



LECTURE 6: Spontaneous Symmetry Breaking (Part II)

Overview:

-Recap of Abelian case

-Ginzburg-Landau

-Higgs Mechanism (non-Abelian case)

(This lecture mostly follows Quigg Chapters 4-5)

②

Higgs Mechanism (Abelian case recap)

We saw that spontaneous breaking of a continuous symmetry leads to massless bosons (Goldstone bosons). We expect one massless boson per broken generator.

We saw however that in the case of a local gauge theory, the massless gauge boson and the massless Goldstone boson conspire to give us a massive gauge boson without the massless Goldstone boson.

In the case we studied, we had before symmetry breaking:

2 scalars: 2 degrees of freedom

1 massless vector boson: 2 degrees of freedom

Total = 4

After breaking we had (explicit in unitary gauge):

1 massive vector boson: 3 degrees of freedom

1 massive Higgs scalar: 1 degree of freedom

Total = 4

Ginzburg Landau Superconductivity

(3)

ψ : macroscopic wave function describing condensate
Free energy of superconductor can be written as:
 $G_{\text{super}}(0) = G_{\text{normal}}(0) + \alpha |\psi|^2 + \beta |\psi|^4$

$$G_{\text{super}}(\mathbf{B}) = G_{\text{super}}(0) + \frac{\mathbf{B}^2}{2} + \frac{1}{2m^*} \psi^* (-i\nabla - e^* \mathbf{A})^2 \psi$$

in weak field approx., field equations derived using $G_{\text{super}}(\mathbf{B})$ lead to massive photon

Meisner Effect:

- Cooper pairs form BEC condensate below $T_c \sim 10^0 - 10^2$ K. Condensate disturbed by EM field
- Short range force, attenuation length $\sim 10^{-6}$ cm
- equivalent to photon acquiring a mass

Electroweak symmetry breaking:

- Higgs condenses below $T_c \sim 10^{15}$ K. Condensate disturbed by gauge bosons
- Short range force, attenuation length $\sim 10^{-18}$ cm
- W/Z bosons acquire mass



Higgs Mechanism (non-Abelian case)

(4)

We will study an $SU(2)$ doublet of complex scalar fields:

$$\varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix}$$

The Lagrangian is: $(\partial_\mu \varphi)^\dagger (\partial^\mu \varphi) - \mu^2 \varphi^\dagger \varphi - \lambda (\varphi^\dagger \varphi)^2$

The covariant derivative: $D_\mu = \partial_\mu + ig \frac{\tau_a}{2} B_\mu^a$

Under infinitesimal transformation: $\varphi(x)' = (1 + \frac{i}{2} \alpha(x) \cdot \tau) \varphi$

$$B_\mu^a' = B_\mu^a - \frac{1}{g} \partial_\mu \alpha^a - \alpha^b \times B_\mu^c$$

We obtain

$$\mathcal{L} = (\partial_\mu \varphi + ig \frac{\tau_a}{2} B_\mu^a \varphi)^\dagger (\partial^\mu \varphi + ig \frac{\tau_a}{2} B^\mu{}^a \varphi) - V(\varphi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$V(\varphi) = \mu^2 (\varphi^\dagger \varphi) + \lambda (\varphi^\dagger \varphi)^2$$

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu - g B_\mu \times B_\nu$$

Higgs Mechanism (non-Abelian case)

(5)

minimum of potential at $\phi^\dagger \phi = \frac{1}{2} (\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) = -\frac{\mu^2}{2\lambda}$

we chose minimum around which to do our expansion: $\phi_3^2 = -\frac{\mu^2}{\lambda} \equiv v^2$ $\phi_1 = \phi_2 = \phi_4 = 0$

We parametrize fluctuations from the vacuum $\phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$ in terms of 4 real scalar fields $\epsilon_1, \epsilon_2, \epsilon_3, \eta$

$$\phi(x) = e^{i\gamma \cdot \epsilon(x)/v} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \eta \end{pmatrix} \approx \frac{1}{\sqrt{2}} \begin{pmatrix} 1 + i\epsilon_3/v & i(\epsilon_1 - \epsilon_2)/v \\ i(\epsilon_1 + \epsilon_2)/v & 1 - i\epsilon_3/v \end{pmatrix} \begin{pmatrix} 0 \\ v + \eta \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} \epsilon_2 + i\epsilon_1 \\ v + \eta - i\epsilon_3 \end{pmatrix}, \quad \text{so } \phi^\dagger \phi =$$

$$\frac{1}{\sqrt{2}} (\epsilon_2 - i\epsilon_1, v + \eta + i\epsilon_3) \frac{1}{\sqrt{2}} \begin{pmatrix} \epsilon_2 + i\epsilon_1 \\ v + \eta - i\epsilon_3 \end{pmatrix} = \epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2 + v^2 + \eta^2 + 2v\eta$$

We know all terms from V will cancel save $\frac{1}{2} m^2 \eta$
→ for small oscillations ↪ massive scalar

Higgs Mechanism (non-Abelian case)

⑥

Let's move to the unitary gauge right away:

$$\varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \eta \end{pmatrix}$$

$$D_\mu \varphi = \frac{1}{\sqrt{2}} \left(\partial_\mu \begin{pmatrix} 0 \\ \eta \end{pmatrix} + \frac{ig}{2} \begin{pmatrix} b_\mu^3 & \sqrt{2} b_\mu^- \\ \sqrt{2} b_\mu^+ & -b_\mu^3 \end{pmatrix} \begin{pmatrix} 0 \\ v + \eta \end{pmatrix} \right)$$

where $b_\mu^\pm = \frac{1}{\sqrt{2}} (b_\mu^1 \pm i b_\mu^2)$

$$|D_\mu \varphi|^2 = \frac{1}{2} \partial_\mu \eta \partial^\mu \eta + \frac{1}{4} g^2 v^2 (b_\mu^+ b^{\mu-} + \frac{1}{2} b_\mu^3 b^{\mu 3})$$

$$+ \frac{1}{4} g^2 \eta^2 (b_\mu^+ b^{\mu-} + \frac{1}{2} b_\mu^3 b^{\mu 3}) + \frac{1}{2} g^2 v \eta (b_\mu^+ b^{\mu-} + \frac{1}{2} b_\mu^3 b^{\mu 3})$$

→ 3 bosons with mass $\frac{g v}{2}$

Higgs Mechanism (non-Abelian case)

⑦

Summary:

we started with: 4 scalars : 4 dof
3 massless bosons: $3 \times 2 = 6$ dof

Total = 10 dof

we end up with: 1 scalar (massive) : 1 dof
3 massive bosons: $3 \times 3 = 9$ dof

Total = 10 dof

In the Standard Model we have 3 massive vector bosons (2 charged, one neutral) and one massless boson (neutral).

Future problem set

Problem set 1

8

Due Date: Wednesday Feb 1st

Problem 1

Analyse the spontaneous breaking of a global $SU(2)$ symmetry for the case of 3 real scalar fields in an $SU(2)$ Triplet: $\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi) \cdot (\partial^\mu \phi) - V(\phi)$$

$$V(\phi) = \frac{1}{2} \mu^2 \phi \cdot \phi + \frac{1}{4} \lambda (\phi \cdot \phi)^2$$

Problem 2

Analyse the spontaneous breaking of a local $SU(2)$ symmetry for the case of 3 real scalar fields in an $SU(2)$ Triplet: $\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$

→ use \mathcal{L} and V from page 4

