LECTURE 17: Hadron Structure (Part 3) and QCD (Part 1)

Overview:

-Deep Inelastic Scattering (charged current)

-Deep Inelastic Scattering (neutral current)

(I used Quigg and Giffiths as references)

Deep Inelastic scattering (charged current)

In your first problem (in problem set #3), you calculate the cross section for the reaction:

$$\nu + N \rightarrow \mu + X$$

→ lepton tensor now has (V-A) structure

The result is:

$$\frac{d^2\sigma^\nu}{dQ^2 d\nu} = \frac{G_F^2}{2\pi}\frac{E'}{E}\left[2W_1^\nu \sin^2\left(\frac{\theta}{2}\right) + W_2^\nu \cos^2\left(\frac{\theta}{2}\right) + W_3^\nu \frac{(E+E')}{M}\sin^2\left(\frac{\theta}{2}\right)\right]$$

In terms of x and y:

$$\frac{d^2\sigma^\nu}{dx\,dy} = \frac{G_F^2}{\pi} ME\left[F_1(x)\, xy^2 + F_2(x)(1-y) \pm F_3\, xy\,(1-y/2)\right]$$

$$F_3 \equiv \nu W_3^\nu(x)$$

Parton model prediction: $\boxed{\sigma_{Total}(\nu N \rightarrow \ell + X) \propto E}$

Deep Inelastic scattering (charged current)

$$\frac{\sigma}{E}(\nu N \to \ell X) \simeq 6 \times 10^{-39} \ \text{cm}^{-2} / \text{GeV}$$

$$\frac{\sigma}{E}(\bar{\nu} N \to \bar{\ell} X) \simeq 3 \times 10^{-39} \ \text{cm}^{-2} / \text{GeV}$$

Also:

$$\frac{x F_3(x)}{F_2(x)} = \begin{array}{ll} +1 & \text{Fermion Target} \\ -1 & \text{anti-F Target} \end{array}$$

Combine with $2x F_1(x) = F_2(x)$ for spin ½ partons

gives:

$$\frac{d^2\sigma}{dx\,dy}(\nu q) = \frac{G_F^2 ME}{\pi} F_2(x)$$

$$\frac{d^2\sigma}{dx\,dy}(\bar{\nu} q) = \frac{G_F^2 ME}{\pi} (1-y)^2 F_2(x)$$

why?

→ reproduces the form of previous xsections with charged current interaction.

Deep Inelastic scattering (charged current)

with the approx.: $\cos\theta_c = 1$

$$F_2^{\nu}(x) = 2x\,(d(x) + \bar{u}(x))$$

$$F_3^{\nu}(x) = 2\,(d(x) - \bar{u}(x))$$

and

$$F_2^{\bar{\nu}}(x) = 2x\,(u(x) + \bar{d}(x))$$

$$F_3^{\bar{\nu}}(x) = 2\,(u(x) - \bar{d}(x))$$

Note that:

$$F_2^{\nu p} + F_2^{\nu n} = 2x\,(u(x) + \bar{u}(x) + d(x) + \bar{d}(x))$$

is proportional to: $\dfrac{F_2^{ep} + F_2^{en}}{F_2^{\nu p} + F_2^{\nu n}} = \dfrac{5}{18}$ (neg. s quarks)

→ ~ gluons carry ~ 50% of proton momentum which does not interact through weak and em forces

Deep Inelastic scattering (charged current)

→ also note that F_3^{ν} measures the difference between quark and anti-quark contributions and F_2 measures the sum.

$$F_3^{\nu p} + F_3^{\nu n} = 2(u - \bar{u} + d - \bar{d})$$

→ baryon number sum rule : $\int_0^1 dx \left(F_3^{\nu p}(x) + F_3^{\nu n}(x)\right) = 6$

Consider a nucleon with $N \equiv \frac{1}{2}(p+n)$

the charged xs are:

$$\frac{d^2\sigma}{dx\,dy}(\nu N \to \mu^- X) = \frac{G_F^2 ME}{\pi} \times \left[u(x) + d(x) + (\bar{u}(x) + \bar{d}(x))(1-y)^2\right]$$

$$\frac{d^2\sigma}{dx\,dy}(\bar{\nu} N \to \mu^+ X) = \frac{G_F^2 ME}{\pi} \times \left[\bar{u}(x) + \bar{d}(x) + (u(x) + d(x))(1-y)^2\right]$$

Deep Inelastic scattering (neutral current)

For neutral currents:

$$\frac{d\sigma^2}{dx\,dy}(\nu N \to \nu X) = \frac{G_F^2 ME}{4\pi} \times \left[(L_u^2 + L_d^2)\left[(u(x)+d(x)) + (\bar{u}(x) + \bar{d}(x))(1-y)^2\right] + (R_u^2 + R_d^2)\left[(u(x)+d(x))(1-y)^2 + (\bar{u}(x)+\bar{d}(x))\right]\right]$$

$$\frac{d\sigma^2}{dx\,dy}(\bar{\nu} N \to \bar{\nu} X) = \ldots$$ ↗ except swap $(1-y)^2$ Terms

if we neglect anti-quark Terms we obtain the ratios:

$$R_\nu \equiv \frac{\sigma(\nu N \to \nu X)}{\sigma(\nu N \to \mu^- X)} = \frac{1}{2} - x_w + \frac{20}{27} x_w^2$$

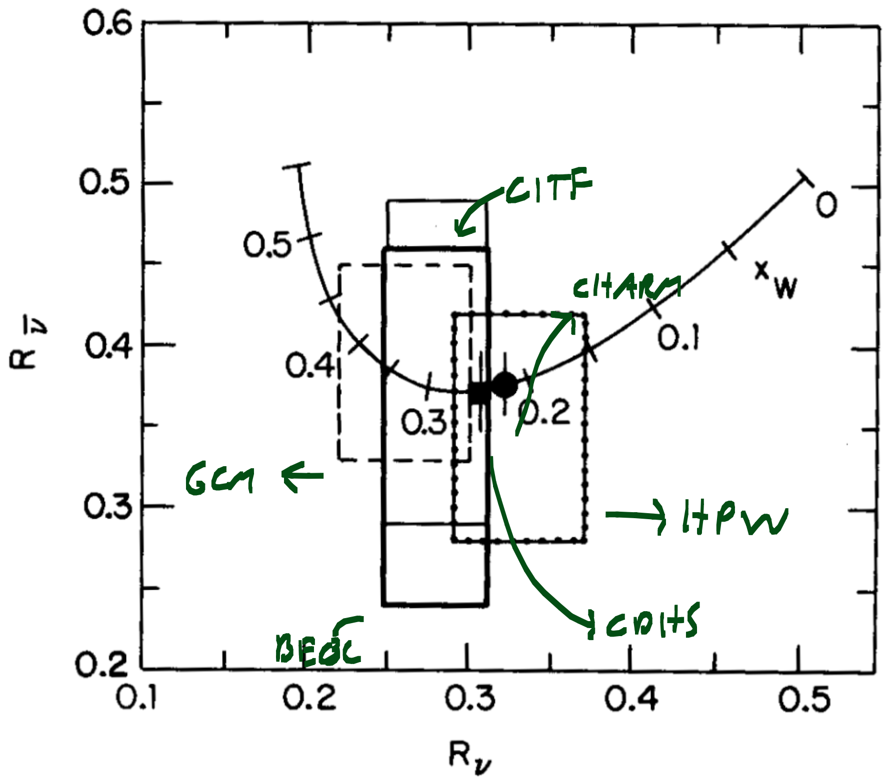
$$R_{\bar{\nu}} \equiv \frac{1}{2} - x_w + \frac{20}{9} x_w^2$$

$x_w \simeq 0.23 \quad \to \quad R_\nu = 0.31$

$R_{\bar{\nu}} = 0.39$

Deep Inelastic scattering (charged current)

Ratios of R_ν and $R_{\bar\nu}$ from various experiments:



We can also observe γ-Z interference effects in deep inelasitic scattering.

$$\frac{d^2\sigma}{dx\,dy}(ep \to eX) = \text{em Term} + \text{weak Term} + \text{interference Term}$$

Deep Inelastic scattering (neutral currents)

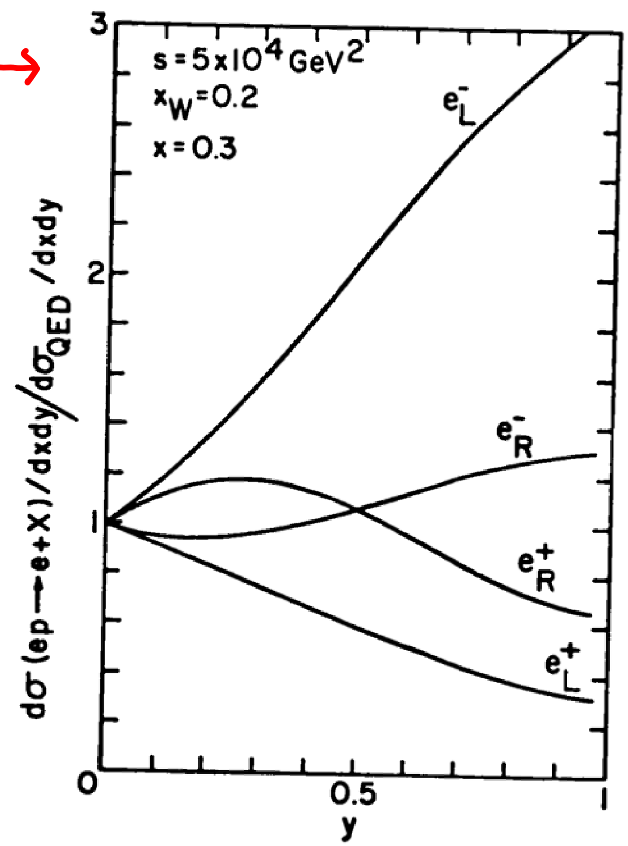
Asymmetry for $Q^2 \ll M_Z^2$:

$$A = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \simeq \frac{-G_F Q^2}{4\pi\alpha\sqrt{2}} \cdot \frac{9}{5}\left(1 - \frac{20}{9}x_w\right)$$

$$\frac{A}{Q^2} \simeq -7 \times 10^{-5}$$ → small, observed by SLAC-YALE collaboration

At higher energies →

effect becomes of order 1



Polarized high energy beams were used at DESY.

HADRON-HADRON INTERACTIONS

$$\sigma(a+b \to c+X) = \sum_{i,j} F_i^{(a)}(x_a)\, F_j^{(b)}(x_b)\, \hat{\sigma}(i+j \to c+X')$$

- Invariant mass of i,j system $M = \sqrt{s\tau}$

  ↳ dimensionless parameter

i,j system longitudinal momentum in hadron-hadron in c.m.:

$$p = x\sqrt{s}/2$$

- $$x_{a,b} = \frac{1}{2}\left[(x^2 + 4\tau)^{1/2} \pm x\right]$$

DRELL-YAN PRODUCTION

$$a+b \to \ell^+\ell^- + X \qquad q+\bar{q} \to \gamma \to \ell^+\ell^-$$

invariant mass M

HADRON-HADRON INTERACTIONS

diff. cross section given by:

$$\frac{d\sigma}{dM^2 dx} = \left(\frac{4\pi\alpha^2}{3M^4}\right) F(\tau, x)$$

$\left(\frac{4\pi\alpha^2}{3M^4}\right)$ ⟶ as in $e^+e^- \to \mu^+\mu^-$

$$F(\tau, x) = \frac{x_a x_b}{(x^2 + 4\tau)^{1/2}} \, g(x_a, x_b)$$

$$\to g(x_a, x_b) = \frac{1}{3} \sum_i e_i^2 \left[q_i^a(x_a) \, \bar{q}_i^b(x_b) + \bar{q}_i^a(x_a) + q_i^b(x_b) \right]$$