

LECTURE 12: Z Boson Physics and Neutral Currents

Overview:

- Higgs Boson (cont.)
- Z width
- Problem set #2 (last problem)

(I used Quigg, Griffiths, my thesis as references)

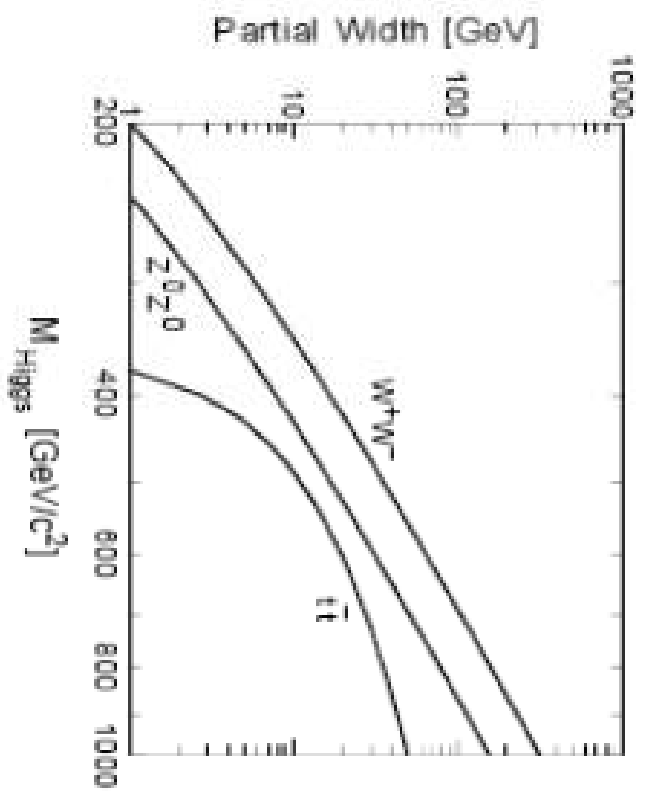
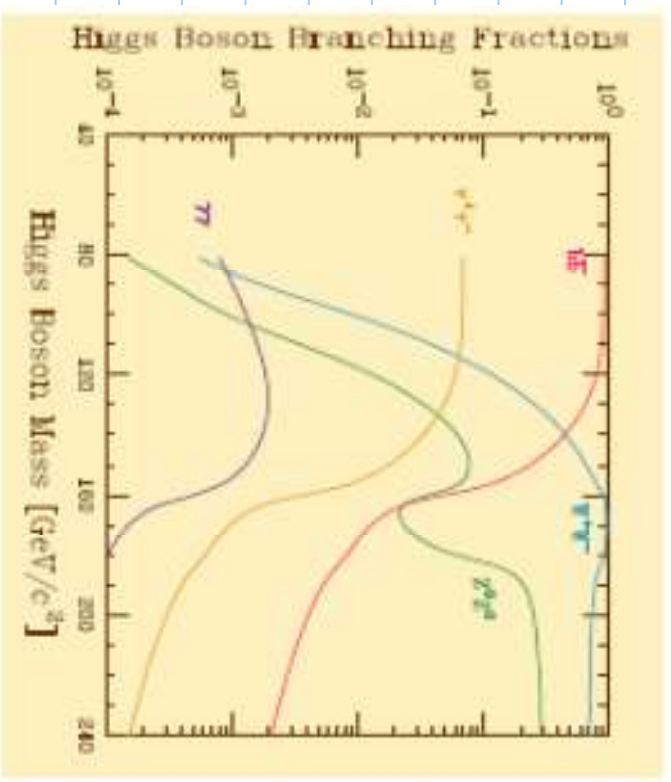
THE Higgs Boson (cont)

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NOTES:

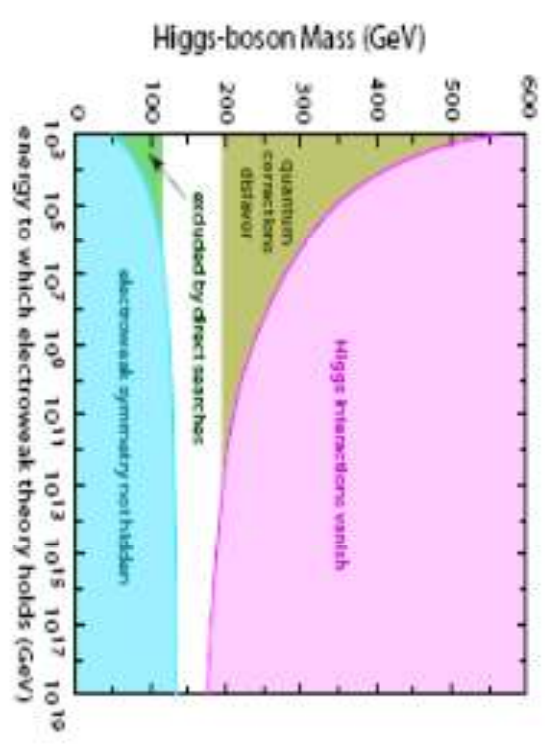
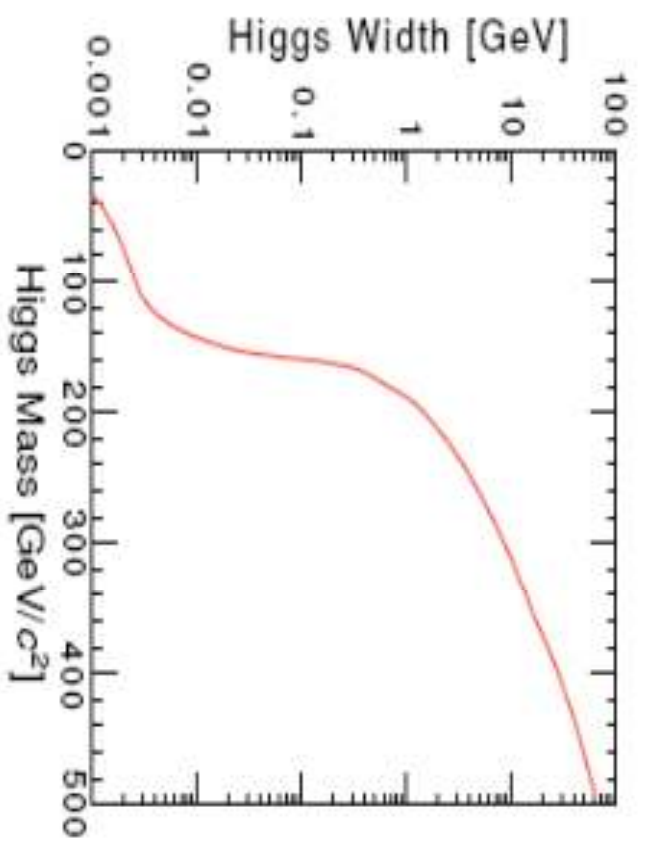
- in the limit where $M_H \gg M_W$, instead of Γ becomes M_H^2 why?

- why do we set a factor of 1/2 for the 2 bosons? (the obvious answer does not tell the full story)



THE HIGGS BOSON (cont.)

③



Theory limits on the Higgs

Unitarity in $W^+W^- \rightarrow W^+W^-$ cross section
 imply that in μH should be $\lesssim 900 \text{ GeV}$
 \rightarrow Higgs or new physics at 1 TeV ...

THE Higgs boson (cont.)

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Triviality: evolution of renormalized coupling given by:

$$\lambda_r(Q) = \frac{\lambda(\nu)}{1 - \frac{3}{2g^2} \lambda(\nu) \log\left(\frac{Q}{\nu}\right)}$$

$\lambda(\nu) \rightarrow 0$ when $Q \rightarrow \infty$ i.e. Trivial Theory
Free Field theory

if Higgs potential valid up to 10^{16} GeV

then $m_H \lesssim 170$ GeV

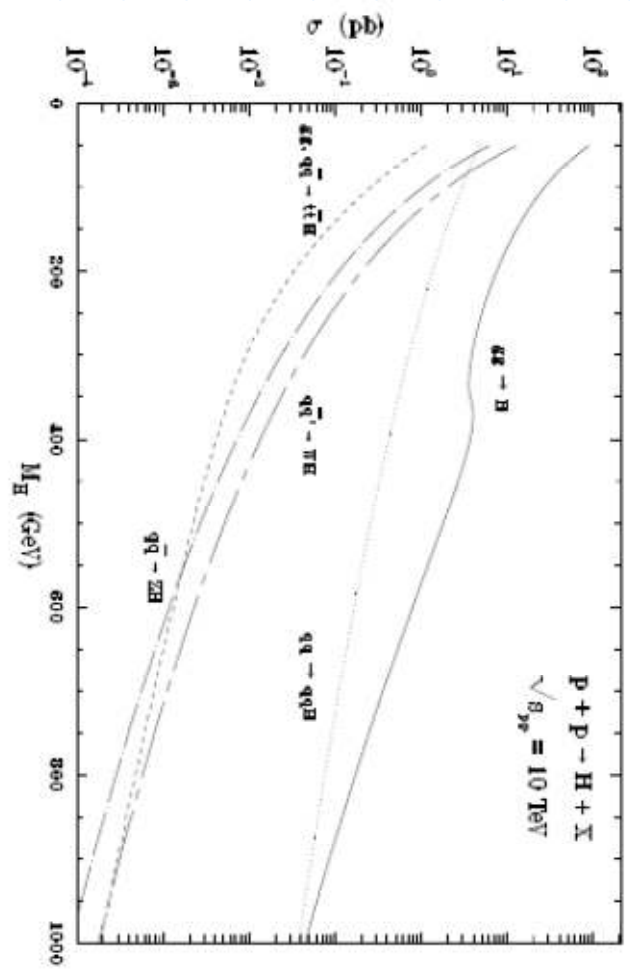
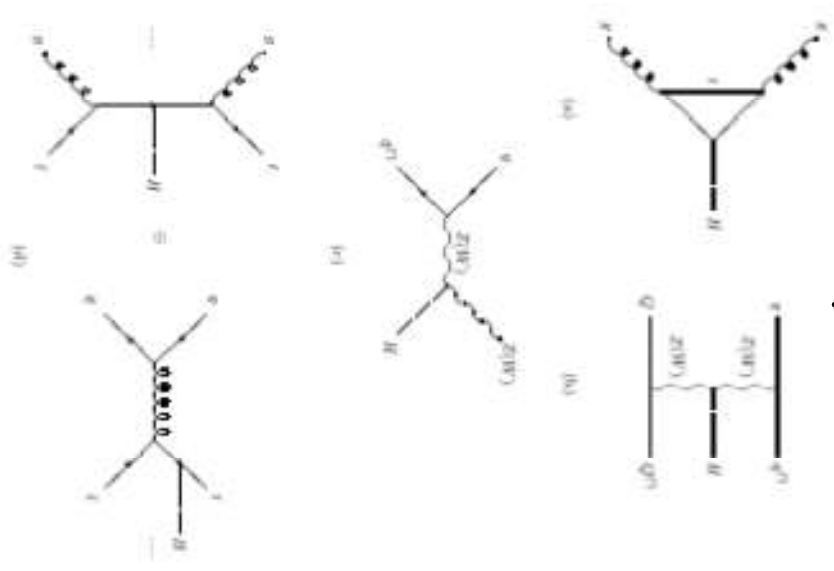
1 TeV $m_H \lesssim 640$ GeV

Vacuum stability:

$$V(\phi) \sim \mu^2 \phi^\dagger \phi + \lambda(Q_0) (\phi^\dagger \phi)^2 + B_7 (\phi^\dagger \phi)^2 \log\left(\frac{Q^2}{Q_0^2}\right)$$

SM valid up to Planck scale $\rightarrow m_H \gtrsim 130$ GeV

Higgs production AT THE LHC

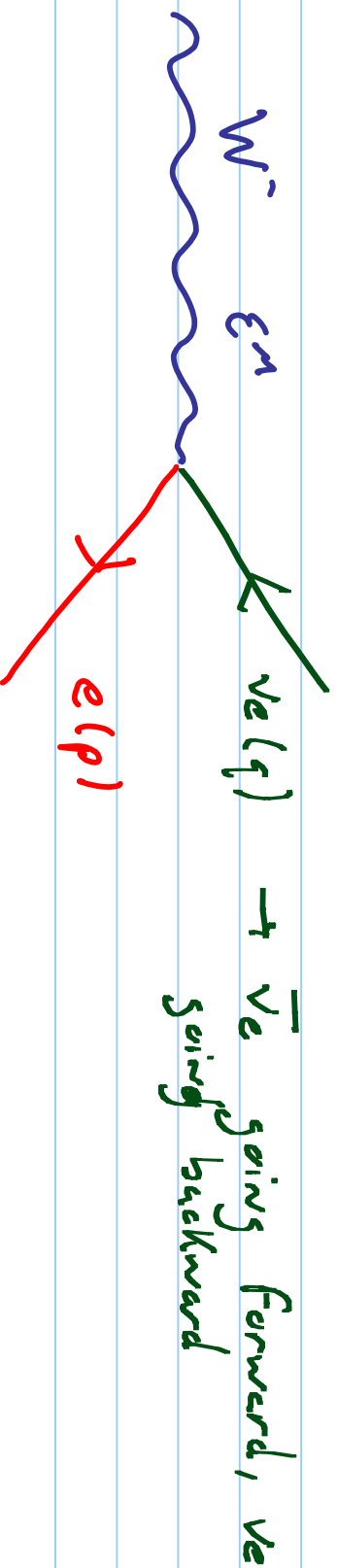


Let's look at slides from Chris Quigg, academic lectures part 5, 6

Z Boson Decay

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A reminder from lecture 8 where we studied $W \rightarrow e\bar{\nu}$:



$$M = -i \left(\frac{G_F M_W^2}{\sqrt{2}} \right)^{1/2} \bar{v}(p) \gamma_\mu (1 - \gamma_5) v(q) \epsilon^\mu$$

$\epsilon^\mu \equiv (0, \hat{\epsilon})$ is the polarization vector of the W

→ we neglect the electron mass

$$\begin{aligned} |M|^2 &= \frac{G_F^2 M_W^2}{\sqrt{2}} \text{Tr} [\not{\epsilon} (1 - \gamma_5) \not{q} (1 + \gamma_5) \not{p}^* \not{p}] \\ &= \frac{G_F^2 M_W^2}{\sqrt{2}} 2 \text{Tr} [(1 + \gamma_5) \not{q} \not{p}^* \not{p}] \end{aligned}$$

W decay

⑦

$$|M|^2 = \frac{8G_F^2 M_W^2}{\sqrt{2}} \left((\varepsilon \cdot q)(\varepsilon^* \cdot p) - (\varepsilon \cdot \varepsilon^*)(p \cdot q) + (\varepsilon \cdot p)(\varepsilon^* \cdot q) \right. \\ \left. + i \varepsilon_{\mu\nu\rho\sigma} \varepsilon^\mu q^\nu \varepsilon^{*\rho} p^\sigma \right)$$

Let's pick the longitudinal polarization for

the W: $\varepsilon^\mu = (0; 0, 0, 1) = \varepsilon^{\mu^*}$ (helicity 0)

→ the $\varepsilon_{\mu\nu\rho\sigma}$ term vanishes

$$p = \frac{M_W}{2} (1, \sin\theta, 0, \cos\theta)$$

$$q = \frac{M_W}{2} (1, -\sin\theta, 0, -\cos\theta)$$

$$|M|^2 = \frac{8G_F^2 M_W^2}{\sqrt{2}} \cdot \frac{M_W^2}{4} \left(-\cos^2\theta - 1 \cdot [1 + \sin^2\theta + \cos^2\theta] - \cos^2\theta \right) \\ = \frac{4G_F^2 M_W^4}{\sqrt{2}} \sin^2\theta$$

W Decay

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$$\frac{d\Gamma}{d\Omega} = \frac{1 M^2}{64 \pi^2 M_W} = \frac{G_F M_W^3}{16 \pi^2 \sqrt{2}} \sin^2 \theta$$

$$d\Gamma = \frac{G_F M_W^3}{16 \pi^2 \sqrt{2}} \int_0^{2\pi} d\phi \int_0^{\pi} \sin \theta \sin^2 \theta d\theta$$

$$\int_0^{\pi} (-\cos \theta + \frac{\cos^3 \theta}{3}) \Big|_0^{\pi} = 1 - \frac{1}{3} - (-1 + \frac{1}{3}) = \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$$

$$\Gamma = \frac{G_F M_W^3}{16 \pi^2 \sqrt{2}} \cdot 2\pi \cdot \frac{4}{3} = \frac{G_F M_W^3}{6 \pi \sqrt{2}}$$

$$= 227 \text{ MeV} \quad (\text{For } M_W = 80.4)$$
$$G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$$

Total width: 2.06 GeV

Z Decay

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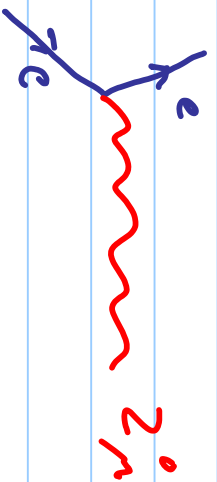
The W decay amplitude

$$M = -i \left(\frac{G_F M_W^2}{\sqrt{2}} \right)^{1/2} \bar{v}(p) \gamma_\mu (1 - \gamma_5) v(q) \epsilon^\mu$$

Needs to be modified for the Z given Feynman rules:



$$-\frac{i}{\sqrt{2}} \left(\frac{G_F M_Z^2}{\sqrt{2}} \right)^{1/2} \bar{v} \gamma_\mu (1 - \gamma_5) \nu$$



$$-\frac{i}{\sqrt{2}} \left(\frac{G_F M_Z^2}{\sqrt{2}} \right)^{1/2} \bar{e} \gamma_\mu \left[2s_w^2 (1 + \gamma_5) + (2s_w^2 - 1) (1 - \gamma_5) \right] e$$

$$s_w = \sin \theta_w$$

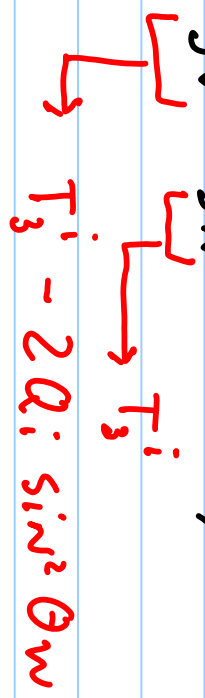
So $Z \rightarrow \nu e \bar{\nu}_e$ is straight forward:

$$|M(Z \rightarrow \nu e \bar{\nu}_e)|^2 = \frac{1}{2} \frac{M_Z^2}{M_W^2} |M(W \rightarrow \nu e \bar{\nu}_e)|^2$$

$$\rightarrow \Gamma(Z \rightarrow \nu \bar{\nu}) = \frac{G_F M_Z^3}{12\pi \sqrt{2}}$$

REMINDER:

$$- \frac{g}{\cos \theta_w} \quad \mathcal{T}_i: \gamma^{\mu i} \gamma^{\nu i} - \gamma^{\mu i} \gamma^{\nu i} \gamma^{\mu i} \gamma^{\nu i} \gamma^{\mu i} \gamma^{\nu i} \gamma^{\mu i} \gamma^{\nu i}$$



i	Q ⁱ	g ^{iA}	g ^{iV}
ν _e ν _μ ν _τ	0	1/2	1/2
e μ τ	-1	-1/2	-1/2 + 2 sin ² θ _w ≈ -0.03
ν _e T	2/3	1/2	1/2 - 4/3 sin ² θ _w ≈ 0.19
dsb	-1/3	-1/2	-1/2 + 2/3 sin ² θ _w ≈ -0.34

$$J_n^{em} = J_n^3 + 1/2 J_n^Y$$

$$J_n^{nc} = J_n^3 - \sin^2 \theta_w J_n^{em}$$

$$g_w = \frac{g_e}{\sin \theta_w}$$

REMINDER (cont.)

Standard Model Lagrangian (cont.)

(10')

As you've done in problem set 1, we parametrize the Higgs doublet

$$\Phi = \exp\left(\frac{i\tau^i \alpha_i}{2v}\right) \begin{pmatrix} 0 \\ (v+\eta)/\sqrt{2} \end{pmatrix}$$

we make a gauge Trans. $\Phi \rightarrow \Phi' = \exp\left(-i\frac{\tau^i \gamma_i}{2v}\right) \Phi$

$$= \frac{(v+\eta)}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

A scalar: $\left| (d_\mu + i\frac{\tau^i}{2} W_\mu^i + i\frac{\gamma_i}{2} B_\mu) \frac{(v+\eta)}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right|^2$

$$= \frac{M^2}{2} (v+\eta)^2 - \frac{\lambda}{4} (v+\eta)^4$$

$$-g W_\mu^3 + g' B_\mu$$

$$\left| \begin{pmatrix} 0 \\ (d_\mu + i\frac{\tau^i}{2} W_\mu^i + i\frac{\gamma_i}{2} B_\mu) \frac{(v+\eta)}{\sqrt{2}} \begin{pmatrix} W_\mu^+ \\ -1 \\ Z_\mu \end{pmatrix} \end{pmatrix} \right|^2$$

$$= -g W_\mu^3 + g' B_\mu$$

$$= \frac{-g}{\cos\theta_w} Z_\mu$$

REMINDER (cont.)

Why do we write W_n^+ as $2(W_n^+ - iW_n^2)$?

→ weak current for lepton l :

$$J_n^+ = \bar{l} \gamma_n (1 - \gamma_5) \nu = 2 \bar{l}_L \gamma_n \nu_L$$

The charged weak current can be written as

$$J_n^+ = \bar{L} \gamma_n \frac{\tau^+}{2} L$$

Explicitly:

$$J_n^+ = \frac{1}{2} (\bar{\nu}_L \bar{l}_L) \gamma_n \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ l_L \end{pmatrix} = \frac{1}{2} (\bar{l}_L \gamma_n \nu_L + \bar{\nu}_L \gamma_n l_L)$$

$$J_n^2 = \frac{1}{2} (\bar{\nu}_L \bar{l}_L) \gamma_n \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ l_L \end{pmatrix} = \frac{i}{2} (\bar{l}_L \gamma_n \nu_L - \bar{\nu}_L \gamma_n l_L)$$

$$J_n^3 = \frac{1}{2} (\bar{\nu}_L \bar{l}_L) \gamma_n \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \nu_L \\ l_L \end{pmatrix} = \frac{1}{2} (\bar{\nu}_L \gamma_n \nu_L - \bar{l}_L \gamma_n l_L)$$

Z DECAY (cont.)

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$$\Gamma(Z \rightarrow e^+e^-) = \Gamma(Z \rightarrow \nu\bar{\nu}) [(2 \sin^2 \theta_w - 1)^2 + (2 \sin^2 \theta_w)^2]$$

$$\begin{aligned} & [2s_w^2 + 2s_w^2 \gamma_f + 2s_w^2 - 2s_w^2 \gamma_f - 1 + \gamma_f] \\ & = [2s_w^2 + (2s_w^2 - 1) - \gamma_f] \end{aligned}$$

$\rightarrow \gamma_f$ contribution will vanish as in W case

We have $M_W \approx 80.4 \text{ GeV}$

$$\Gamma(W \rightarrow e\nu) \approx 225 \text{ GeV}$$

$$M_Z \approx 91.2 \text{ GeV}$$

$$\Gamma(Z \rightarrow e^+e^-) \approx 84 \text{ GeV}$$

\leftarrow -note

$$\Gamma(Z \rightarrow \nu\bar{\nu}) \approx 166 \text{ GeV}$$

How do we measure $Z \rightarrow \nu\bar{\nu}$?

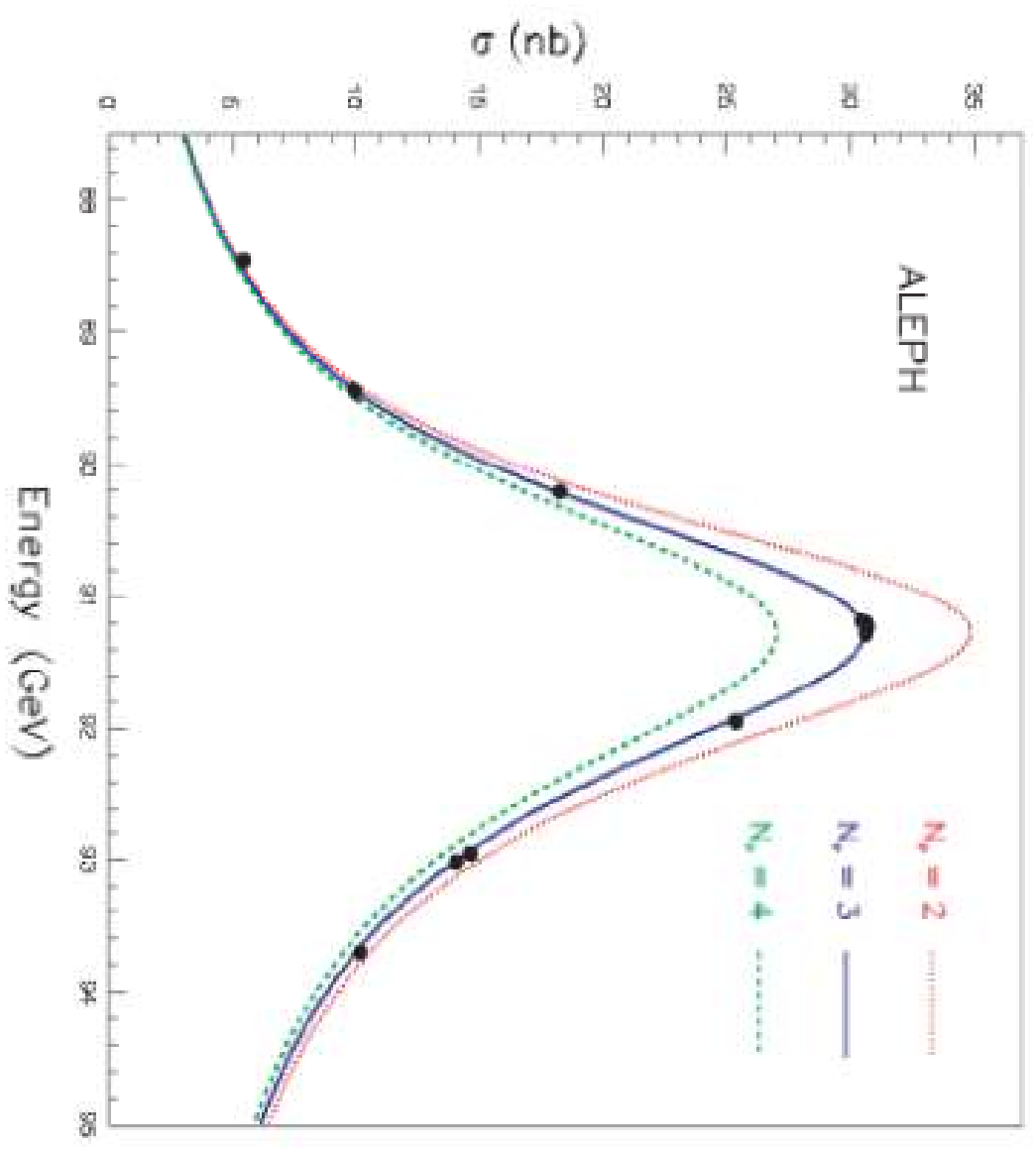
Z decay (cont.)

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Most precise method is "indirect": look at
2 linesshape:

Obtain Γ_{TOT} from
linesshape. Subtract
 $\Gamma_{had}, \Gamma_{e,\mu,\tau}$ and
get $\Gamma_{inv} \sim 500 \text{ MeV}$

\Rightarrow 3 neutrino
families below
 $\frac{M_Z}{2}$

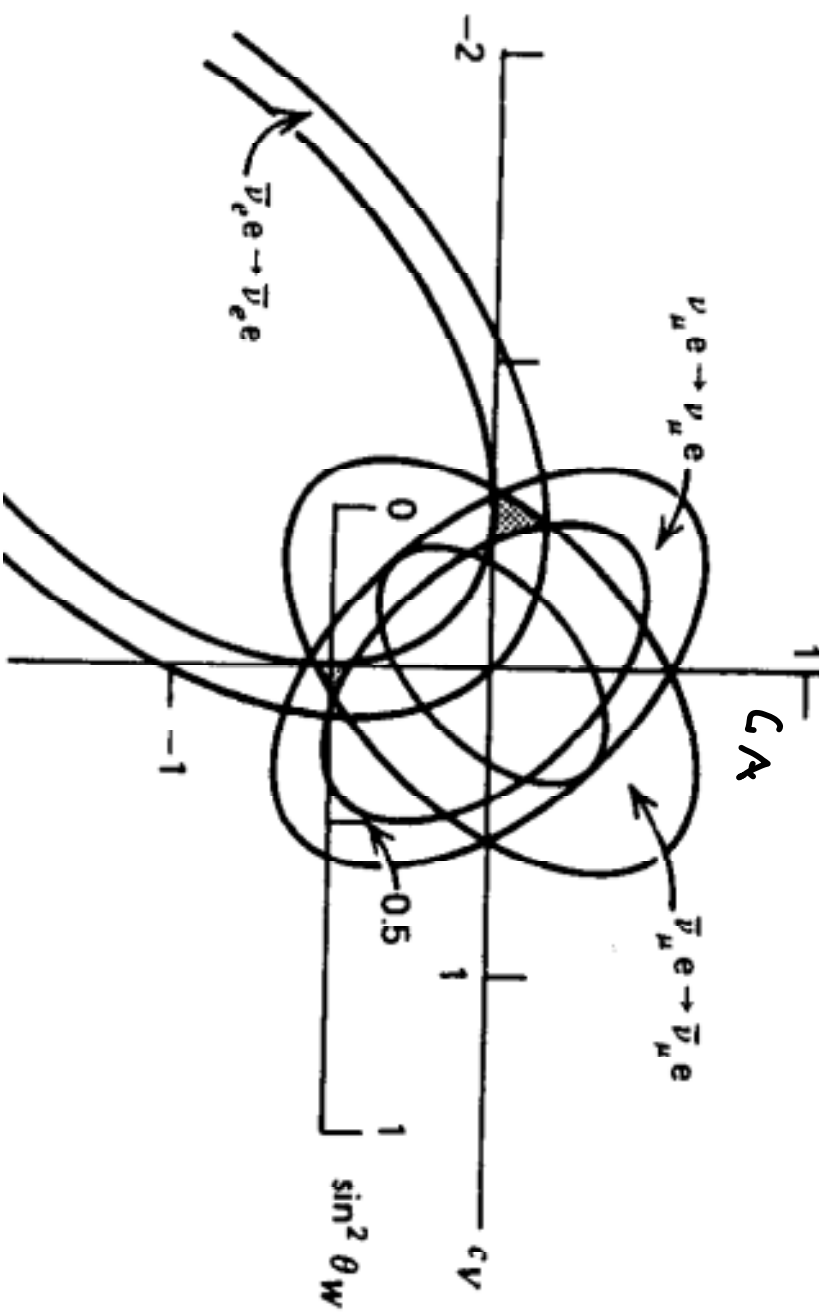


2 Boson Physics

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Problem set #2 problem #3

→ Calculate σ (etc $\rightarrow Z \rightarrow \mu^+ \mu^-$)

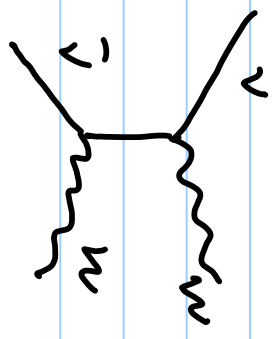


PROBLEM SET #2 due MARCH 14th noon

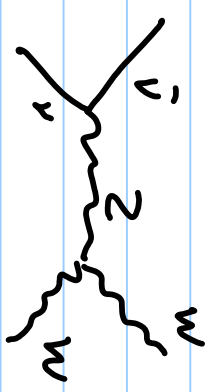
PROBLEM 1

In lecture 9 we saw that the $\bar{\nu} \rightarrow W W$ cross section was not well behaved at high energies:

$$\sigma \sim \frac{G_F^2 s}{3\pi}$$



We know we were missing one diagram:



$$M_2 = \frac{-ig^2}{4(s-M_Z^2)} \bar{\nu} \gamma_\mu (1-\gamma_5) \nu (g_{\mu\nu} - \frac{p_\mu p_\nu}{M_Z^2}) \times \epsilon_+^{*\alpha} \epsilon_-^{*\beta}$$

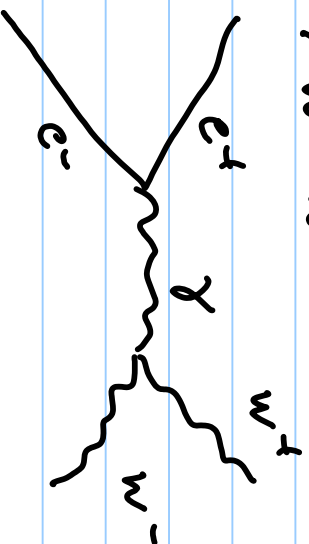
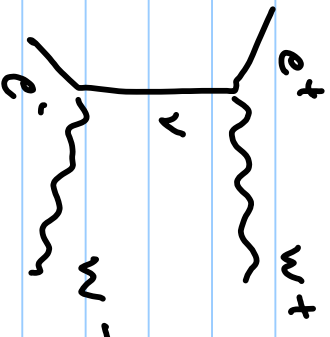
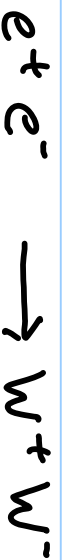
$$[g_{\mu\alpha}(k_- - k_+)^\nu + g_{\alpha\nu}(k_+ + p)_\beta - g_{\beta\nu}(k_- + p)_\alpha]$$

PROBLEM SET 2, PROBLEM #1:
show that extra diagram cures our problem.

PROBLEM #2

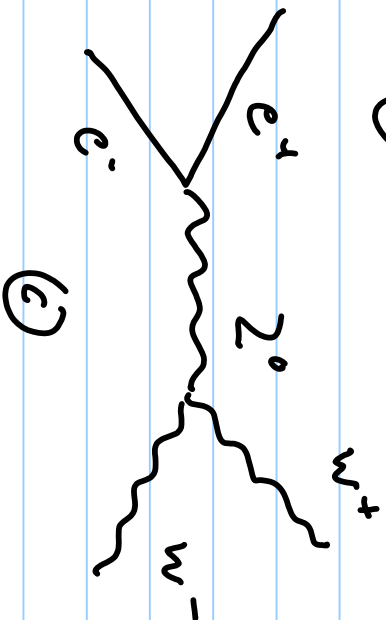
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Another interesting case is provided by the reaction

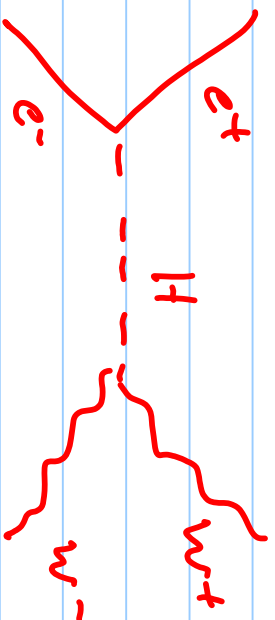


(a)

(b)



(d)



PROBLEM SET 2, PROBLEM #2 : show that you need diagram (d) To cancel the divergence in the cross section (you'll need To keep the electron mass).

problem #3

Calculate σ (etc $\rightarrow Z \rightarrow \mu + \mu^-$)