LECTURE 22: CP Violation (Part I)

Overview:

- Overview
- Meson mixing

(I used Griffiths, Burgess, and Cheng Li as references)

Recap

weak eigenstates $\neq$ mass eigenstates

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

W's couple to : $\begin{pmatrix} u \\ d' \end{pmatrix}$, $\begin{pmatrix} c \\ s' \end{pmatrix}$

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix} \qquad \begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ -d\sin\theta_c + s\cos\theta_c \end{pmatrix}$$

With 3 generations

$$\begin{pmatrix} u \\ d' \end{pmatrix} \quad \begin{pmatrix} c \\ s' \end{pmatrix} \quad \begin{pmatrix} T \\ b' \end{pmatrix} , \qquad \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{Td} & V_{Ts} & V_{Tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

First doublet : $\begin{pmatrix} u \\ V_{ud}\, d + V_{us}\, s + V_{ub}\, b \end{pmatrix}$

Recap (cont.)

If CKM matrix was real, it would be orthogonal, couplings would be real.

A general 3x3 unitary matrix can be parameterized by 3 real angles and 6 complex phases.

If we start with:

$$q_1 = \begin{pmatrix} u \\ U_{11} d + U_{12} s + U_{13} b \end{pmatrix}$$

$$U_{11} = R_{11} e^{i\delta} \quad , \quad R_{11} \text{ real}$$

we can redefine u-quark field:

$$u \to u' = u e^{-i\delta} \quad \text{which gives:}$$

$$q_1 = e^{i\delta} \begin{pmatrix} u' \\ R_{11} d + U'_{12} s + U'_{13} b \end{pmatrix}$$

RECAP (cont.)

We can factor out complex phases for U_{21}, U_{31} too

We can also absorb two other phases U_{21} and U_{31} by redefining s, b fields

→ down to 13 parameters (9 R_{ij}, 4 phases)

→ normalization of each state reduces this by 3

→ orthogonality of 6 states reduces this by 6

→ need 3 parameters for 3x3 real orthogonal matrix

→ one independent phase

→ $n(n-1)/2$ angles

→ $(n-1)(n-2)/2$ indep. phases

C and P

$\theta - \tau$ puzzle :

$$\theta \to \pi^+ + \pi^0 \qquad P = +1$$

$$\tau \to \pi^+ + \pi^0 + \pi^0 \qquad P = -1$$
$$\pi^+ + \pi^+ + \pi^-$$

θ, τ same particle (K^+)

$\longrightarrow$ parity not conserved

$C \to$ "charge conjugation" changes all internal quantum numbers (charge, baryon and lepton number strangeness, etc.) but leaves mass, spin the same.

$\pi^+ \to \mu^+ + \nu_\mu \to$ left handed μ^+

$\pi^- \to \mu^- + \bar{\nu}_\mu \to$ no left-handed μ^-

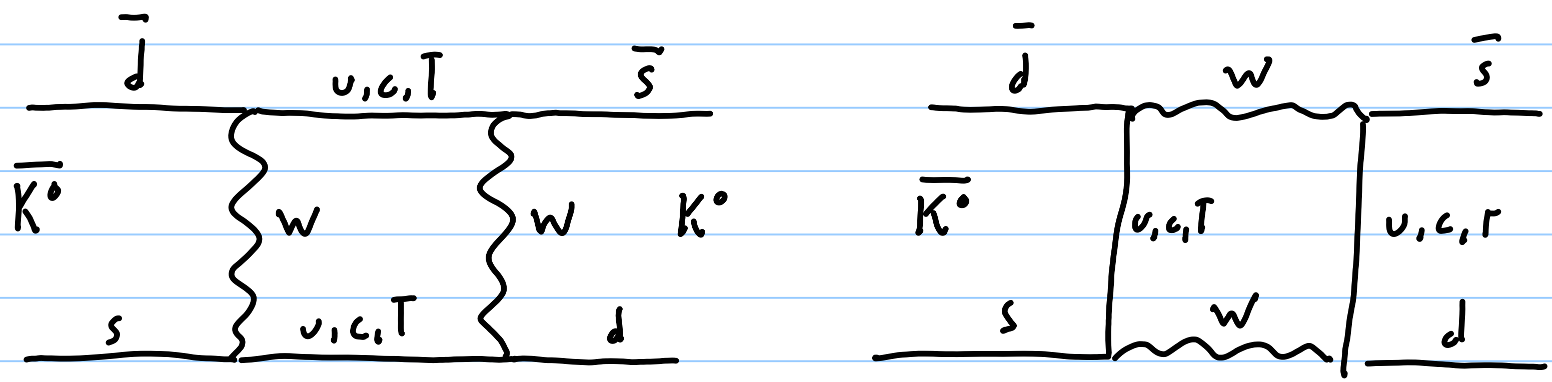
C and P

→ weak interactions not invariant under C either.

→ What about CP?

CP invariance has interesting implications on neutral mesons.

For Kaons:



→ When we turn weak interactions on, $\overline{K^0}$ and K^0 will mix

We have to deal will superposition of K^0 and $\overline{K^0}$

CP Violation

$P|K^0\rangle = -|K^0\rangle \quad , \quad P|\bar{K}_0\rangle = -|K^0\rangle$

$C|K^0\rangle = |\bar{K}^0\rangle \quad , \quad C|\bar{K}_0\rangle = |K^0\rangle$

$CP|K^0\rangle = -|\bar{K}^0\rangle \quad , \quad CP|\bar{K}^0\rangle = -|K^0\rangle$

$\rightarrow$ normalized eigenstates of CP:

$$|K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right) \quad , \quad K_2 = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)$$

$$CP|K_1\rangle = +|K_1\rangle \quad , \quad CP|K_2\rangle = -|K_2\rangle$$

If CP is conserved in weak interactions:

K_1 will only decay into $CP = +1$ state

K_2 will only decay into $CP = -1$ state

CP Violation (cont.)

→ both states have $C = +1$

K_1 has $P = +1$
K_2 has $P = -1$

→ Decay to two pions gives state $P = +1$

→ Decay to three pions gives state $P = -1$

⇒ K_1 will decay to 2 pions and K_2 will decay to 3 pions

⇒ K_1 lifetime $<$ K_2 lifetime

$$\tau_{K_1} = 0.9 \times 10^{-10}$$

$$\tau_{K_2} = 5 \times 10^{-8}$$

$$m_2 - m_1 = 3.5 \times 10^{-6}\ \text{eV} \ !$$

CP Violation (cont.)

If we start with beam of K^0, weak interactions will turn it into linear combination K^0 and $\bar{K}^0$ $\rightarrow$ K_1 and K_2

$\rightarrow$ K_1 component will decay in first few centimetres

$\rightarrow$ K_2 component will be the only component left after a few metres

$\Longrightarrow$ Decays well away from initial K^0 production should all go to 3 pions (if CP conserved)

Observation: $\sim \frac{1}{500}$ K_2 decays go to $\pi^+\pi^-$

$\Rightarrow$ CP not conserved

CP Violation (cont.)

So the long-lived Kaon is not a perfect CP eigenstate. We should write

$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(|K_2\rangle + \varepsilon|K_1\rangle\right)$$

→ Note that the short component can be regenerated if K_L been interacts with matter

$\overline{K^0}$ and K^0 components interact differently in material e.g. $\overline{K^0}\,p \to \Sigma^0\pi^+$ $s \leftrightarrow u$ exchange

similar process doesn't exist for K^0

$\Rightarrow$ $\overline{K^0}$ component can be absorbed

$\Rightarrow$ regenerates K_1 !

Meson Mixing (details...)

Mixing changes flavour by 2 units (G_F^2 process)

Decay rate is also $\propto G_F^2$ so can't be neglected.

So we introduce small imaginary part in meson mass:

$$E_K = \sqrt{K^2+m^2} \approx \sqrt{K^2+m^2} - \frac{i\Gamma(K)}{2}$$

$$\Gamma(K) = \frac{m\Gamma}{\sqrt{K^2+m^2}} \longrightarrow \text{decay rate for moving particle}$$

Charge conjugate field $\neq$ complex conjugate field with energy having a complex contribution

$$\bar{\varphi}(x) = \int \frac{d^3K}{2\pi^3\, 2K_0} \left[\bar{a}_K e^{iKx} + a_K^* e^{-iKx} \right]$$

$$\varphi(x) = \int \frac{d^3K}{2\pi^3\, 2K_0} \left[a_K e^{iKx} + \bar{a}_K^* e^{-iKx} \right]$$

$$\bar{\varphi}(x) \neq \varphi^* \qquad (Kx = E_K \cdot T - \vec{K} \cdot x)$$

Meson Mixing (cont.)

Most general quadratic Lagrangian for complex scalar field:

$$\mathcal{L} = -\partial_\mu \bar{\varphi}\, \partial^\mu \varphi - A\, \bar{\varphi}\varphi - \frac{1}{2}\left(B\varphi^2 + C\,\bar{\varphi}^2 \right)$$

$$= -\partial_\mu \bar{\varphi}\, \partial^\mu - \frac{1}{2} (\varphi, \bar{\varphi}) \begin{pmatrix} B & A \\ A & C \end{pmatrix} \begin{pmatrix} \varphi \\ \bar{\varphi} \end{pmatrix}$$

– We would have flavour conservation if $C = B = 0$

$$\varphi \to e^{i\alpha}\varphi \quad , \quad \bar{\varphi} = e^{-i\alpha}\bar{\varphi} \qquad \text{(U(1) symmetry)}$$

– Unitary evolution if $\varphi^* = \bar{\varphi}$

$$\to A = A^* \quad , \quad B = C^*$$

– CP conservation:

CP takes $\varphi \to \pm\bar{\varphi}$

$$\varphi^+ = \frac{(\varphi + \bar{\varphi})}{\sqrt{2}} \quad , \quad \varphi^- = i\frac{(\bar{\varphi} - \varphi)}{\sqrt{2}} \qquad \text{CP eigenstates}$$

$$\Rightarrow B = C$$

None of the above holds for weak interactions

Meson Mixing (cont.)

The mass matrix can be diagonalized by redefining the field:

$$\varphi = z\psi \ , \quad \bar{\varphi} = z^{-1}\psi \ , \quad z = \left(\frac{C}{B}\right)^{1/4}$$

Propagation eigenstates:

$$\psi_+ = \frac{1}{\sqrt{2}}(\bar{\psi} + \psi) = \frac{1}{\sqrt{2}}\left(z\bar{\varphi} + \frac{\varphi}{z}\right)$$

$$\psi_- = \frac{i}{\sqrt{2}}(\bar{\psi} - \psi) = \frac{i}{\sqrt{2}}\left(z\bar{\varphi} - \frac{\varphi}{z}\right)$$

$$\mu_\pm^2 \approx m_\pm^2 - i m_\pm \Gamma_\pm = A + \sqrt{BC}$$

Using $\quad A = |A| e^{-i\alpha} \ , \quad B = |B| e^{-i\beta} \ , \quad C = |C| e^{-i\gamma}$

$$m_\pm^2 = |A| \cos\alpha \pm \sqrt{|BC|} \cos\left[\tfrac{1}{2}(\beta + \gamma)\right]$$

$$m_\pm \Gamma_\pm = |A| \sin\alpha \pm \sqrt{|BC|} \sin\left[\tfrac{1}{2}(\beta + \gamma)\right]$$