LECTURE 13: Z Boson Physics and Neutral Currents (Part 2)

Overview:

-A closer look at the structure of the neutral current

-Neutral currents and neutrino scattering

-Forward-backward asymmetries

(I used Quigg and Halzen-Martin as references)

NEUTRAL CURRENT

We wrote the NC as:

$$-\frac{g}{2\cos\theta_W} \sum_{f_i = \nu, l} \bar{f}_i \gamma^\mu \left(g_V^i - g_A^i \gamma^5 \right) f_i Z_\mu$$

$g_A^i \rightarrow T_3^i$

$g_V^i \rightarrow T_3^i - 2Q_i \sin^2\theta_W$

Electrons: $T_3 = -1/2$, $Q = -1$

neutrinos: $T_3 = 1/2$, $Q = 0$

$\Rightarrow$ electrons: $\frac{-g}{2\cos\theta_W} \bar{e} \gamma^\mu \left[-1/2 + 2\sin^2\theta_W + \frac{1}{2}\gamma^5 \right] e Z_\mu$ ①

we could write: ① $= \frac{+g}{2\cos\theta_W} \cdot \frac{1}{2} \cdot \bar{e} \gamma^\mu \left[1 - 4\sin^2\theta_W - \gamma^5 \right] e Z_\mu$

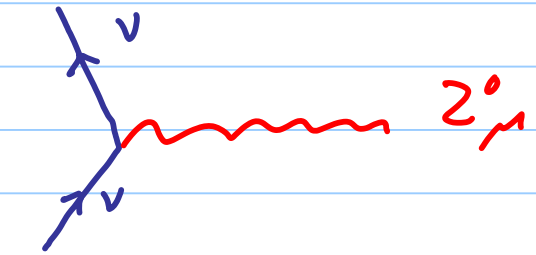
or $= \frac{-g}{2\cos\theta_W} \frac{1}{2} \bar{e} \gamma^\mu \left[-1 + 4\sin^2\theta_W + \gamma^5 \right] e Z_\mu$

$= \frac{-g}{2\cos\theta_W} \cdot \frac{1}{2} \cdot \bar{e} \gamma^\mu \left[\underbrace{2\sin^2\theta_W (1+\gamma^5)}_{R_e} + \underbrace{(2\sin^2\theta_W - 1)(1-\gamma^5)}_{L_e} \right] e Z_\mu$

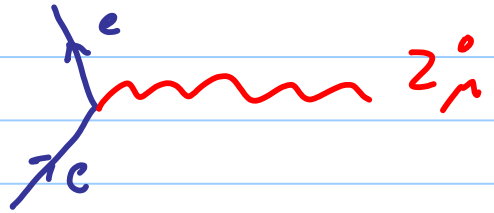
$\Rightarrow$ neutrinos: $\frac{-g}{2\cos\theta_W} \bar{\nu} \gamma^\mu \frac{1}{2} (1-\gamma^5) \nu Z_\mu$

NEUTRAL CURRENT (cont.)

FROM WHICH WE GET THE VERTEX FACTORS:



$$\frac{-i}{\sqrt{2}}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)^{1/2} \bar{\nu}\gamma_\mu(1-\gamma_5)\nu$$



$$\frac{-i}{\sqrt{2}}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)^{1/2} \bar{e}\gamma_\mu\left[2s_W^2(1+\gamma_5) + (2s_W^2-1)(1-\gamma_5)\right]e$$

$s_W = \sin\theta_W$

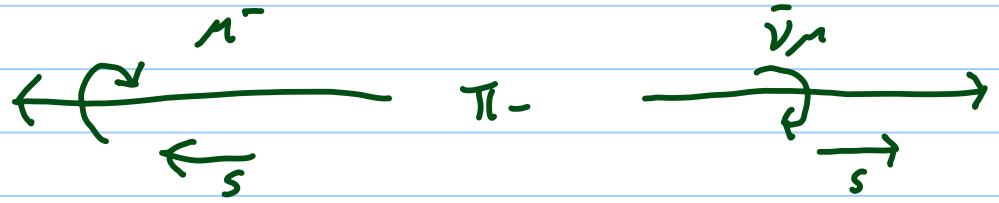
Pion decay interlude:

Pion decay width: $\Gamma = \frac{f_\pi^2}{\pi M_\pi^3}\left(\frac{g_W}{4M_W}\right)^4 m_\ell^2 (M_\pi^2 - m_\ell^2)^2$

$$\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} = 1.3 \times 10^{-4}$$

why?

Hint: pion has spin 0, $\bar{\nu}$ is right-handed $\Rightarrow$ electron must be right-handed...



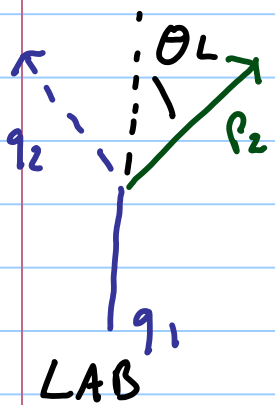
→ have To Take some care when we Talk about "handedness" and helicity. Helicity is not a Lorentz invariant. Also note that the required addition of ν_R in the SM has almost no impact on the results we've obtained so far since m_ν is so small.

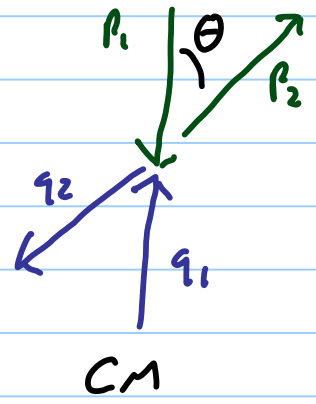
Consider The reactions:

$\bar{\nu}_e e \to \bar{\nu}_e e$] FROM REACTORS

$\bar{\nu}_\mu e \to \bar{\nu}_\mu e$] FROM ACCELERATORS

$\nu_\mu e \to \nu_\mu e$





NEUTRAL CURRENTS (cont.)

4 vectors: in LAB

$q_1^\mu = (E, 0, 0, E)$

$p_1^\mu = (m, 0, 0, 0)$

$q_2^\mu = (E + m - E', -p\sin\theta_L, 0, -p\cos\theta_L)$

$p_2^\mu = (E', p\sin\theta_L, 0, p\cos\theta_L)$

$E' = \sqrt{p'^2 + m^2} \equiv yE$

in CM

$q_1^\mu = (p^*, 0, 0, p^*)$

$p_1^\mu = (w^*, 0, 0, -p^*)$

$q_2^\mu = (p^*, -p^*\sin\theta, 0, -p^*\cos\theta)$

$p_2^\mu = (w^*, p^*\sin\theta, 0, p^*\cos\theta)$

Invariants: in LAB

$(p_1 \cdot q_1) = (p_2 \cdot q_2) = mE$

$(p_1 \cdot q_2) = (p_2 \cdot q_1) \approx mE(1-y)$

$(p_1 \cdot p_2) = myE$

$(q_1 \cdot q_2) = m(E' - m) \simeq mEy$

in CM

$= p^*(w^* + p^*) \approx 2p^{*2}$

$= p^*(w^* - p^*\cos\theta) \approx p^{*2}(1 - \cos\theta)$

$\simeq p^{*2}(1 + \cos\theta)$

$= p^{*2}(1 + \cos\theta)$

NEUTRAL CURRENTS (CONT.)

NOTE FROM THE ABOVE THAT

$$mE \approx 2p^{2*}$$

$$p^{*2}(1-\cos\theta) \simeq mE(1-y) \Rightarrow \boxed{1-\cos\theta = 2(1-y)}$$

In Quigg's book, a general matrix element for charged-current interaction is studied: $\Sigma M_i = \Sigma C_i \bar{\nu} O_i e \bar{e} O_i (1-\gamma_5)\nu$
O_i can be vector, scalar, axial vector, Tensor, pseudoscalar. We use some of the results in the following.

V-A interaction for $\nu_e e \to \nu e$ (we'll deal with constants and propagators and other details later)

$$M = \bar{u}_\nu \gamma_\mu (1-\gamma_5) u_e \; \bar{u}_e \gamma_\mu (1-\gamma_5) u_\nu$$

$$|M|^2 = Tr[\gamma_\mu (1-\gamma_5)(\not{p}_1 + m)\gamma_\nu (1-\gamma_5)\not{q}_2]$$

$$\times Tr[\gamma^\mu (1-\gamma^5)\not{q}_1 \gamma^\nu (1-\gamma^5)(\not{p}_2 + m)]$$

NEUTRAL CURRENTS (cont.)

TRACE 1: $2\mathrm{Tr}\left[(1+\gamma_5)\gamma_\nu \not{q}_2 \gamma_\mu (\not{p}_1 + m)\right]$

$= 8\left(q_{2\nu} p_{1\mu} - g_{\mu\nu}(q_2\cdot p_1) + q_{2\mu} p_{1\nu}\right) - 8\,\varepsilon_{\mu\nu\rho\sigma} q_2^\rho p_1^\sigma$

TRACE 2: $2\mathrm{Tr}\left[(1+\gamma^5)\gamma^\mu \not{q}_1 \gamma^\nu (\not{p}_2 + m)\right]$

$= 8\left(q_1^\mu p_2^\nu - g^{\mu\nu}(q\cdot p_2) + q_1^\nu p_2^\mu + 8\,\varepsilon^{\mu\nu\kappa\lambda} q_{1\kappa} p_{2\lambda}\right.$

$|\mathcal{M}|^2 = 128\left(q_1\cdot q_2\, p_1\cdot p_2 + q_1\cdot p_1\, q_2\cdot p_2\right)$

$-64\, i\,\varepsilon_{\mu\nu\rho\sigma} q_2^\rho p_1^\sigma \left(q_1^\nu p_2^\mu + q_1^\mu p_2^\nu\right)$

$+64\, i\,\varepsilon^{\mu\nu\kappa\lambda}\, q_{1\kappa} p_{2\lambda}\left(q_{2\nu} p_{1\mu} + q_{2\mu} p_{1\nu}\right)$ $= 0$ (3 indep. momenta)

$+64\,\varepsilon_{\mu\nu\rho\sigma}\,\varepsilon^{\mu\nu\kappa\lambda}\, q_2^\rho p_1^\sigma q_{1\kappa} p_{2\lambda}$

$= 128\left(q_1\cdot p_1\, q_2\cdot p_2 - q_1\cdot q_2\, p_1\cdot p_2\right)$

$\Rightarrow |\mathcal{M}|^2 = 256\, q_1\cdot p_1\, q_2\cdot p_2 = 256\,(mE)^2$

NEUTRAL CURRENTS

$$\frac{d\sigma}{d\Omega_{cm}} = \frac{|M|^2}{64\pi^2 s}, \qquad s = (p_1+q_1)^2 \simeq 2(p_1\cdot q_1)$$

$$= 4p^{*2} = 2mE$$

note factor of $\frac{1}{2}$ for average spin of e

$$\sigma = \frac{4mE}{\pi} \qquad \text{note that} \qquad \frac{d\sigma}{dy} = \frac{4mE}{\pi}$$

$$\frac{d\sigma}{dy} = 4\pi \frac{d\sigma}{d\Omega}$$

V-A interaction for $\bar{\nu}_e e \to \bar{\nu}_e e$

we need to change $q_1 \longleftrightarrow q_2$

this give $|M|^2 = 256\, q_2\cdot p_1\, q_1\cdot p_2$

$$= 256\,(mE)^2\,(1-y)^2$$

NEUTRAL CURRENTS (CONT.)

$$\frac{d\sigma}{dy} = 256\,(mE)^2\,(1-y)^2 \cdot \frac{4\pi}{64\pi^2 \cdot 2mE} \cdot \frac{1}{2}$$

$$= \frac{4mE}{\pi}(1-y)^2 \quad , \quad \sigma = \int_0^1 \frac{4mE}{\pi}(1-y)^2$$

$$= \frac{4mE}{3\pi}$$

V+A for $\nu_\mu e \to \nu_\mu e$

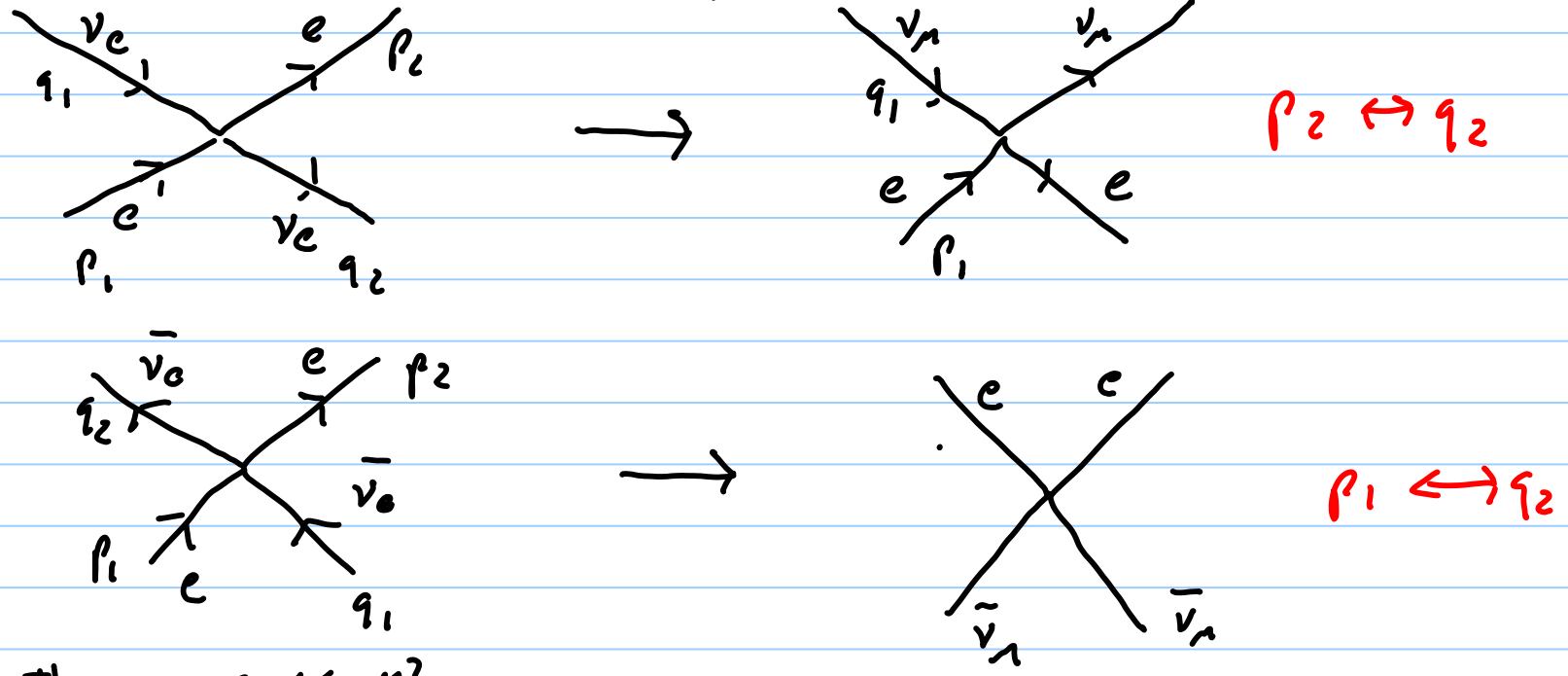
$$M = \bar{u}_\nu \gamma_\mu (1+\gamma_5) \nu_e \; \bar{u}_e \gamma_\mu (1-\gamma_5) \nu_e$$

This changes the sign of the $\varepsilon_{\mu\nu\kappa\sigma}$ Term

which gives $|M|^2 = 256\, q_1 \cdot q_2\, p_1 \cdot p_2 = 256\,(mE)^2 y^2$

$$\frac{d\sigma}{dy} = \frac{4mE}{\pi} y^2 \quad \to \quad \sigma = \frac{4mE}{3\pi}$$

Now, To get actual SM processes with $s \ll M_Z^2$:



with $s \ll M_Z^2$

We get:

$$\frac{\pi}{mE}\frac{d\sigma}{dy}(\nu_\mu e \to \nu_\mu e) \qquad \frac{\pi}{mE}\frac{d\sigma}{dy}(\bar{\nu}_\mu e \to \bar{\nu}_\mu e)$$

$\lvert V-A\rvert^2$	4	$4(1-y)^2$
$\lvert V+A\rvert^2$	$4(1-y)^2$	4

Neutral currents (cont.)

Using $\frac{-i}{\sqrt{2}}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)^{1/2} \bar{e}\gamma_\mu [R_e(1+\gamma_5) + L_e(1-\gamma_5)]e$

The processes $e\nu_\mu \to e\nu_\mu$, $e\bar{\nu}_\mu \to e\bar{\nu}_\mu$ give:

$$\frac{d\sigma}{dy}(\nu_\mu e) = \frac{4mE}{\pi}\frac{G_F^2}{8}\left[L_e^2 + R_e^2(1-y)^2\right]$$

(cross terms cancel $\to$ $(1-\gamma_5)(1+\gamma_5) = 0$)

$$= \frac{G_F^2 mE}{2\pi}\left[(2\sin^2\theta_W - 1)^2 + 4\sin^4\theta_W (1-y)^2\right]$$

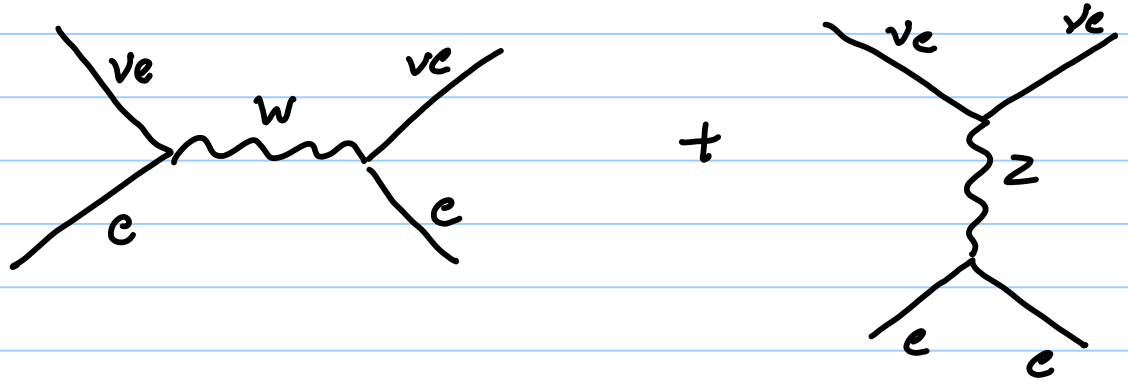
$$\frac{d\sigma}{dy}(\bar{\nu}_\mu e) = \frac{G_F^2 mE}{2\pi}\left[L_e^2(1-y)^2 + R_e^2\right]$$

$$\sigma(\nu_\mu e) = \frac{G_F^2 mE}{2\pi}\left(L_e^2 + \frac{R_e^2}{3}\right)$$

$$\sigma(\bar{\nu}_\mu e) = \frac{G_F^2 mE}{2\pi}\left(\frac{L_e^2}{3} + R_e^2\right)$$

NEUTRAL CURRENTS AND NEUTRINO SCATTERING

$\nu_e e \to \nu_e e$: Two contributions



Fierz reordering Theorem:

$$-[\bar{e}\gamma^\mu(1-\gamma^5)\nu_e][\bar{\nu}_e\gamma_\mu(1-\gamma_5)e] = [\bar{\nu}_e\gamma^\mu(1-\gamma^5)\nu_e][\bar{e}\gamma_\mu(1-\gamma_5)e]$$

↳ note sign

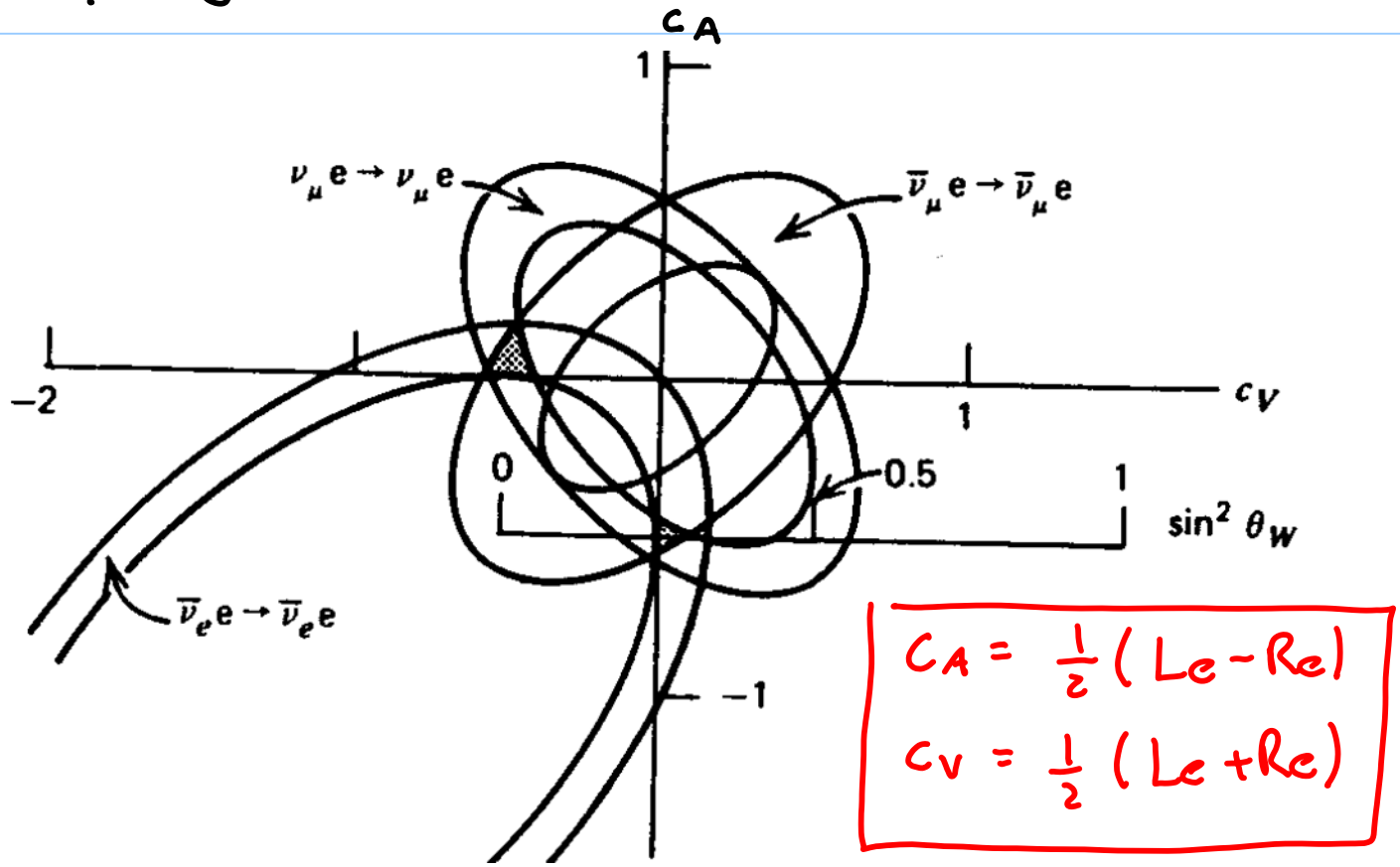
$\nu_e e \to \nu_e e$ is obtained from $\nu_\mu e \to \nu_\mu e$ with: $L_e \to L_e + 2$

and $\bar{\nu}_e e$ is obtained from $\bar{\nu}_\mu e \to \bar{\nu}_\mu e$ with

we have: $\sigma(\bar{\nu}_e e \to \bar{\nu}_e e) = \frac{G_F^2 m E}{2\pi}\left[\left(\frac{L_e+2}{3}\right)^2 + R_e^2\right]$

Neutral currents (cont.)

Putting everything together with experimental results we get:



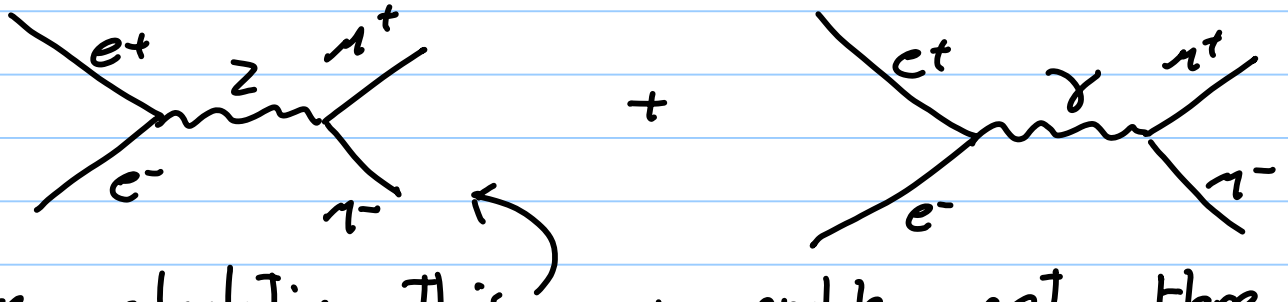
$$c_A = \frac{1}{2}(L_e - R_e)$$

$$c_V = \frac{1}{2}(L_e + R_e)$$

Note that there are two solution.
How do we determine which one is correct?

NEUTRAL CURRENTS (CONT.)

We can study $e^+e^- \to \mu^+\mu^-$ for example. Two diagrams contribute:

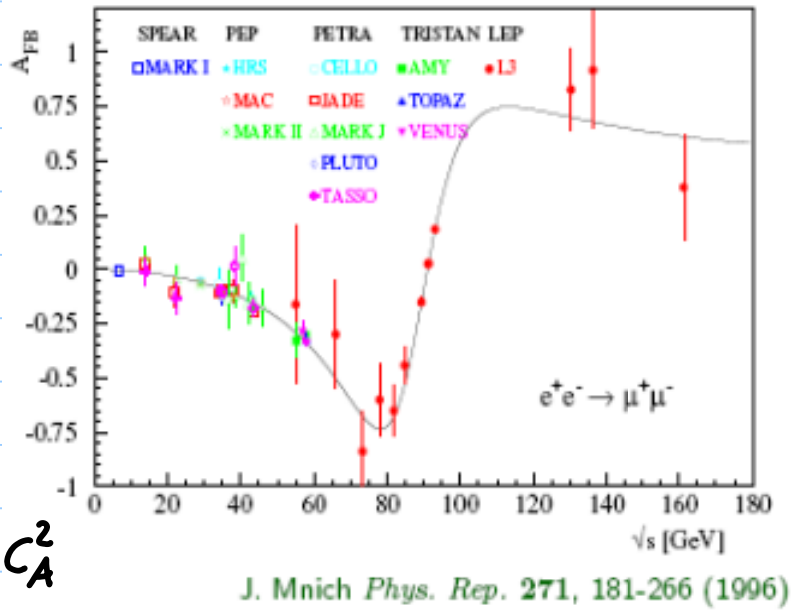


You are calculating this in problem set. Here we need the γ and γZ terms too.

We then measure the forward-backward asymmetry defining $z = \cos\theta_{cm}$

$$A_{FB} \equiv \frac{\int_0^1 dz \frac{d\sigma}{dz} - \int_{-1}^0 dz \frac{d\sigma}{dz}}{\int_{-1}^1 dz \frac{d\sigma}{dz}}$$

$$\lim_{s/M_Z^2 \to 0} A_{FB} \propto (R_e - L_e)(R_\mu - L_\mu) \propto C_A^2$$



J. Mnich *Phys. Rep.* **271**, 181-266 (1996)

Resolves ambiguity, measures $\sin^2\theta_W$