e2} U_{\mu 2} \sin^2 \Delta_{21} - 4 U_{\mu 1} U_{e3} U_{\mu 3} \sin^2 \Delta_{31} - 4 U_{e2} U_{\mu 2} U_{e3} U_{\mu 3} \sin^2 \Delta_{32}$$

$$\text{with } \Delta_{ji} = \frac{(m_j^2 - m_i^2)L}{4E} = \frac{\Delta m_{ji}^2 L}{4E}$$

$$\text{with } \Delta_{31} \approx \Delta_{32}$$

$$P(\nu_e \rightarrow \nu_\mu) \approx -4 U_{e1} U_{\mu 1} U_{e2} U_{\mu 2} \sin^2 \Delta_{21} + 4 U_{e3}^2 U_{\mu 3}^2 \sin^2 \Delta_{32}$$

Can also simplify:

$$P(\nu_e \rightarrow \nu_e) \approx 1 - 4 U_{e1}^2 U_{e3}^2 \sin^2 \Delta_{21} - 4(1 - U_{e3}^2) U_{e3}^2 \sin^2 \Delta_{32}$$

Parametrization of PMNS Matrix

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Use θ_{12} , θ_{23} , θ_{13} and a complex phase δ . Use $\epsilon_{ij} = \sin \theta_{ij}$
 $c_{ij} = \cos \theta_{ij}$

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{atmospheric}} \times \begin{pmatrix} c_{13} & 0 & s_{13} e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{-i\delta} & 0 & c_{13} \end{pmatrix} \times \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{solar}}$$

Measurements: Δm_{21}^2 , Δm_{32}^2 \rightarrow absolute value of masses
 θ_{12} , θ_{13} , θ_{23} δ

Experiments:

- sources of neutrinos: $(\bar{\nu}_e)$
 - nuclear reactors
 - cosmic rays hitting atmosphere \rightarrow produces shower of neutrinos
 - Accelerator beams
 - the sun

- Detection: charged and neutral weak current on atomic electrons and nucleus

Reactor Experiments

(13)

Reactors produce large flux of $\bar{\nu}_e$ and this flux is proportional to reactor power output which is well known

Using $\Delta_{31} \approx \Delta_{32}$ we have

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \approx 1 - 4|U_{e1}|^2|U_{e2}|^2 \sin^2 \Delta_{21} - 4|U_{e3}|^2 \left[|U_{e1}|^2 + |U_{e2}|^2 \right] \sin^2 \Delta_{32}$$

Use unitarity relations (see Appendix A) and get:

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \approx 1 - 4|U_{e1}|^2|U_{e2}|^2 \sin^2 \Delta_{21} - 4|U_{e3}|^2 \left[1 - |U_{e3}|^2 \right] \sin^2 \Delta_{32}$$

in terms of PMNS parametrization of previous slide:

$$\begin{aligned} P(\bar{\nu}_e \rightarrow \bar{\nu}_e) &\approx 1 - 4(c_{12}c_{13})^2 (s_{12}c_{13})^2 \sin^2 \Delta_{21} - 4s_{13}^2 (1-s_{13}^2) \sin^2 \Delta_{32} \\ &= 1 - \cos^4(\theta_{13}) \sin^2(2\theta_{12}) \sin^2 \left(\frac{\Delta_{M_{21}}^2 L}{4E\nu} \right) \\ &\quad - \sin^2(2\theta_{13}) \sin^2 \left(\frac{\Delta_{M_{32}}^2 L}{4E\nu} \right) \end{aligned}$$

$$\theta_{12} \sim 30^\circ, \quad \theta_{23} \sim 45^\circ, \quad \theta_{13} \sim 10^\circ, \quad \Delta_{M_{21}}^2 \sim 8 \times 10^{-5} \text{ eV}^2, \quad \Delta_{M_{32}}^2 \sim 3 \times 10^{-3} \text{ eV}^2$$

Daya Bay \rightarrow close range ($\sim 0.5 \text{ km}$), Kamland \rightarrow longer range ($\sim 100 \text{ km}$)

Long baseline experiments

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Examples: MINOS, T2K, and future HyperK, DUNE

For MINOS, we have a pure μ neutrino beam and we measure the disappearance of ν_μ . Note that we anticipate that $\nu_\mu \rightarrow \nu_\tau$ will dominate but we are below threshold to produce τ leptons.

Again with $\Delta_{31} \approx \Delta_{32}$

$$P(\nu_\mu \rightarrow \nu_\mu) \approx 1 - 4|U_{\mu 1}|^2|U_{\mu 3}|^2 \sin^2 \Delta_{21} - 4|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2) \sin^2 \Delta_{32}$$

\rightarrow with $L \sim 700 \text{ Km}$ and $E_\nu > 1 \text{ GeV}$, first term Δ_{21} is very small

$$\rightarrow P(\nu_\mu \rightarrow \nu_\mu) \approx 1 - 4|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2) \sin^2 \Delta_{32}$$

$$\begin{aligned} \text{giving: } & 1 - 4 \sin^2(\theta_{23}) \cos^2(\theta_{13}) [1 - \sin^2(\theta_{23}) \cos^2(\theta_{13})] \sin^2 \Delta_{32} \\ & = 1 - [\sin^2(2\theta_{23}) \cos^4(\theta_{13}) + \sin^2(2\theta_{13}) \sin^2(\theta_{23})] \sin^2 \Delta_{32} \end{aligned}$$

Fixed L , measure oscillation probability vs E_ν (spectrum distortion)

Solar neutrinos

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We get $\frac{\Delta m_{12}^2 L}{4E} \approx \frac{\Delta m_{32}^2 L}{4E} \gg 1$

→ oscillate quickly relative Δm_{12}^2 term which leads to an average value for both terms

this leads to $P(\nu_e \rightarrow \nu_e) \approx 1 - \sin^2 2\theta_{12} \sin^2 \left(\frac{\Delta m_{12}^2 L}{4E} \right)$

Neutrino masses

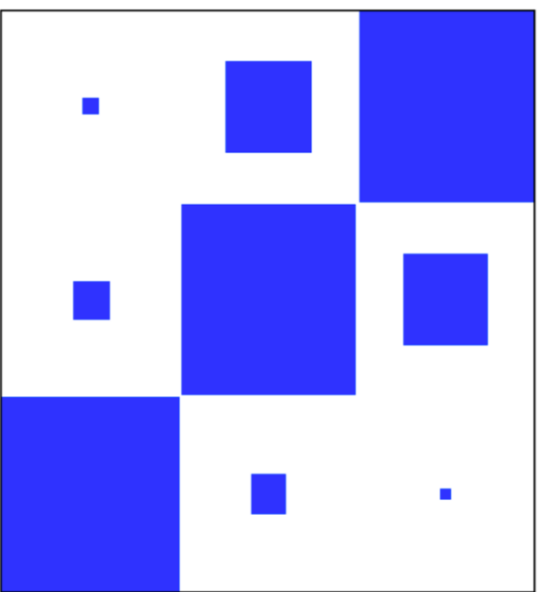
→ Dirac and Majorana masses covered in future presentation

→ current limits from cosmology : $\sum m_\nu < 0.17 \text{ eV}$ @ 95% CL
from tritium decay : $m_\nu < 1.1 \text{ eV}$ @ 90% CL

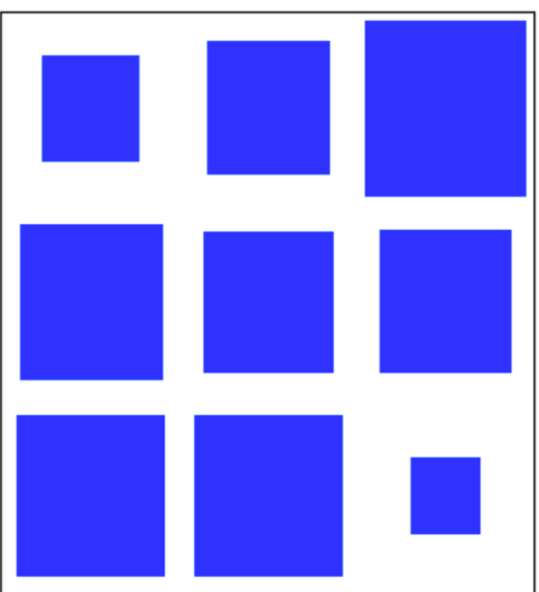
↳ KATRIN from 2019

Neutrino Mixing

Quark mixing



Neutrino mixing



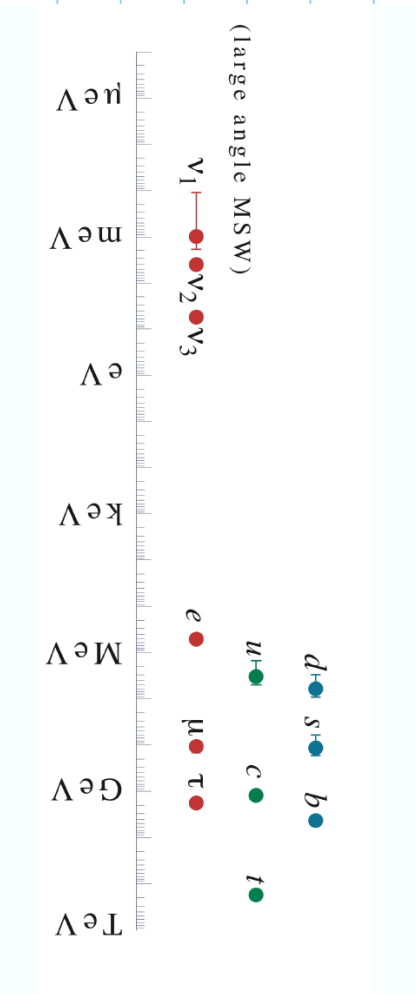
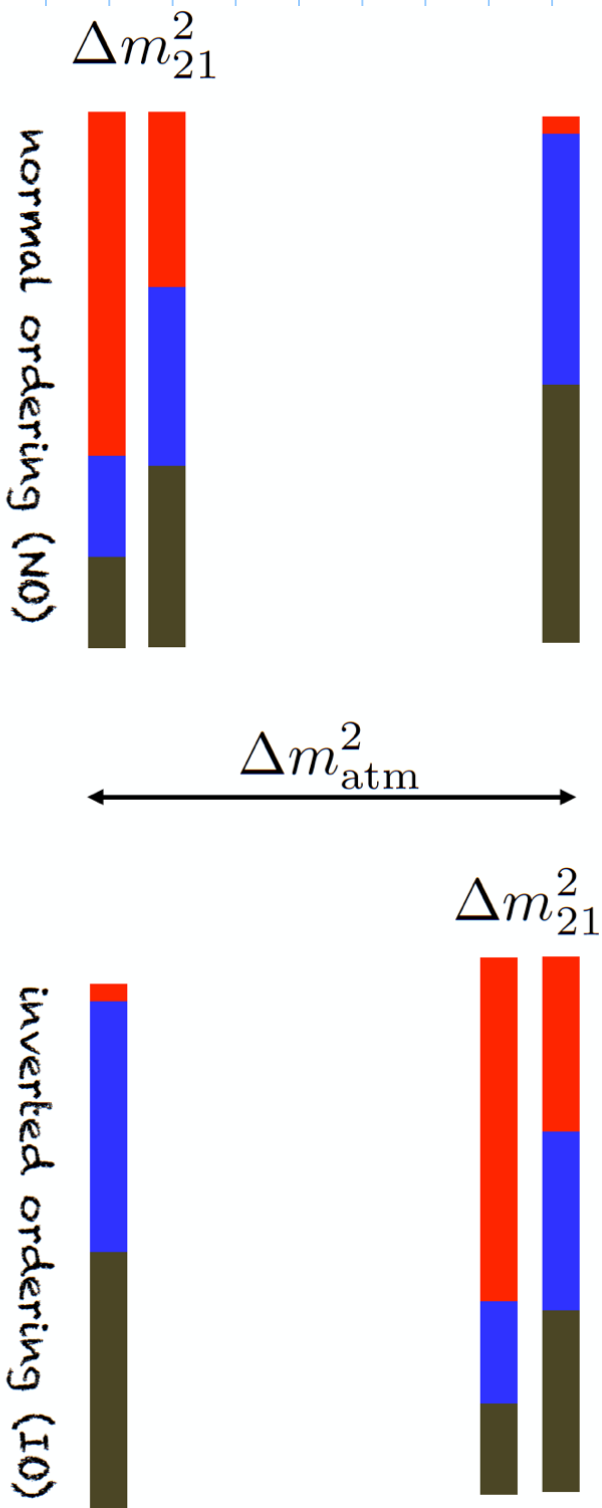
$$\begin{pmatrix} 0.8 & 0.5 & 0.16 \\ -0.4 & 0.5 & -0.7 \\ -0.4 & 0.5 & 0.7 \end{pmatrix}$$

NUFIT 4.1 (2019)

$$|U|_{3\sigma}^{\text{w/o SK-atm}} = \begin{pmatrix} 0.797 \rightarrow 0.842 & 0.518 \rightarrow 0.585 & 0.143 \rightarrow 0.156 \\ 0.244 \rightarrow 0.496 & 0.467 \rightarrow 0.678 & 0.646 \rightarrow 0.772 \\ 0.287 \rightarrow 0.525 & 0.488 \rightarrow 0.693 & 0.618 \rightarrow 0.749 \end{pmatrix}$$

$$|U|_{3\sigma}^{\text{with SK-atm}} = \begin{pmatrix} 0.797 \rightarrow 0.842 & 0.518 \rightarrow 0.585 & 0.143 \rightarrow 0.156 \\ 0.243 \rightarrow 0.490 & 0.473 \rightarrow 0.674 & 0.651 \rightarrow 0.772 \\ 0.295 \rightarrow 0.525 & 0.493 \rightarrow 0.688 & 0.618 \rightarrow 0.744 \end{pmatrix}$$

Masses and Mixing



Latest global Fit performed in 2019 (NuFit)

with SK atmospheric data

	Normal Ordering (best fit)	
	bfp $\pm 1\sigma$	3σ range
$\sin^2 \theta_{12}$	0.310 $^{+0.013}_{-0.012}$	0.275 \rightarrow 0.350
$\theta_{12}/^\circ$	33.82 $^{+0.78}_{-0.76}$	31.61 \rightarrow 36.27
$\sin^2 \theta_{23}$	0.563 $^{+0.018}_{-0.024}$	0.433 \rightarrow 0.609
$\theta_{23}/^\circ$	48.6 $^{+1.0}_{-1.4}$	41.1 \rightarrow 51.3
$\sin^2 \theta_{13}$	0.02237 $^{+0.00066}_{-0.00065}$	0.02044 \rightarrow 0.02435
$\theta_{13}/^\circ$	8.60 $^{+0.13}_{-0.13}$	8.22 \rightarrow 8.98
$\delta_{CP}/^\circ$	221 $^{+39}_{-28}$	144 \rightarrow 357
$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	7.39 $^{+0.21}_{-0.20}$	6.79 \rightarrow 8.01
$\frac{\Delta m_{3l}^2}{10^{-3} \text{ eV}^2}$	+2.528 $^{+0.029}_{-0.031}$	+2.436 \rightarrow +2.618

Unitarity Conditions

Appendix A

$$\begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix} \cdot \begin{pmatrix} V_{e1}^* & V_{\mu 1}^* & V_{\tau 1}^* \\ V_{e2}^* & V_{\mu 2}^* & V_{\tau 2}^* \\ V_{e3}^* & V_{\mu 3}^* & V_{\tau 3}^* \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$V_{e1} V_{e1}^* + V_{e2} V_{e2}^* + V_{e3} V_{e3}^* = 1$$

$$V_{\mu 1} V_{\mu 1}^* + V_{\mu 2} V_{\mu 2}^* + V_{\mu 3} V_{\mu 3}^* = 1$$

$$V_{\tau 1} V_{\tau 1}^* + V_{\tau 2} V_{\tau 2}^* + V_{\tau 3} V_{\tau 3}^* = 1$$

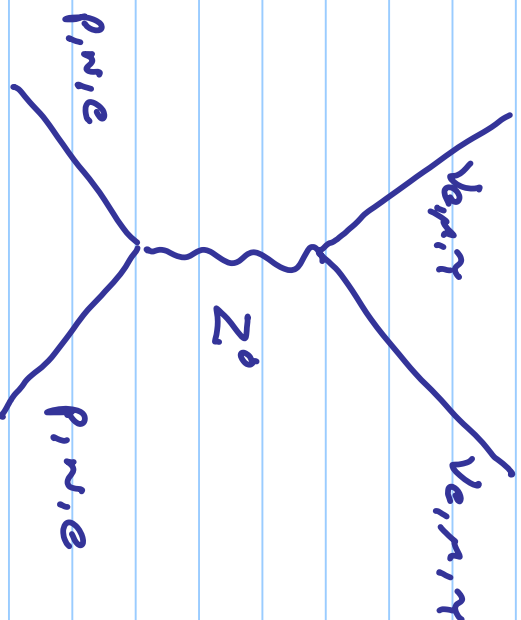
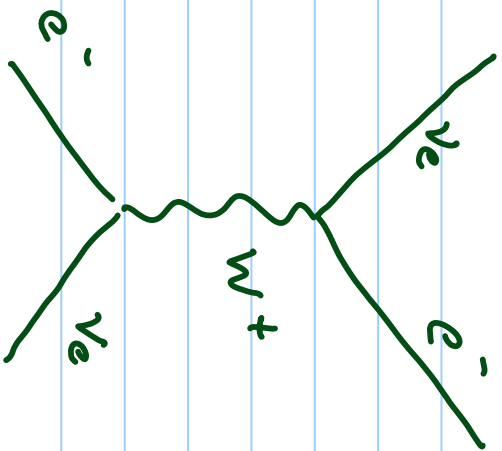
$$V_{e1} V_{\mu 1}^* + V_{e2} V_{\mu 2}^* + V_{e3} V_{\mu 3}^* = 0$$

$$V_{e1} V_{\tau 1}^* + V_{e2} V_{\tau 2}^* + V_{e3} V_{\tau 3}^* = 0$$

$$V_{\mu 1} V_{\tau 1}^* + V_{\mu 2} V_{\tau 2}^* + V_{\mu 3} V_{\tau 3}^* = 0$$

Neutrino oscillations in Matter

Appendix B



$$H_{cc} = \frac{G_F}{\sqrt{2}} [\bar{e}\gamma_\mu(1-\gamma_5)\nu_e] [\bar{\nu}_e\gamma^\mu(1-\gamma_5)e] \quad \text{Fierz} \rightarrow$$

$$= \frac{G_F}{\sqrt{2}} [\bar{e}\gamma_\mu(1-\gamma_5)e] [\bar{\nu}_e\gamma^\mu(1-\gamma_5)\nu_e] \quad (\text{low } E \text{ neutrinos})$$

$$H_{\text{eff}}(\nu_e) = \langle H_{cc} \rangle_e \equiv \bar{\nu}_e \nu_e \nu_e \nu_e$$

↳ integrated over all e variables

Unpolarized medium with zero total momentum, relevant term is $\langle \bar{e}\gamma_0 e \rangle = \langle e^T e \rangle = N_e \rightarrow$ number density

Neutrino oscillations in Matter (cont.)

$$\rightarrow | \nu_e \rangle_{cc} \equiv | \nu_{cc} \rangle = \sqrt{2} | \nu_e \rangle$$

$$| \nu_e \rangle_{nc} = - \frac{G_F N_n}{\sqrt{2}} \quad (\text{protons, electrons cancel off})$$

$$| \nu_n \rangle_{nc} = | \tilde{\nu}_n \rangle_{nc} = - \frac{G_F N_n}{\sqrt{2}}$$

More convenient to work in Flavor basis because effective potentials are diagonal in this basis.

For the Two-Flavor case, in the absence of matter:

$$i \left(\frac{d}{dt} \right) | \nu_n \rangle = H_n | \nu_n \rangle, \quad H_n \text{ is diagonal}$$

$$i \left(\frac{d}{dt} \right) | \nu_p \rangle = H_p | \nu_p \rangle = U H_n U^\dagger | \nu_p \rangle$$

$E_i \approx p + m_i^2 / 2E$

$$i \left(\frac{d}{dt} \right) \begin{pmatrix} | \nu_e \rangle \\ | \nu_n \rangle \end{pmatrix} = \begin{pmatrix} \left(p + \frac{m_1^2 + m_2^2}{4E} \right) - \frac{\Delta m^2 \cos 2\theta_0}{4E} & \frac{\Delta m^2 \sin 2\theta_0}{4E} \\ \frac{\Delta m^2 \sin 2\theta_0}{4E} & \left(p + \frac{m_1^2 + m_2^2}{4E} \right) - \frac{\Delta m^2 \cos 2\theta_0}{4E} \end{pmatrix} \begin{pmatrix} | \nu_e \rangle \\ | \nu_n \rangle \end{pmatrix}$$

Neutrino oscillations in Matter (cont.)

Extra Terms on the diag. can only modify the common phase of the neutrino states \Rightarrow we can omit them

$$\text{We get } i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2 \cos 2\theta_0}{4E} & \frac{\Delta m^2 \sin 2\theta_0}{4E} \\ \frac{\Delta m^2 \sin 2\theta_0}{4E} & \frac{\Delta m^2 \cos 2\theta_0}{4E} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

With matter present:

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2 \cos 2\theta_0 + \sqrt{2} G_F N_e}{4E} & \frac{\Delta m^2 \sin 2\theta_0}{4E} \\ \frac{\Delta m^2 \sin 2\theta_0}{4E} & \frac{\Delta m^2 \cos 2\theta_0}{4E} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

Note the $G_F N_e$ Term is common to both and on diag. so we can discard it (overall phase)

Neutrino oscillations in Matter (cont.)

In matter with constant density ($N_e = \text{const}$),
diag. of Hamiltonian gives following eigenstates:

$$\nu_A = \nu_e \cos \theta + \nu_\mu \sin \theta$$

$$\nu_B = -\nu_e \sin \theta + \nu_\mu \cos \theta$$

$$\tan 2\theta = \frac{\Delta m^2}{2E} \sin 2\theta_0$$

$$\frac{\Delta m^2 \cos 2\theta_0 - \sqrt{2} G_F N_e}{2E}$$

$$P(\nu_e \rightarrow \nu_\mu; L) = \sin^2 2\theta \sin^2 \left(\frac{\pi L}{L_m} \right)$$

$$L_m \text{ (oscillation length in matter)} = \frac{2\pi}{E_A - E_B}$$

$$E_A - E_B = \sqrt{\left(\frac{\Delta m^2 \cos 2\theta_0 - \sqrt{2} G_F N_e}{2E} \right)^2 + \left(\frac{\Delta m^2}{2E} \right) \sin^2 2\theta_0}$$

Neutrino masses

Appendix C

Now that we accept that neutrinos have mass, we need to add neutrino mass terms to our SM Lagrangian:

$$\mathcal{L}_\nu = \frac{-y_\nu}{\sqrt{2}} \bar{\nu}_L \nu_R + \frac{y_\nu}{\sqrt{2}} \bar{\nu}_R \nu_L - \frac{y_\nu}{\sqrt{2}} h | \bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L |$$

$\underbrace{\hspace{15em}}_{\text{Dirac Mass Term}}$
 $\underbrace{\hspace{15em}}_{\text{coupling and interaction with Higgs boson}}$

\Rightarrow implies we have a right-handed neutrino

But neutrinos are electrically neutral particles right handed neutrinos are singlets \rightarrow same for left-handed anti-neutrinos

Charge Conjugation could leave neutrino field unchanged particle fields and anti-particle field would be the same

$$\psi_L \xrightarrow{C} C \bar{\psi}_L^T = \psi_L^c \quad \psi_R = \psi_L^c \quad \psi = C \bar{\psi}^T \quad \bar{\psi} = \psi^c$$

Neutrino masses cont.

with $\nu_R = \nu_L^c = C \bar{\nu}_L^T$

Majorana mass term: $M (\bar{\nu}_L^c \nu_L + \bar{\nu}_L \nu_L^c)$

or $M (\bar{\nu}_R^c \nu_R + \bar{\nu}_R \nu_R^c)$

→ respects local gauge invariance

Most general mass term will include both mass terms:

$$\mathcal{L}_{DM} = -1/2 [m_b \bar{\nu}_L \nu_R + m_b \bar{\nu}_R^c \nu_L^c + M \bar{\nu}_R^c \nu_R] + h.c.$$

with $\mathcal{L}_b = -1/2 m_b (\bar{\nu}_L \nu_R + \bar{\nu}_R^c \nu_L^c) + h.c.$

$$\Rightarrow \bar{\nu}_L \nu_R = \bar{\nu}_R^c \nu_L^c$$

$$\mathcal{L}_{DM} = -1/2 (\bar{\nu}_L \bar{\nu}_R^c) \begin{pmatrix} 0 & m_b \\ m_b & M \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} + h.c.$$

Neutrino masses (cont.)

Eigenvalues of the matrix:

$$M_{\pm} = \frac{M \pm M \sqrt{1 + 4m_b^2/M^2}}{2} \quad \text{if } M \gg m$$

$$M_{\pm} \approx \frac{1}{2} M \pm \frac{1}{2} \left(M + \frac{2m_b^2}{M} \right)$$

so $M_{+} \approx M$ or M_{ν}

$$\text{and } m^{-} = \frac{m_b^2}{M} \quad \text{or } m_{\nu}$$

Dirac mass "scale" suppressed by M mass scale
this would imply M_{ν} in the 10^{11} GeV range

Assignment Due March 29th

1- The imaginary components of the PMNS Matrix provide a possible source of CP violation in the Standard Model.

The relative magnitude of the CP violation neutrino oscillations is given by:

$$P(\nu_e \rightarrow \nu_\mu) - P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$$

This can be shown to be equal to:

$$16 |m_{e1}^* m_{\mu 1} m_{e2} U_{\mu 2}^*| \sin \Delta_{12} \sin \Delta_{13} \sin \Delta_{23}$$

13.30

13.4 Derive equation (13.30) in the following three steps:

a) By writing the oscillation probability $P(\nu_e \rightarrow \nu_\mu)$ as

$$P(\nu_e \rightarrow \nu_\mu) = 2 \sum_{k < j} \text{Re} \left\{ U_{ek}^* U_{\mu k} U_{ej} U_{\mu j}^* \left[e^{i(\phi_j - \phi_k)} - 1 \right] \right\},$$

→ Continued on next page

Assignment 7 (cont.)

and writing $\Delta_{ij} = (\phi_i - \phi_j)/2$, show that

$$P(\nu_e \rightarrow \nu_\mu) = -4 \sum_{i < j} \operatorname{Re}\{U_{ei}^* U_{\mu i} U_{ej} U_{\mu j}^*\} \sin^2 \Delta_{ij} + 2 \sum_{i < j} \operatorname{Im}\{U_{ei}^* U_{\mu i} U_{ej} U_{\mu j}^*\} \sin 2\Delta_{ij}.$$

b) Defining $-J \equiv \operatorname{Im}\{U_{e1}^* U_{\mu 1} U_{e3} U_{\mu 3}^*\}$, use the unitarity of the PMNS matrix to show that

$$\operatorname{Im}\{U_{e1}^* U_{\mu 1} U_{e3} U_{\mu 3}^*\} = -\operatorname{Im}\{U_{e2}^* U_{\mu 2} U_{e3} U_{\mu 3}^*\} = -\operatorname{Im}\{U_{e1}^* U_{\mu 1} U_{e2} U_{\mu 2}^*\} = -J.$$

c) Hence, using the identity

$$\sin A + \sin B - \sin(A + B) = 4 \sin\left(\frac{A}{2}\right) \sin\left(\frac{B}{2}\right) \sin\left(\frac{A+B}{2}\right),$$

show that

$$P(\nu_e \rightarrow \nu_\mu) = -4 \sum_{i < j} \operatorname{Re}\{U_{ei}^* U_{\mu i} U_{ej} U_{\mu j}^*\} \sin^2 \Delta_{ij} + 8J \sin \Delta_{12} \sin \Delta_{13} \sin \Delta_{23}.$$

d) Hence show that

$$P(\nu_e \rightarrow \nu_\mu) - P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu) = 16 \operatorname{Im}\{U_{e1}^* U_{\mu 1} U_{e2} U_{\mu 2}^*\} \sin \Delta_{12} \sin \Delta_{13} \sin \Delta_{23}.$$

e) Finally, using the current knowledge of the PMNS matrix determine the maximum possible value of $P(\nu_e \rightarrow \nu_\mu) - P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$.

2 - How can we determine whether we have a "normal" hierarchy vs an inverted hierarchy using long baseline experiments e.g. T2K?