LECTURE 19: QCD (Part II)

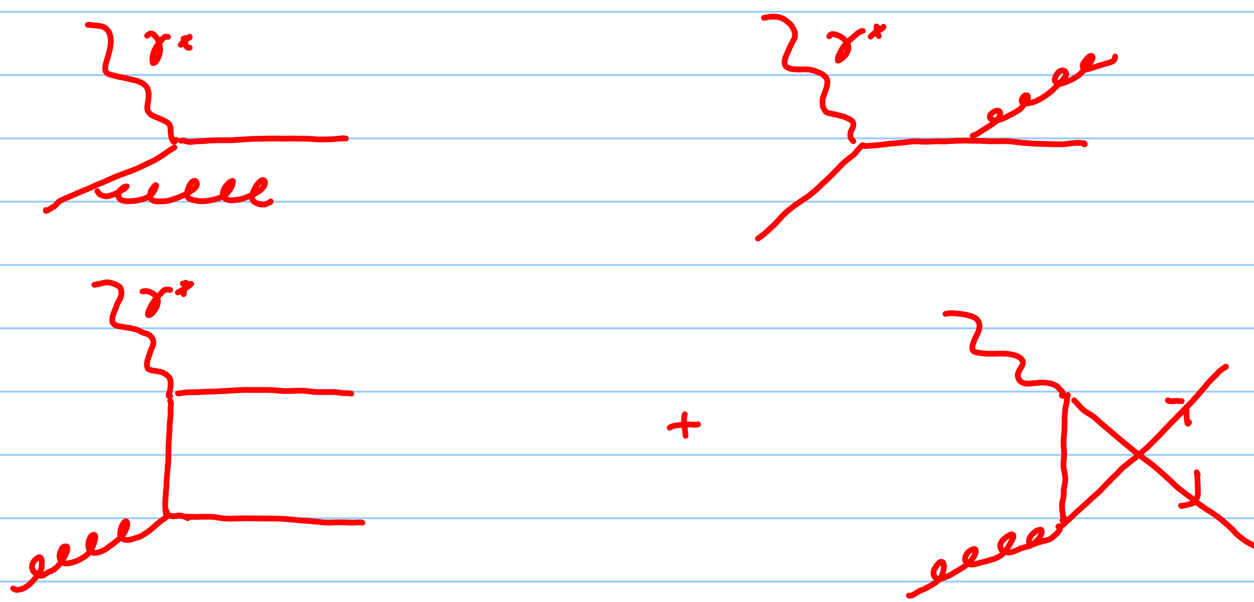
Overview:

- QCD corrections to DIS
- Gluon emission

(I used Halzen-Martin and Quigg as a reference)

DIS and QCD

Adding QCD corrections will change previous result:



DIS and QCD (cont.)

Corrections will change previous results:

→ scaling will not be true

→ quark direction not colinear with photon

We had:

$$F_1 = M W_1(\nu, Q^2)$$

$$F_2 = \nu W_2(\nu, Q^2)$$

$$\nu \equiv \frac{p \cdot q}{M} \quad , \quad Q^2 = -q^2$$

→ F_1, F_2 no longer scale since they will be functions of both ν and Q^2 instead of the ratio: $x = \dfrac{Q^2}{2M\nu}$

DIS and QCD (cont.)

We can interpret the structure functions in terms of virtual photon - proton cross sections. In DIS

$$2F_1 = \frac{\sigma_T}{\sigma_0}$$

$$\sigma_0 \equiv \frac{4\pi^2\alpha}{2MK} \approx \frac{4\pi^2\alpha}{s}$$

$$\frac{F_2}{x} = \frac{\sigma_T + \sigma_L}{\sigma_0}$$

→ xs for long. and transversely polarized virtual photons.

We need to express the above as virtual photon - parton xs:

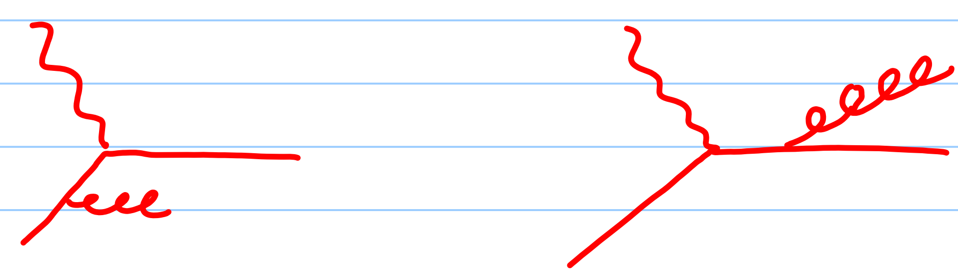
γ^* - proton	γ^* - parton
p	$p_i = yp$
$x = \dfrac{Q^2}{2p\cdot q}$	$z = \dfrac{Q^2}{2p_i\cdot q} = \dfrac{x}{y}$

DIS and QCD (cont.)

$$\left(\frac{\sigma_T}{\sigma_0}(x,Q^2)\right)_{\gamma^* p} = \sum_i \int_0^1 dz \int_0^1 dy\, f_i(y)\, \delta(x-zy) \left(\frac{\hat\sigma_T}{\hat\sigma_0}(z,Q^2)\right)_{\gamma^* i}$$

after integration: $\displaystyle \frac{\sigma_T}{\sigma_0}(x,Q^2) = \sum_i \int_x^1 \frac{dy}{y} f_i(y) \left(\frac{\hat\sigma_T}{\hat\sigma_0}\left(\frac{x}{y},Q^2\right)\right)$

Gluon emission cross section $\gamma^* q \to q g$



→ similar to Compton scattering : $\gamma^* e \to e\gamma$

$$\overline{|M|}^2 = 32\pi^2\alpha^2 \left(\frac{-u}{s} - \frac{s}{u} + \underbrace{2\frac{tQ^2}{su}}_{\text{virtual photon result}}\right)$$

DIS and QCD (cont.)

For $\gamma^* q \to q g$, we substitute

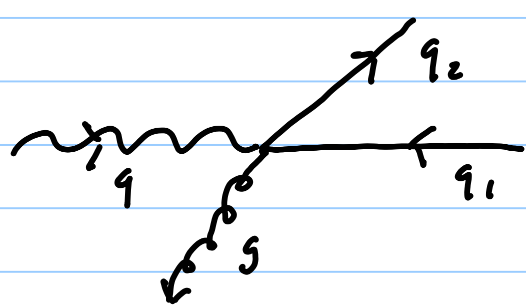
- $\alpha^2 \to e_i^2 \alpha \alpha_s$
- colour factor $\left(4/3 \to \frac{1}{3} \cdot 8 \cdot \underbrace{\frac{1}{2}}_{\text{convention}}\right)$
- $u \longleftrightarrow t$

Some Kinematics:

$\gamma^* q_1 \to q_2 g$

$q\,(q_0, 0, 0, K)$ $\qquad$ $q_2\,(K', K'\sin\theta, 0, K'\cos\theta)$

$q_1\,(K, 0, 0, -K)$ $\qquad$ $g\,(K', -K'\sin\theta, 0, K'\cos\theta)$



$\hat{s} = (q + q_1)^2 = (q_2 + g)$

$\hat{t} = (q - q_2)^2 = (g - q_1)^2$

$\hat{u} = (q_1 - q_2)^2$

DIS and QCD (cont.)

$$-\hat{t} - \hat{u} = Q^2 + 2K' q_0 + 2KK' = Q^2 + 4K'^2 = Q^2 + s$$

$$q_0 = 2K' - K$$

$$\hat{s} = (q_0 + K)^2 = q_0^2 + K^2 + 2q_0 K$$

$$(q_0^2 - K^2) = -Q^2$$

$$\Rightarrow \hat{s} = 2K^2 + 2q_0 K - Q^2$$

$$\hat{s} = (g + q_2)^2 = 4K'^2$$

$$\hat{t} = (q - q_2)^2 = -2KK'(1 - \cos\theta)$$

$$\hat{u} = -2KK'(1 + \cos\theta)$$

$$p_T = K' \sin\theta$$

$$\hat{s}\hat{t}\hat{u} = 4K'^2 (2KK')^2 \sin^2\theta = (4K'K)^2 p_T^2 = (\hat{t} + \hat{u})^2 p_T^2$$
$$= (\hat{s} + Q^2)^2 p_T^2$$

DIS and QCD (cont.)

when $-\hat{t} \ll \hat{s} \longrightarrow -\hat{u} = \hat{s} + Q^2$

$$\Rightarrow p_T^2 = \frac{\hat{s}\hat{t}\hat{u}}{(\hat{s}+Q^2)^2} \longrightarrow p_T^2 = \frac{s(-\hat{t})}{(s+Q^2)}$$

$p_T = k' \sin\theta \quad , \quad p_T^2 = k'^2 \sin^2\theta$

$$d p_T^2 = 2 k'^2 \sin\theta \cos\theta \, d\theta = \frac{\hat{s}}{2} \sin\theta \cos\theta \, d\theta$$

$d\Omega = \sin\theta \, d\theta \, d\varphi$, after φ int.: $d\Omega = 2\pi \sin\theta \, d\theta$

$$\cos\theta \approx 1 \rightarrow d\Omega = 2\pi \, dp_T^2 \cdot \frac{2}{\hat{s}} = \frac{4\pi \, dp_T^2}{\hat{s}}$$

$$\frac{d\hat{\sigma}}{d\Omega} = \frac{1}{64\pi^2 s} \frac{p_f}{p_i} |M|^2$$

$$\Rightarrow \frac{d\hat{\sigma}}{dp_T^2} = \frac{4\pi}{\hat{s}} \frac{1}{64\pi^2 \hat{s}} = \frac{1}{16\pi \hat{s}^2} |M|^2$$

DIS and QCD (cont.)

$$|M|^2 = 32\pi^2 (e_i^2 \alpha\alpha_s) \frac{4}{3} \left(\frac{-\hat{t}}{\hat{s}} - \frac{\hat{s}}{\hat{t}} + \frac{2\hat{u}Q^2}{\hat{s}\hat{t}} \right)$$

with $-\hat{t} \ll s$

$$\frac{d\hat{\sigma}}{dp_T^2} = \frac{8\pi e_i^2}{3} \frac{\alpha\alpha_s}{\hat{s}^2} \cdot \left(\frac{1}{-\hat{t}}\right) \left[\hat{s} - \frac{2\hat{u}Q^2}{\hat{s}} \right]$$

$$-\hat{u} = \hat{s} + Q^2$$

$$\rightarrow \frac{8\pi}{3} e_i^2 \frac{\alpha\alpha_s}{\hat{s}^2} \cdot \left(\frac{1}{-\hat{t}}\right) \left[\hat{s} + \frac{2(\hat{s}+Q^2)Q^2}{\hat{s}} \right]$$

$$= \frac{4\pi^2\alpha}{\hat{s}} \cdot \frac{e_i^2}{2\pi} \cdot \alpha_s \cdot \frac{4}{3} \left[\frac{\hat{s}}{-\hat{t}\hat{s}} + \frac{2(\hat{s}+Q^2)Q^2}{-\hat{s}\hat{t}\hat{s}} \right]$$

$$p_T^2 = \frac{\hat{s}(-\hat{t})}{\hat{s}+Q^2}$$

$$\frac{4\pi^2\alpha}{\hat{s}} \cdot \frac{e_i^2}{2\pi} \alpha_s \cdot \frac{4}{3} \left[\frac{[\hat{s}+Q^2]}{-\hat{t}\hat{s}} \frac{\hat{s}}{[s+Q^2]} + \frac{2(\hat{s}+Q^2)Q^2}{-\hat{s}\hat{t}\hat{s}} \right]$$

DIS and QCD (cont.)

$$= \frac{4\pi^2\alpha}{\hat{s}}\frac{e_i^2}{2\pi}\alpha_s\frac{4}{3}\left[\frac{1}{p_T^2}\frac{\hat{s}}{(s+Q^2)} + \frac{2}{p_T^2}\frac{Q^2}{\hat{s}}\right]$$

$$= \frac{4\pi^2\alpha}{\hat{s}}\frac{e_i^2}{2\pi}\alpha_s\frac{4}{3}\frac{1}{p_T^2}\underbrace{\left[\frac{\hat{s}}{(s+Q^2)} + 2\frac{Q^2}{\hat{s}}\right]}$$

$$= \frac{\hat{s}^2 + 2Q^2(s+Q^2)}{\hat{s}(\hat{s}+Q^2)} = \frac{\hat{s}^2 + 2Q^2 s + 2Q^4}{\hat{s}(\hat{s}+Q^2)}$$

$$z = \frac{Q^2}{2p_i\cdot q} = \frac{Q^2}{\hat{s}+Q^2}, \quad z^2 = \frac{Q^4}{\hat{s}^2+Q^4+2\hat{s}Q^2}$$

$$z^2+1 = \frac{Q^4 + \hat{s}^2+Q^4+2\hat{s}Q^2}{\hat{s}^2+Q^4+2\hat{s}Q^2} = \frac{2Q^4+\hat{s}^2+2sQ^2}{(\hat{s}+Q^2)^2}$$

$$1-z = \frac{\hat{s}+Q^2-Q^2}{\hat{s}+Q^2} = \frac{\hat{s}}{\hat{s}+Q^2}$$

$$\Rightarrow \quad \frac{z^2+1}{1-z}, \quad P_{qq}(z) \equiv \frac{4}{3}\frac{z^2+1}{1-z}$$

DIS and QCD (cont.)

$$\frac{d\hat{\sigma}}{dp_T^2} = e_i^2 \, \hat{\sigma}_0 \, \frac{1}{p_T^2} \, \frac{\alpha_s}{2\pi} \, P_{qq}(z)$$

$\hat{\sigma}_0 \rightarrow \frac{4\pi^2\alpha}{\hat{s}}$ $\qquad P_{qq}(z) \rightarrow \frac{4}{3}\,\frac{z^2+1}{1-z}$

$z = 1 \rightarrow$ divergence (soft massless emission)

$p_T^2 \rightarrow 0 \rightarrow$ colinear divergence

we used $-\hat{t} \ll \hat{s} \rightarrow$ qq dominant