

LECTURE 9: Weak Interactions (Part 3) and the Standard Model (Part 1)

Overview:

- $\nu \nu \rightarrow W W$ scattering
- Construction of the Standard Model

(I used Quigg and Novaes as references)

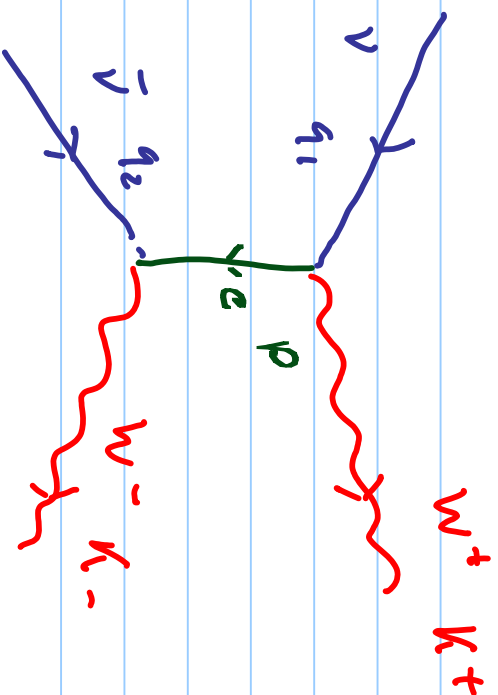
$\bar{\nu} \rightarrow W^+ W^-$ scattering

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Recall that we had essentially solved our cross section of problems at high energies with the introduction of a heavy gauge boson and its associated propagator.

In fact we have other problems that will require the addition of new particles. Here's an example:

$\bar{\nu} \rightarrow W^+ W^-$



$$M = -i \frac{G_F M_W^2}{\sqrt{2}} \bar{\nu} \not{\epsilon}^* (1 - \gamma_5) \frac{(\not{p} + \not{n})}{p^2 + M^2} \not{\epsilon}^* (1 - \gamma_5) \nu$$

$\bar{v}v \rightarrow W W$ scattering continued

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$$q_1 \equiv (Q, 0, 0, Q)$$

$$q_2 = (Q, 0, 0, -Q)$$

$$K_+ = (Q, K \sin \theta, 0, K \cos \theta)$$

$$K_- = (Q, -K \sin \theta, 0, -K \cos \theta)$$

$\xi_{\pm}^A = (0, \hat{\xi}_{\pm}) \rightarrow W$ pol. in W rest frame

$$\Rightarrow \text{in } CA \text{ frame: } \xi_{\pm}^A = \left[\frac{\vec{K}_{\pm} \cdot \vec{\xi}_{\pm}}{M_W}, \xi_{\pm}^1 + \frac{\vec{K}_{\pm} \cdot \vec{\xi}_{\pm}}{M_W(Q \pm M_W)} \right]$$

For longitudinal pol.: $\xi_{\mu}^{\pm} = \left(\frac{K}{M_W}, \frac{Q K_{\pm}}{M_W} \right)$

at very high energies: $Q \simeq K \gg M_W$

$$\rightarrow \xi_{\pm} \simeq \frac{K_{\pm}}{M_W}$$

$\bar{\nu} \nu \rightarrow W^+ W^-$ scattering (continued)

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$$\mathcal{M} = \frac{-i G_F}{\sqrt{2}} \bar{\nu} \not{k}_- (1 - \gamma_5) \frac{(\not{p} + m)}{p^2 - m^2} \not{k}_+ (1 - \gamma_5) \nu$$

With the Dirac equation we get:

$$\not{k}_+ \nu = 0 \quad \text{and} \quad \bar{\nu} \not{q}_2 = 0$$

so we can replace \not{k}_+ with: $\not{k}_+ - \not{q}_1 = -\not{p}$
and \not{k}_- with: $\not{k}_- - \not{q}_2 = \not{p}$

we neglect the mass of the electron (m):

$$\text{this gives } \frac{i G_F}{\sqrt{2}} \bar{\nu} \not{p} (1 - \gamma_5) \frac{\not{p} \not{p}}{p^2} (1 - \gamma_5) \nu$$

$$\not{p} \not{p} = p^2 \Rightarrow \mathcal{M} = \sqrt{2} i G_F \bar{\nu} \not{p} (1 - \gamma_5) \nu$$

$$\begin{aligned} |\mathcal{M}|^2 &= 2 G_F^2 \text{Tr} \left[\not{q}_2 \not{p} (1 - \gamma_5) \not{q}_1 \not{p} (1 - \gamma_5) \right] \\ &= 4 G_F^2 \text{Tr} \left[(1 - \gamma_5) \not{q}_2 \not{p} \not{q}_1 \not{p} \right] \end{aligned}$$

$\bar{v}v \rightarrow W^+W^-$ scattering (continued)

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$$|M|^2 = 16 G_F^2 [(q_2 \cdot p)(q_1 \cdot p) - (q_2 \cdot q_1)(p \cdot p) + (q_2 \cdot p)(q_1 \cdot p)]$$

note that the $\epsilon_{\mu\nu\alpha\beta}$ term = 0

$$= 16 G_F^2 [2 (q_2 \cdot p)(q_1 \cdot p) - \underbrace{p^2}_{2Q^2} (q_2 \cdot q_1)]$$

$$p = (0, K \sin \theta, 0, Q - K \cos \theta)$$

$$q_1 \cdot p = (Q, 0, 0, Q) \cdot p = QK \cos \theta - Q^2$$

$$q_2 \cdot p = -QK \cos \theta + Q^2$$
$$2 (q_1 \cdot p)(q_2 \cdot p) = -2Q^2 K^2 \cos^2 \theta - \cancel{2Q^4} + \cancel{4Q^3 K \cos \theta}$$

$$p^2 = -K^2 \sin^2 \theta - K^2 \cos^2 \theta + 2KQ \cos \theta - Q^2$$

$$= -K^2 - Q^2 + 2KQ \cos \theta$$

$$p^2 \cdot 2Q^2 = -2Q^2 K^2 - \cancel{2Q^4} + \cancel{4KQ} \cos \theta$$

$$\text{result: } 2Q^2 K^2 (1 - \cos^2 \theta) = 2Q^2 K^2 \sin^2 \theta$$

$$|M|^2 = 32 G_F^2 Q^2 K^2 \sin^2 \theta$$

$\bar{\nu} \nu \rightarrow W^+ W^-$ scattering (continued)

⑥

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} |M|^2 = \frac{1}{64\pi^2 s} \cdot 32 G_F^2 Q^2 K^2 \sin^2 \theta$$

integration yields: $2\pi \cdot \frac{4}{3}$ (see W decay, lecture 8)

$$s \approx 4 Q^2, \quad Q \approx K$$

$$\Rightarrow s^2 = 16 Q^4$$

$$\sigma = \frac{G_F^2}{64\pi^2 s} \cdot 2s^2 \cdot 2\pi \cdot \frac{4}{3} = \frac{G_F^2 s}{12\pi}$$

↪ $\frac{G_F^2 s}{3\pi}$ in Quiss?
 ~ correct...

Again a badly behaved cross section...

We'll see that we need to add the Z boson
to tame the high energy behaviour.

The Standard Model

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Before we start, a few notes and comments:

$$v_R = \frac{1}{2} (1 + \gamma_5) v, \quad v_L = \frac{1}{2} (1 - \gamma_5) v$$

$$v(p, s) \equiv C \bar{v}^\dagger(p, s) = i \gamma_2 v^*(p, s)$$

$$v_R = \frac{1}{2} (1 - \gamma_5) v, \quad v_L = \frac{1}{2} (1 + \gamma_5) v$$
$$\underbrace{\quad}_{\equiv L} \quad \underbrace{\quad}_{\equiv R}$$

$$\bar{\psi}_L = (L \psi)^\dagger \gamma_0 = \psi^\dagger L^\dagger \gamma_0 = \psi^\dagger L \gamma_0 = \psi^\dagger \gamma_0 R = \bar{\psi} R$$

$$\bar{\psi}_R = \bar{\psi} L$$

note that mass term mixes right and left-handed

components: $\bar{\psi} \psi = \bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R$

The Standard Model (cont.)

$$\bar{y}_i y_i = (\bar{y}_R + \bar{y}_L) (y_R + y_L)$$

$$= (\bar{y}_L + \bar{y}_R) (Ry + Ly)$$

$$= \bar{y}_L R y + \bar{y}_L L y + \bar{y}_R R y + \bar{y}_R L y$$

↪ 0

↪ 0

$$= \bar{y}_R y_L + \bar{y}_L y_R$$

For an interaction:

$$\bar{y}_i y_j z = (\bar{y}_L + \bar{y}_R) \gamma_j (Lz + Rz)$$

$$= \bar{y}_L \gamma_j Lz + \bar{y}_L \gamma_j Rz + \bar{y}_R \gamma_j Lz + \bar{y}_R \gamma_j Rz$$

$$= \bar{y}_L R \gamma_j z + \bar{y}_L L \gamma_j z + \bar{y}_R R \gamma_j z + \bar{y}_R L \gamma_j z$$

↪ 0

↪ 0

$$= \bar{y}_R \gamma_j z + \bar{y}_L \gamma_j z$$

The Standard Model (cont.)

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For V-A weak current:

$$\frac{1}{2} \bar{\psi} \gamma^\mu (1 - \gamma^5) \psi = \bar{\psi} \gamma^\mu L \psi = \bar{\psi} \gamma^\mu L^2 \psi$$
$$= \bar{\psi}_L \gamma^\mu \psi_L$$

We will now select a gauge group for the EM and weak interactions.

→ weak current for lepton ℓ :

$$j_\mu^\dagger = \bar{\ell} \gamma_\mu (1 - \gamma^5) \nu = 2 \bar{\ell}_L \gamma_\mu \nu_L$$

We introduce left-handed isospin doublet ($T = 1/2$)

$$L = \begin{pmatrix} \nu \\ \ell \end{pmatrix}_L = \begin{pmatrix} L_\nu \\ L_\ell \end{pmatrix} = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \quad \text{with } T_3 = \pm 1/2$$

The Standard Model (cont.)

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We'll consider the neutrinos massless for now.

So we'll accommodate the right-handed part of the charged lepton in a weak isospin singlet ($T=0$)

$$R_L = \ell_R$$

The charged weak current can be written as

$$J_\mu^i = \bar{L} \gamma_\mu \frac{\tau^i}{2} L$$

Explicitly:

$$J_\mu^1 = \frac{1}{2} (\bar{\nu}_L \bar{\ell}_L) \gamma_\mu \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} = \frac{1}{2} (\bar{\ell}_L \gamma_\mu \nu_L + \bar{\nu}_L \gamma_\mu \ell_L)$$

$$J_\mu^2 = \frac{1}{2} (\bar{\nu}_L \bar{\ell}_L) \gamma_\mu \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} = \frac{i}{2} (\bar{\ell}_L \gamma_\mu \nu_L - \bar{\nu}_L \gamma_\mu \ell_L)$$

$$J_\mu^3 = \frac{1}{2} (\bar{\nu}_L \bar{\ell}_L) \gamma_\mu \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} = \frac{1}{2} (\bar{\nu}_L \gamma_\mu \nu_L - \bar{\ell}_L \gamma_\mu \ell_L)$$

The Standard Model (cont.)

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Charged weak current can be written in terms

$$\text{of } J^1 \text{ and } J^2 : J_\mu^+ = 2(J_\mu^1 - iJ_\mu^2)$$

\rightarrow will couple to W_μ^-

J_μ^3 will involve a neutral current. We define

the hypercharge current:

$$J_\mu^Y = -(\bar{L}\gamma_\mu L + 2\bar{R}\gamma_\mu R)$$

$$= -(\bar{\nu}_L\gamma_\mu\nu_L + \bar{l}_L\gamma_\mu l_L + 2\bar{l}_R\gamma_\mu l_R) \leftarrow$$

the Q_N current is given by:

$$J_\mu^{\text{em}} = -\bar{l}\gamma_\mu l = -(\bar{\nu}_L\gamma_\mu l_L + \bar{l}_R\gamma_\mu l_R) = J_\mu^3 + \frac{1}{2}J_\mu^Y$$

We have $\frac{1}{2}(\bar{\nu}_L\gamma_\mu\nu_L - \bar{l}_L\gamma_\mu l_L) \Rightarrow$ we need:
 $-\frac{1}{2}(\bar{\nu}_L\gamma_\mu\nu_L), -\frac{1}{2}(\bar{l}_L\gamma_\mu l_L), (\bar{l}_R\gamma_\mu l_R)$ so

The Standard Model (cont.)

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Note that T_3 and Q do not commute with

$T_{1,2}$ but the "charges" associated with J^i and J^y

$$T^i = \int d^3x J_0^i \quad \text{and} \quad Y = \int d^3x J_0^y$$

$$[T^i, T^j] = i \epsilon^{ijk} T^k, \quad [T^i, Y] = 0$$

$$Q = T^3 + \frac{1}{2} Y$$

→ weak hypercharge of the doublet: $Y_L = -1$

" " of the singlet: $Y_R = -2$

So our gauge group is $SU(2)_L \otimes U(1)_Y$

$$SU(2)_L \rightarrow W_n^1, W_n^2, W_n^3$$

$$U(1)_Y \rightarrow B_\mu$$

The Standard Model (cont.)

Field strength Tensors:

$$W_{\mu\nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g \epsilon^{ijk} W_\mu^j W_\nu^k$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

We get $\mathcal{L}_{gauge} = -\frac{1}{4} W_{\mu\nu}^i W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$

We get leptons (free Lagrangian): $\bar{R} : \not{\partial} R + \bar{L} : \not{\partial} L$
 $= \bar{R}_R i \not{\partial} R + \bar{L}_L i \not{\partial} L + \bar{\nu}_L i \not{\partial} \nu_L$
 $= \bar{R} i \not{\partial} R + \bar{\nu} i \not{\partial} \nu$

NO mass term. mixes right and left handed components and would break gauge invariance.

The Standard Model (cont.)

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Left and right-handed components transform differently
 \Rightarrow will break gauge invariance e.g.:

$$\begin{pmatrix} \nu \\ e \end{pmatrix}'_L = \exp(-i\alpha \cdot \frac{\tau}{2}) \begin{pmatrix} \nu_e \\ e \end{pmatrix}$$

We now introduce Fermion-gauge coupling

using the gauge-covariant derivative:

$$L = \bar{\psi}_m + i\frac{g}{2} \tau^i W_m^i + i\frac{g'}{2} Y B_m$$

$$R = \bar{\psi}_m + i\frac{g'}{2} Y B_m \quad g \rightarrow SU(2), \quad Y_{Le} = -1$$

$$g' \rightarrow U(1), \quad Y_{R2} = -2$$

$$\mathcal{L}_{lep.} = \mathcal{L}_{lep.} + \bar{L} i \gamma_m \left(i\frac{g}{2} \tau^i W_m^i + i\frac{g'}{2} Y B_m \right) L \\ + \bar{R} i \gamma_m \left(i\frac{g'}{2} Y B_m \right) R$$

