LECTURE 20 and 21: QCD (Part III, IV)

Overview:

-Scaling Violations

-Gluon Pair Production

-3-Jet Events

-Jet Algorithms

(I used Halzen-Martin and Quigg along with talk by Gavin Salam)

Scaling Violations

Last lecture we obtained: $\frac{d\hat{\sigma}}{dp_T^2} \simeq e_i^2 \hat{\sigma}_0 \frac{1}{p_T^2} \frac{\alpha_s}{2\pi} P_{qq}(z)$

with $P_{qq} = \frac{4}{3}\left(\frac{1+z^2}{1-z}\right)$

What is the contribution of gluon emission to the structure functions?

$$\hat{\sigma}(\gamma^* q \to qg) = \int_{\mu^2}^{\hat{s}/4} dp_T^2 \frac{d\hat{\sigma}}{dp_T^2}$$

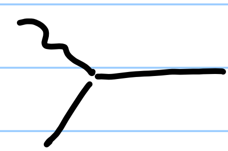
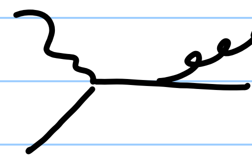
$$= e_i^2 \hat{\sigma}_0 \int_{\mu^2}^{\hat{s}/4} \frac{dp_T^2}{p_T^2} \frac{\alpha_s}{2\pi} P_{qq}(z)$$

$$= e_i^2 \hat{\sigma}_0 \left(\frac{\alpha_s}{2\pi} P_{qq}(z) \log \frac{Q^2}{\mu^2} \right)$$

$\rightarrow$ $p_{T\,max}^2 = \frac{\hat{s}}{4} = Q^2 \frac{(1-z)}{4z}$ $\rightarrow$ $\log \frac{\hat{s}}{4} \approx \log Q^2$, Q^2 large

Scaling Violations

$\rightarrow$ μ introduced To regularize $p_\perp^2 \rightarrow 0$ divergence

$\frac{F_2(x, Q^2)}{x}$ = |  |^2 + |  +  |^2

$$= \sum_q e_q^2 \int_x^1 \frac{dy}{y}\, q(y) \left(\delta\left(1 - \frac{x}{y}\right) + \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{y}\right) \log \frac{Q^2}{\mu^2} \right)$$

$$q(y) \equiv f_q(y)$$

$\longrightarrow$ $\log Q^2$ Term will introduces scaling violations

$\longrightarrow$ violation implies gluon emission

Now we will Try To obtain quark probability distributions From The above

Scaling Violations

$$\frac{F_2(x,Q^2)}{x} = \sum_q e_q^2 \int_x^1 \frac{dy}{y} \left(q(y) + \Delta q(y,Q^2)\right) \delta\left(1-\frac{x}{y}\right)$$

$$= \sum_q e_q^2 \left(q(x) + \Delta_q(x,Q^2)\right)$$

with

$$\Delta_q(x,Q^2) = \frac{\alpha_s}{2\pi} \log\left(\frac{Q^2}{\mu^2}\right) \int_x^1 \frac{dy}{y} q(y) P_{qq}\left(\frac{x}{y}\right)$$

→ quark densities depend on Q^2

→ our ability to resolve partons increases with Q^2

we can rewrite expression above as:

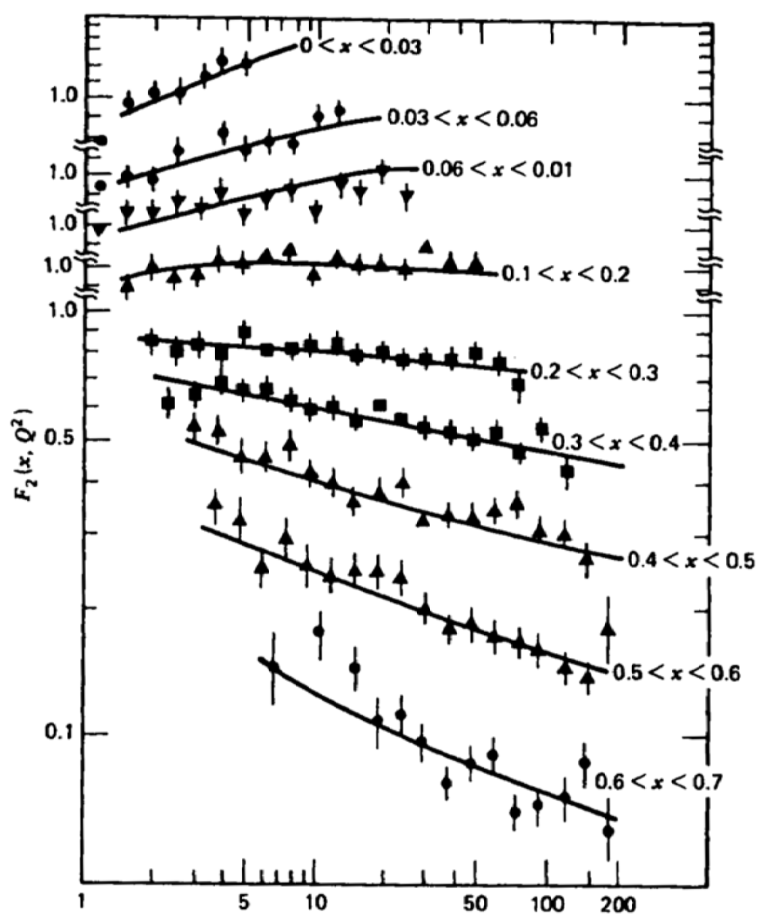
$$\frac{d}{d\log Q^2} q(x,Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} q(y,Q^2) P_{qq}\left(\frac{x}{y}\right)$$

Altarelli-Parisi evolution equation

→ Now "DGLAP"

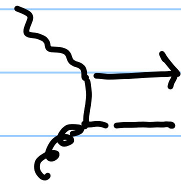
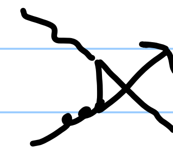
Altarelli-Parisi Equation (DGLAP)

→ a quark with momentum fraction x could have come from parent quark with momentum y (which radiated a gluon). This is associated with prob. $P_{qq}(x/y)$.



Gluon Pair Production

$\rightarrow \gamma^* g \rightarrow q\bar{q}$

$\left.\dfrac{F_2(x, Q^2)}{x}\right|_{\gamma^* g \rightarrow q\bar{q}} = \Big|$  $+$  $\Big|^2$

$$= \sum_q e_q^2 \int_x^1 \frac{dy}{y}\, g(y) \frac{\alpha_s}{2\pi} P_{qg}\left(\frac{x}{y}\right) \log\left(\frac{Q^2}{\mu^2}\right)$$

$g(y)$: gluon density in proton

$P_{gq}(z) = \left(z^2 + (1-z)^2\right)$

↳ prob. that produces $q\bar{q}$ pair with q having z fractional momentum.

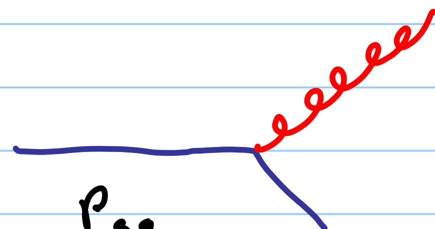
Evolution Equations

Putting everything together, we get:

$$\frac{dq_i}{d\log Q^2} = \frac{\alpha^2}{2\pi}\int_x^1 \frac{dy}{y}\left(q_i(y, Q^2) P_{qq}\left(\frac{x}{y}\right) + g(y, Q^2)\, P_{qg}\left(\frac{x}{y}\right)\right)$$

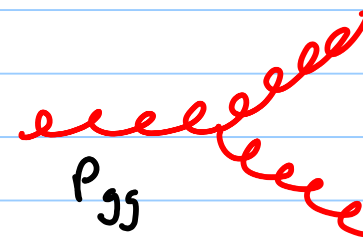
$$\frac{d}{d\log Q^2}\, g(x, Q^2) = \sum_i q_i(y, Q^2) \quad + g(y, Q^2)$$

i.e.



P_{gq}

$+$



P_{gg}

$$\frac{dg(x, Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\left(\sum_i q_i(y, Q^2) P_{gq}\left(\frac{x}{y}\right) + g(x, Q^2) P_{gg}\left(\frac{x}{y}\right)\right)$$

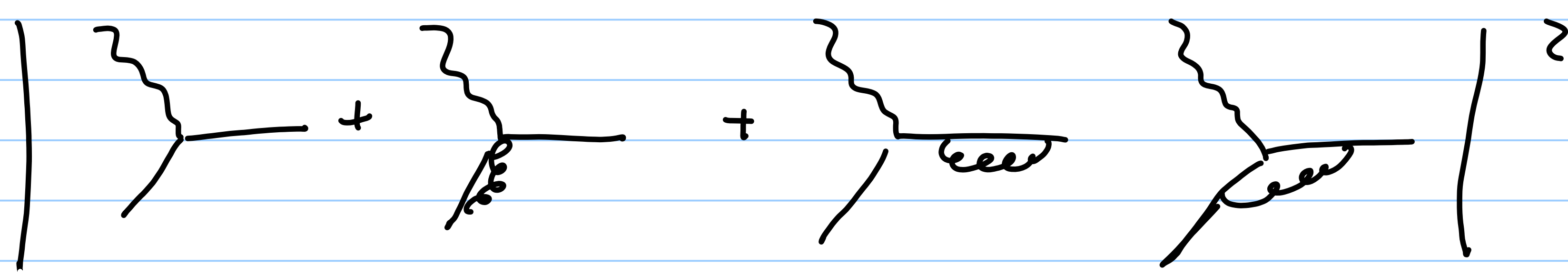
Evolution Equations (cont.)

Recall that: $q(x, Q^2) + \Delta q(x, Q^2) = \int_0^1 dy \int_0^1 dz \; q(y, Q^2)\, \mathcal{P}_{qq}(z, Q^2) \times \delta(x - zy)$

$$\mathcal{P}_{qq} \equiv \delta(1-z) + \frac{\alpha_s}{2\pi} P_{qq}(z) \log\left(\frac{Q^2}{\mu^2}\right)$$

prob. of finding a quark "inside" a quark with momentum fraction z of parent quark to first order in α_s.

Note that:



Terms cancel divergence at $z = 1$!

$e^+e^- \to q\bar{q} + X$

$\longrightarrow$ Fragmentation functions

$\longrightarrow$ $q\bar{q}g$ i.e. 3-jet events

Recall that:

$$\sigma(e^+e \to \text{hadrons}) = \sum_q \sigma(e^+e^- \to \text{hadrons})$$
$$= 3\sum_q e_q^2\, \sigma(e^+e^- \to \mu^+\mu^-)$$

$$\Rightarrow R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\sum_q e_q^2$$

$$R = \frac{11}{3}$$

$O(\alpha_s)$ we'll get $\quad R = 3\sum_q e_q^2\left(1 + \frac{\alpha_s(Q^2)}{\pi}\right)$

FRAGMENTATION

We can write differential cross section as

$$\frac{d\sigma}{dz}(e^-e^+ \to hX) = \sum_q \sigma(e^+e^- \to q\bar{q})\left[D_q^h(z) + D_{\bar{q}}^h(z)\right]$$

$$z \equiv \frac{E_h}{E_q} = \frac{2E_h}{Q}$$

$$\sum_h \int_0^1 z\, D_q^z(z)\, dz = 1 \quad , \quad \sum_q \int_{z_{min}}^1 \left[D_q^h(z) + D_{\bar{q}}^h(z)\right] dz = N_h$$

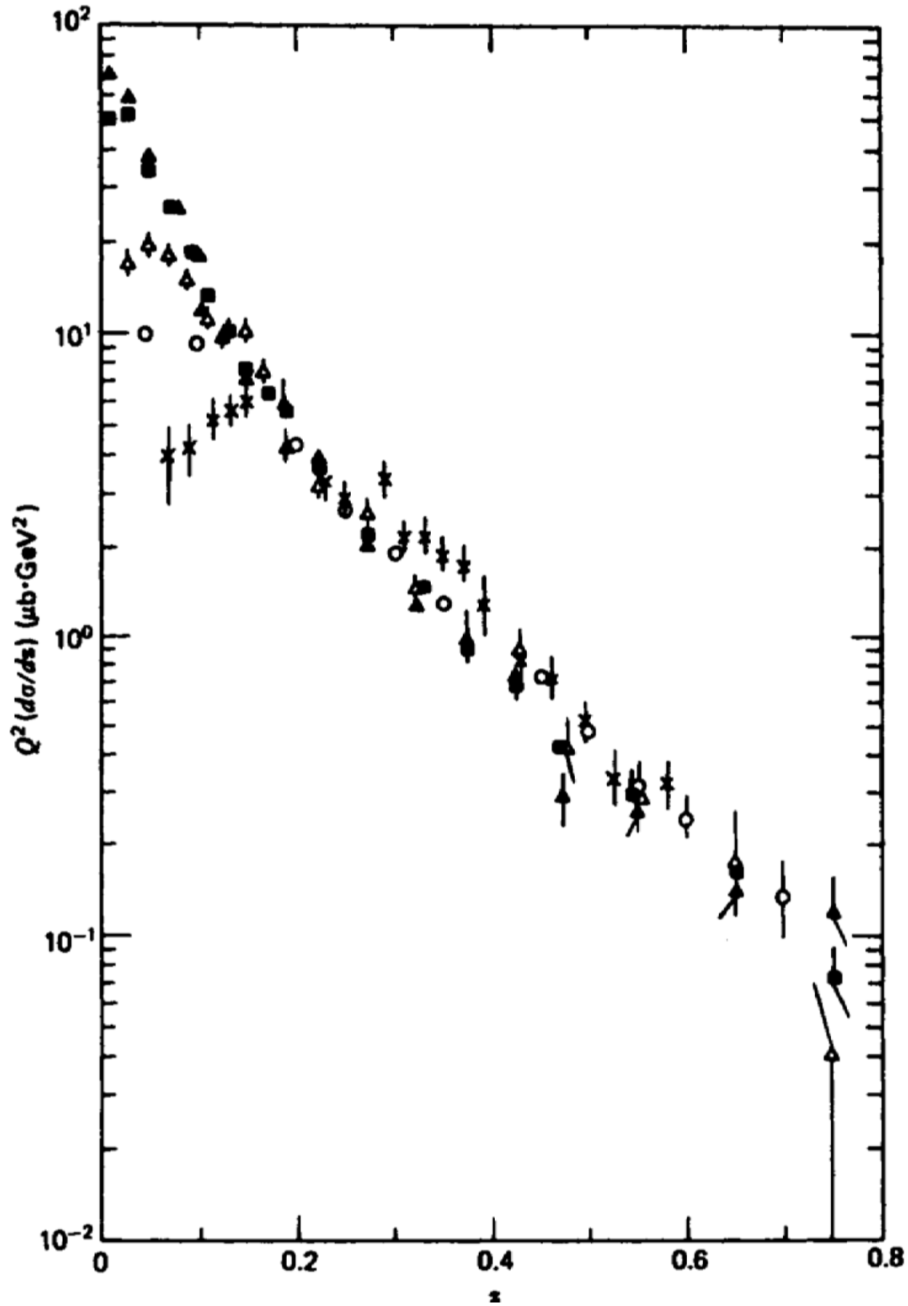
$$z_{min} = 2m_h/Q$$

Using previous results, we get:

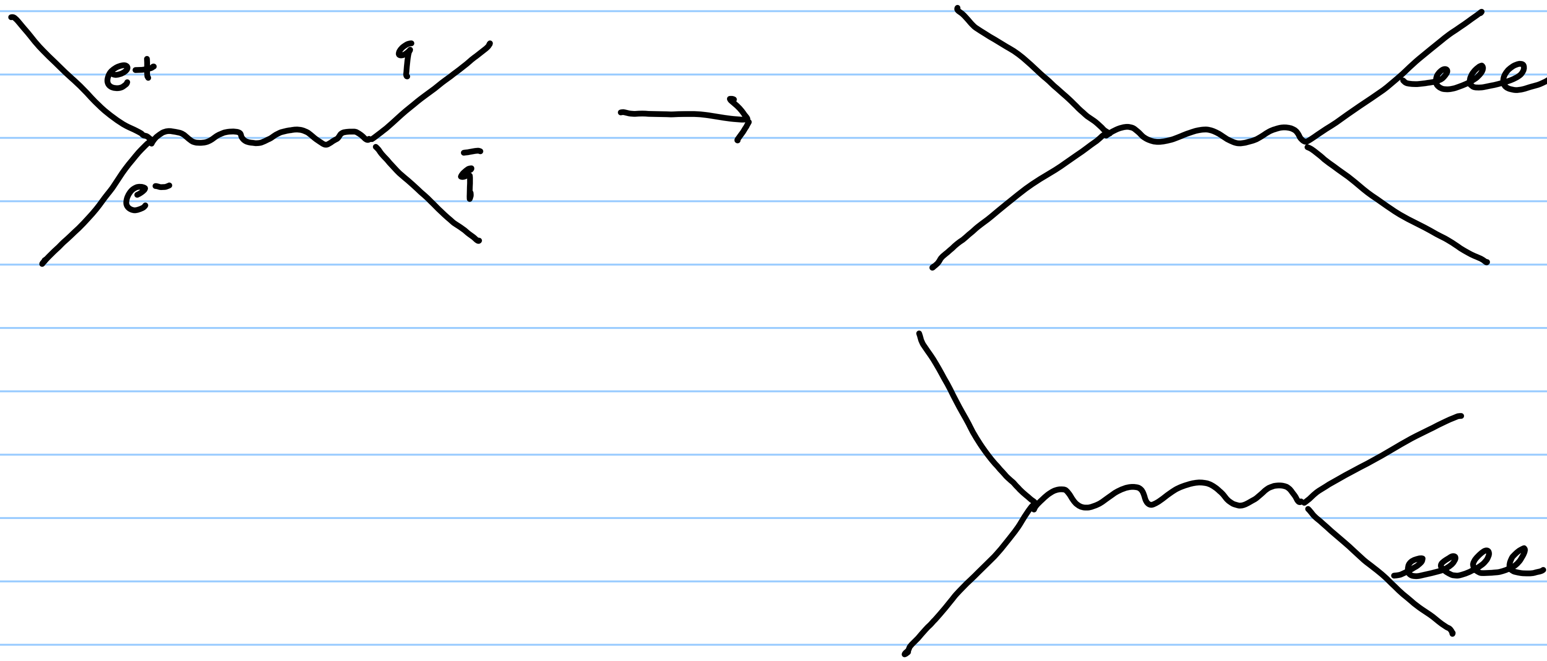
$$\frac{1}{\sigma}\frac{d\sigma}{dz}(e^+e^- \to hX) = \frac{\sum_q e_q^2\left[D_q^h(z) + D_{\bar{q}}^h(z)\right]}{\sum_q e_q^2}$$

$$= F(z) \qquad \to \text{predicted to scale}$$

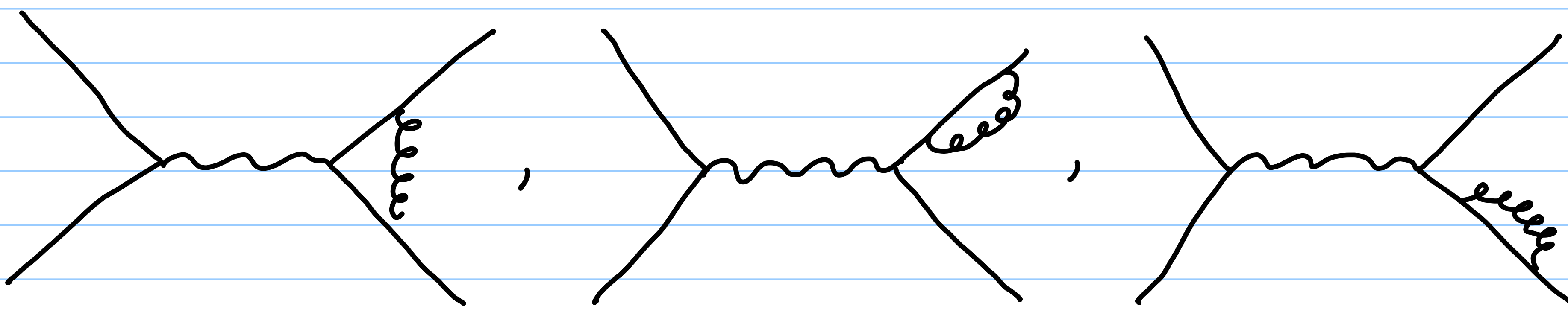
Fragmentation (cont.)



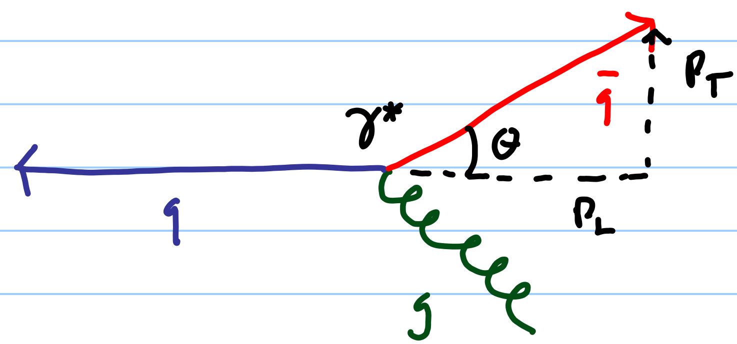
three-jet Events



+ other corrections



Three-Jet Events



$$x_q = \frac{2E_q}{Q}, \quad \bar{x}_q = \frac{2E_{\bar{q}}}{Q}, \quad x_g = \frac{2E_g}{Q}, \quad x_T = \frac{2p_T}{Q}$$

4-momentum fractions:

$(x_q, 0, 0, -x_q)$

$(x_{\bar{q}}, x_T, 0, x_L)$

$(x_g, -x_T, 0, x_q - x_L)$

variables ordered relative to most energetic jets "thrust axis"

Energy conservation $\rightarrow$ $x_q + x_{\bar{q}} + x_q = 2$

Three-Jet Events (cont.)

$$x_{\bar{q}}^2 - x_T^2 - x_L^2 = 0$$

$$x_g^2 - x_T^2 - (x_L - x_g)^2 = 0$$

$$\rightarrow \quad x_T^2 = \frac{4}{x_q^2}(1-x_q)(1-x_{\bar{q}})(1-x_g)$$

We will calculate the cross section for the case where: $x_q > x_{\bar{q}} > x_g$

$$\frac{d\sigma}{dx_{\bar{q}}\, dp_T^2} = \sigma(e^+e^- \rightarrow q\bar{q})\, \gamma_{\bar{q}\bar{q}}(x_{\bar{q}}, p_T^2)$$

↳ Prob. $\bar{q}$ emits g with momentum fraction $(1-x_{\bar{q}})$

$$\sigma(e^-e^+ \rightarrow q\bar{q}) = \frac{4\pi\alpha^2}{Q^2} e_q^2 \,, \quad \gamma_{\bar{q}\bar{q}} = \gamma_{qq} = \frac{\alpha_s}{2\pi}\frac{1}{p_T^2} P_{qq}(x_{\bar{q}})$$

$$\rightarrow \quad \frac{1}{\sigma}\frac{d\sigma}{dx_{\bar{q}}\, dx_T^2} = \frac{\alpha_s}{2\pi}\frac{1}{x_T^2} P_{qq}(x_{\bar{q}})$$

3-Jet Events (cont.)

Using previous result for P_{qq}:

$$\frac{1}{\sigma}\frac{d\sigma}{dx_T^2} = \frac{2\alpha_s}{2\pi}\frac{1}{x_T^2}\int_{x_{\bar{q}\,min}}^{x_{\bar{q}\,max}} dx\,\frac{4}{3}\left(\frac{1+x^2}{1-x}\right)$$

largest allowed value for $x_{\bar{q}}$ corresponds to $x_{\bar{q}} = x_q$

$x_g = x_T$

$x_{q\,min} = x_{\bar{q}\,max} \simeq 1 - \frac{x_T}{2}$, $1+x^2 \simeq 2$

we get:

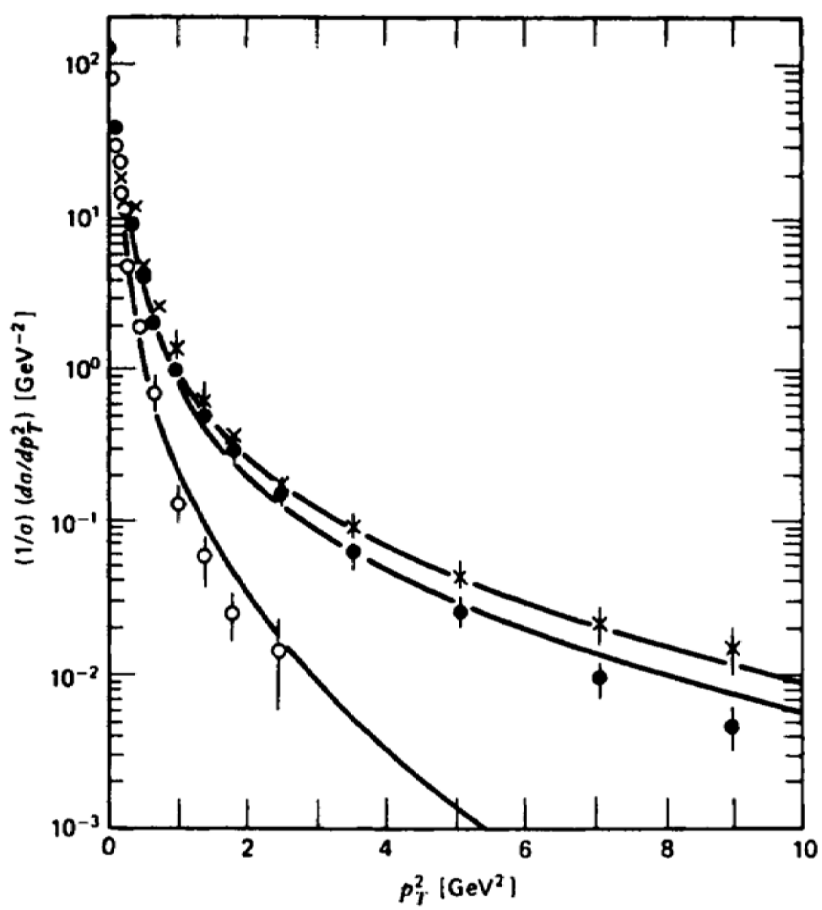
$$\frac{1}{\sigma}\frac{d\sigma}{dx_T^2} \simeq \frac{4\alpha_s}{3\pi}\frac{1}{x_T^2}\int_{x_{\bar{q}\,min}}^{1-x_T/2}\frac{dx}{1-x}$$

$$\boxed{\simeq \frac{1}{\sigma}\frac{d\sigma}{dx_T^2} \simeq \frac{4\alpha_s}{3\pi}\frac{1}{x_T^2}\log\left(\frac{1}{x_T^2}\right)}$$

"exact" result : $\frac{1}{\sigma}\frac{d\sigma}{dx_q\,dx_{\bar{q}}} = \frac{2\alpha_s}{3\pi}\frac{x_q^2 + x_{\bar{q}}^2}{(1-x_q)(1-x_{\bar{q}})}$

3-Jet Events

$$x_T \equiv \frac{2p_T}{Q} \quad \rightarrow \quad \boxed{\frac{1}{\sigma}\frac{d\sigma}{dp_T^2} \sim \alpha_s \frac{1}{p_T^2} \log\left(\frac{Q^2}{4p_T^2}\right)}$$



QCD corrections To $e^+e^- \to$ hadrons

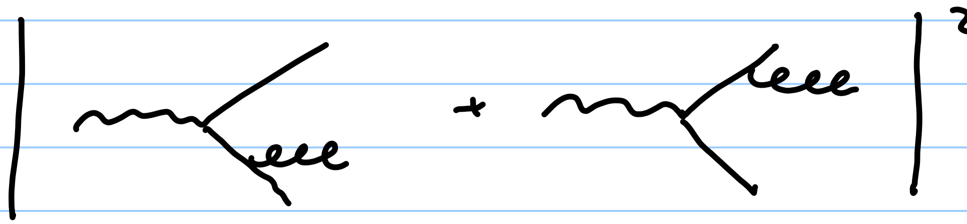
$$\frac{1}{\sigma}\frac{d\sigma}{dx_q\,dx_{\bar{q}}} = \frac{2\alpha_s}{3\pi}\,\frac{x_q^2 + x_{\bar{q}}^2}{(1-x_q)(1-x_{\bar{q}})}$$

$$1 - x_q = \frac{s}{Q^2} = \frac{2\,p_{\bar{q}}\cdot p_g}{Q^2} = \frac{2}{Q^2}E_{\bar{q}}E_g\,(1-\cos\theta_{\bar{q}g})$$

$$R \equiv \frac{\sigma(e^-e^+ \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\sum_q e_q^2 + \text{correction}$$

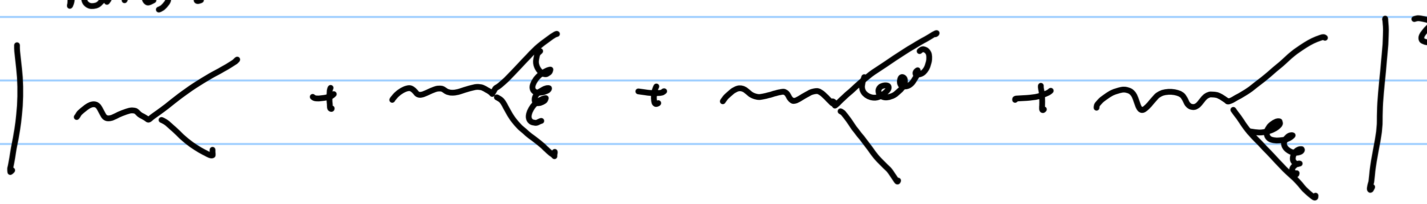
diverges...

So :



$$\sigma_R = \sigma_q \frac{\alpha_s}{2\pi}\frac{4}{3}\left[\log^2\left(\frac{m_g}{Q}\right) + 3\log\left(\frac{m_g}{Q}\right) - \frac{\pi^2}{3} + 5\right.$$

Other Terms :



$$\sigma_v = \sigma_q \frac{\alpha_s}{2\pi}\frac{4}{3}\left[-\log^2\left(\frac{m_g}{Q}\right) - 3\log\left(\frac{m_g}{Q}\right) + \frac{\pi^2}{3} - 7/2\right]$$

QCD corrections to $e^+e^- \to$ hadrons

$$\sigma = \sigma_R + \sigma_V = \sigma_q \frac{\alpha_s}{\pi}$$

$$\Rightarrow \quad R = 3 \sum_q e_q^2 \left[1 + \frac{\alpha_s(Q^2)}{\pi} \right]$$

A word on jet algorithms (see Salam)