LECTURE 5: Spontaneous Symmetry Breaking (Part I)

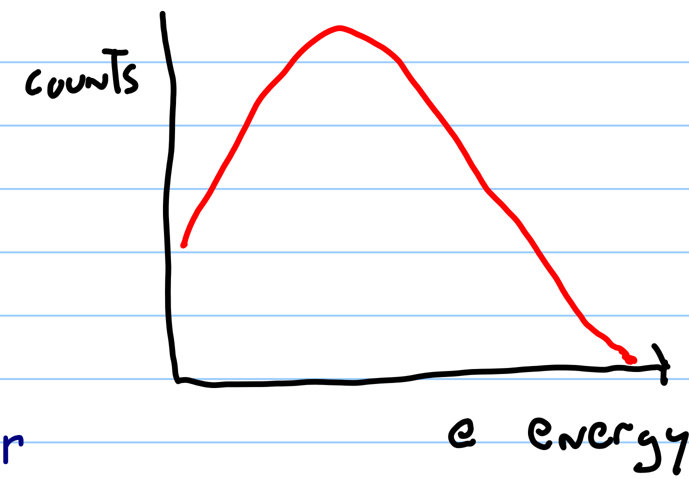
Overview:

- Motivation
- SSB for discrete symmetry
- SSB for continuous symmetry
- Higgs Mechanism (Abelian case)

(This lecture mostly follows Quigg Chapters 4-5)

Motivation (some history)

~1910s: observation that electron spectrum from beta decay is not continuous

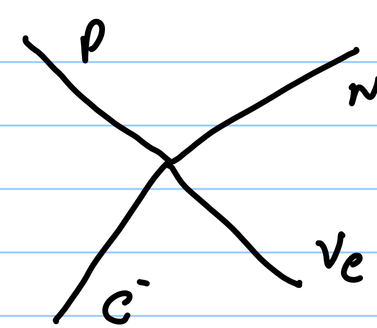


~1930s: Pauli proposes the neutrino to explain spectrum

$$N \to p\, e\, \bar{\nu}_e$$

Fermi proposes field theory for beta decay in analogy to photon emitted from atom

$$e J_\mu A^\mu$$



$$G(\bar{u}_N \gamma_\mu u_p)(\bar{u}_{\nu_e} \gamma^\mu v_e)$$

current - current interaction

For an em interaction we would have: $\boxed{\frac{e^2}{q^2}} J_1^\mu J_{2\mu}$

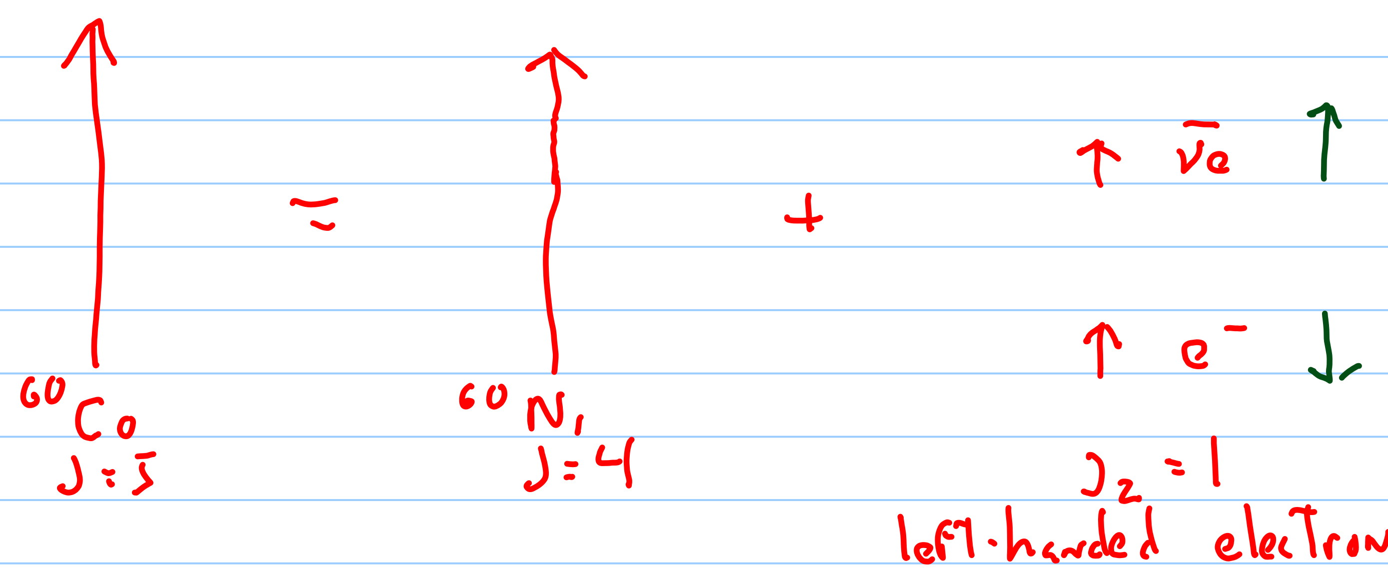
~1950s: theta-tau puzzle. Two particles with same width and mass but with two different decay processes:

$$\Theta^+ \rightarrow \pi^+ + \pi^0 \qquad 0^+$$
$$\tau^+ \rightarrow \pi^+ + \pi^+ + \pi^- \qquad 0^-$$

Parity violated??

Lee and Yang propose test of spacial parity in weak interactions

Wu et al. confirm parity is violated (violation is maximal)



← open paranthesis

[Previous result tells us how we need to write the weak current:

$$\bar{u}_e \gamma^\mu \tfrac{1}{2}(1-\gamma^5)\, u_\nu$$

↳ selects left-handed neutrino

In Weyl (chiral rep.):

$$\gamma^k = \begin{pmatrix} 0 & \sigma \\ -\sigma & 0 \end{pmatrix}, \quad \gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}$$

$$\tfrac{1}{2}(1-\gamma^5) = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$$

For massless particles, $H\psi = (\alpha\cdot p + \beta m)\psi$

gives decoupled equations: $E\chi = -\sigma\cdot p\,\chi$
$E\varphi = +\sigma\cdot p\,\varphi$

with $E = |p|$ we have

$$\underbrace{\sigma \cdot \hat{p}}_{\text{helicity state operator}}\, \chi = -\chi$$

↳ describes a negative helicity neutrino

Now $\underbrace{\frac{1}{2}(1-\gamma^5)}_{\text{selects } \nu_L} u_\nu = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \chi \\ \varphi \end{pmatrix} = \begin{pmatrix} \chi \\ 0 \end{pmatrix}$

We could absorb $\frac{1}{2}(1-\gamma^5)$ into spinor:

$$\frac{1}{2}(1-\gamma^5)\, u = u_L$$

Note that: $\gamma_\mu \left(\frac{1-\gamma_5}{2}\right) = \left(\frac{1+\gamma_5}{2}\right) \gamma_\mu \left(\frac{1-\gamma_5}{2}\right)$

So we could write $\bar{u}_{e_L} \gamma_\mu u_{\nu_L}$

For EM interactions we could write:

$$j^{em}_{\mu} = -\bar{e}\gamma_\mu e = -(\bar{e}_L + \bar{e}_R)\gamma_\mu(e_L + e_R)$$

$$= -\bar{e}_L\gamma_\mu e_L - \bar{e}_R\gamma_\mu e_R$$

using $\quad u = \left(\frac{1-\gamma_5}{2}\right)u + \left(\frac{1+\gamma_5}{2}\right)u = u_L + u_R$

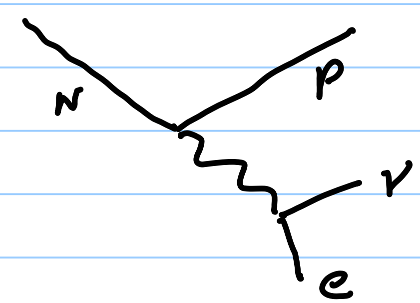
note that $\quad \bar{e}_L\gamma_\mu e_R = 0$

So, where did the right-handed terms go for the weak interaction?

Basis for W' searches in the context of $SU(2)_R \times SU(2)_L$ theories ← close parenthesis

1950s continued:

Schwinger, Lee, Yang propose intermediate vector boson



→ carries electric charge
→ not produced in weak decays

→ massive boson

1960s : Goldstone shows that massless bosons appear when global symmetry is spontaneously broken

Higgs and others give example of field theory with SSB with massive vector boson, no Goldstone boson

Spontaneous Symmetry Breakdown

Continuous symmetries of the Lagrangian lead to conservation laws

Approximate conservation laws can arrise if Lagrangian is imperfectly symmetric

OR

Lagrangian can be exactly invariant under some symmetry but the dynamics imply a vacuum that is not invariant under the symmetry

Examples include buckling needle, ferromagnet etc.

Remember that explicit mass terms violate local gauge invariance of the Lagrangian. We need a massive boson and we want to keep local gauge invariance (renorm.).

SSB of discrete symmetry

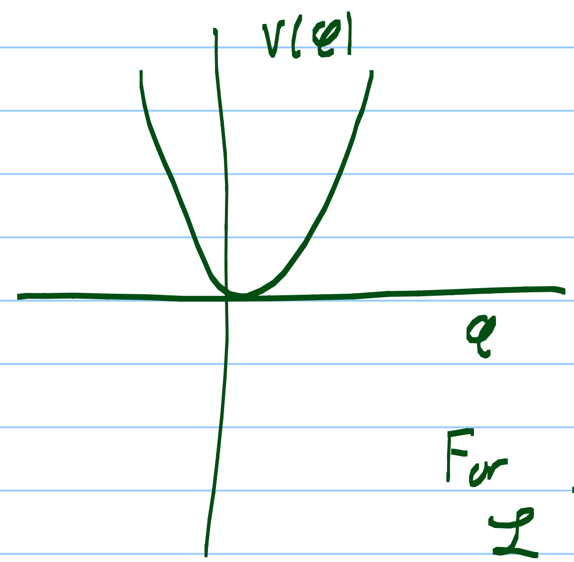
We consider a real scalar field

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)(\partial^\mu \varphi) - V(\varphi)$$

- we take $V(\varphi) = V(-\varphi)$
- $\mathcal{L}$ invariant under $\varphi \to -\varphi$

Let $V(\varphi) = \frac{1}{2}\mu^2 \varphi^2 + \frac{1}{4}|\lambda|\varphi^4$

i)



minimum at $\varphi = 0$

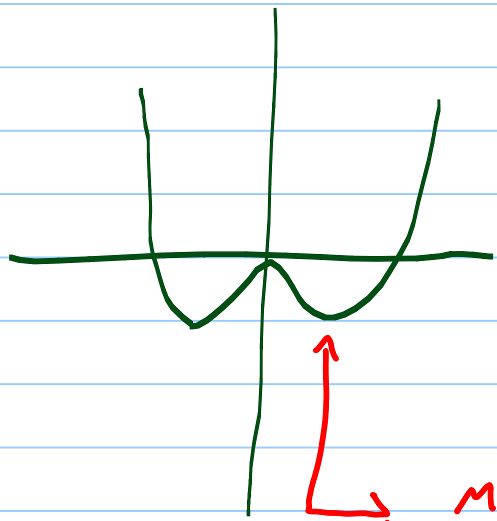
$\langle \varphi_0 \rangle = 0$ → "vev"

For small oscillations about minimum:

$$\mathcal{L}_{so} = \frac{1}{2}\left[\partial_\mu \varphi \partial^\mu \varphi - \mu^2 \varphi\right]$$

SSB of discrete symmetry (cont.)

ii)



$\mu^2 < 0$

$$V(\varphi) = -\frac{1}{2}|\mu^2|\varphi^2 + \frac{1}{4}|\lambda|\varphi^4$$

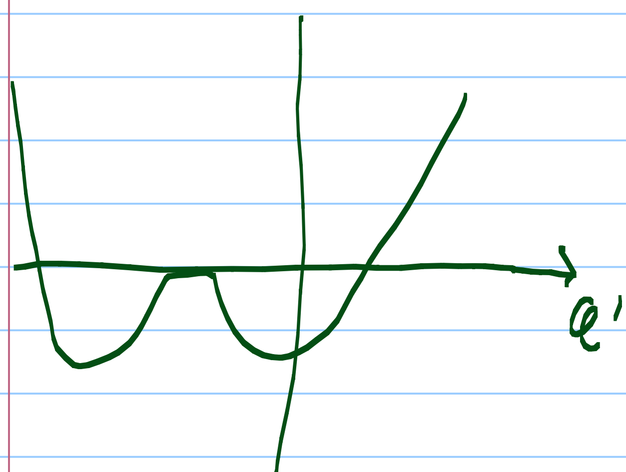
$$\frac{dV}{d\varphi} = 0 \rightarrow |\mu^2|\varphi = |\lambda|\varphi^3$$

minimum $\rightarrow$ $\varphi = \pm\sqrt{\frac{|\mu^2|}{|\lambda|}} \equiv \pm v$

Pick $\langle\varphi_0\rangle = +v$ and define shifted field:

$$\varphi' = \varphi - \langle\varphi\rangle_0 = \varphi - v$$

$$\langle\varphi'\rangle_0 = 0$$



SSB of discrete symmetry (cont.)

Lagrangian for shifted field:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi')(\partial^\mu \varphi') - \frac{|\mu^2|}{2}(\varphi' + v)^2 + \frac{1}{4}|\lambda|(\varphi' + v)^4$$

$$|\lambda| = \frac{|\mu^2|}{v^2}$$

$$\to : V = -\frac{|\mu^2|}{2}\Big[(\varphi'^2 + v^2 + 2\varphi' v) - \frac{1}{2v^2}(\varphi'^2 v^2 + v^2\varphi'^2 + 2\varphi' v^3 + 2v^3\varphi' + 4\varphi'^2 v^2 + \ldots)\Big]$$

$$= -\frac{|\mu^2|}{2}\Big[(\cancel{\varphi'^2} + v^2 + 2\cancel{\varphi'} v - \cancel{\varphi'^2} - 2\cancel{\varphi'} v - 2\varphi'^2 + \ldots)$$

$$= -\frac{|\mu^2|}{2}(2\varphi'^2 + \tfrac{1}{2}v^2) = V$$

$$\Rightarrow \mathcal{L}_{s_0} = \frac{1}{2}\Big[\partial_\mu \varphi' \partial^\mu \varphi' - 2|\mu^2|\varphi'^2\Big]$$

$\to$ particle with mass$^2 = 2|\mu^2|$

SSB of a continuous symmetry

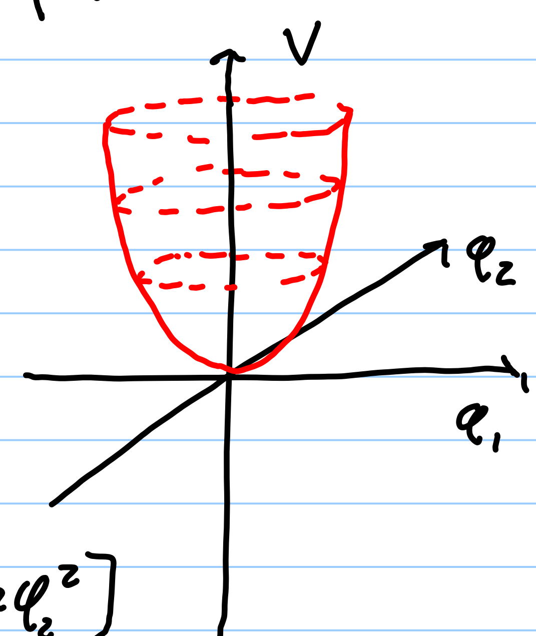
Two scalar fields $\varphi \equiv \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$

$$\mathcal{L} = \frac{1}{2}\left[\partial_\mu \varphi_1 \partial^\mu \varphi_1 + \partial_\mu \varphi_2 \partial^\mu \varphi_2\right] - V(\varphi_1^2 + \varphi_2^2)$$

$\mathcal{L}$ invariant under $SO(2)$:

$$\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} \to \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$$

$$V(\varphi^2) = \frac{1}{2}\mu^2 \varphi^2 + \frac{1}{4}|\lambda|(\varphi^2)^2$$



if $\mu^2 > 0$ minimum at $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

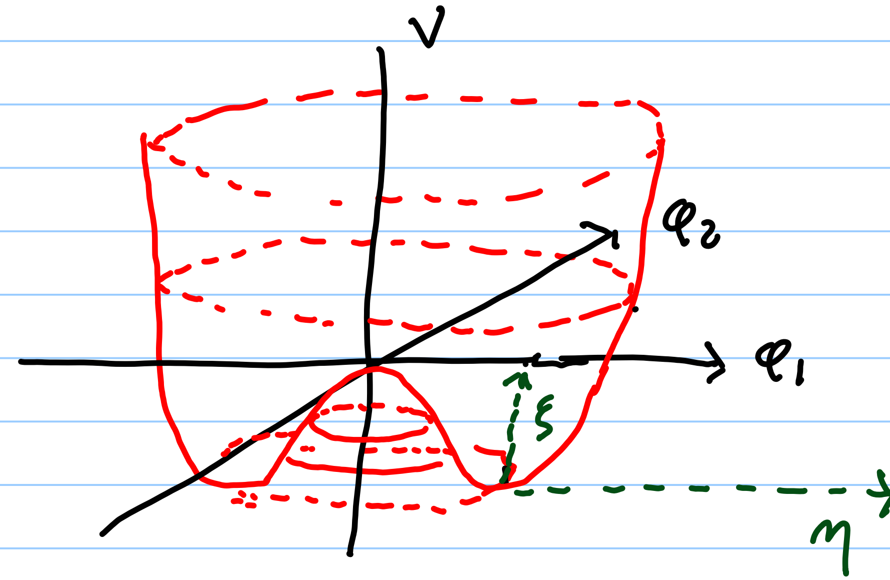
small oscillations $\mathcal{L}_0 =$

$$1/2\left[\partial_\mu \varphi_1 \partial^\mu \varphi_1 - \mu^2 \varphi_1^2\right] + \frac{1}{2}\left[\partial_\mu \varphi_2 \partial^\mu \varphi^2 - \mu^2 \varphi_2^2\right]$$

SSB of a continuous symmetry (cont.)

if $\mu^2 < 0$

$\langle \varphi \rangle_0^2 = -\frac{\mu^2}{|\lambda|} \equiv v^2$



select vacuum state

$\langle \varphi \rangle_0 = \begin{pmatrix} v \\ 0 \end{pmatrix}$

Define shifted field $\varphi' \equiv \varphi - \langle \varphi \rangle_0 \equiv \begin{pmatrix} \eta \\ \xi \end{pmatrix}$

$\mathcal{L}$ in terms of shifted field becomes, with $\varphi = \begin{pmatrix} \eta + v \\ \xi \end{pmatrix}$

$$\frac{1}{2}\left[\partial_\mu \eta \, \partial^\mu \eta + \partial_\mu \xi \, \partial^\mu \xi\right] - \underbrace{\left[-\frac{|\mu^2|}{2}\varphi^2 + \frac{1}{4}|\lambda|(\varphi^2)^2\right]}_{V}$$

SSB of a continuous symmetry (cont.)

$$V = -\frac{|\mu^2|}{2}\begin{pmatrix}\eta+v\\ \xi\end{pmatrix}^2 + \frac{1}{4}\frac{|\mu^2|}{v^2}\left[\begin{pmatrix}\eta+v\\ \xi\end{pmatrix}^2\right]^2$$

$$= -\frac{|\mu^2|}{2}\Big[(\cancel{\eta^2} + v^2 + \cancel{2v\eta} + \cancel{\xi^2}) - \frac{1}{2v^2}\big(\cancel{\eta^2 v^2} + \cancel{v^2\eta^2} + 4v^2\eta^2$$
$$+ \cancel{v^2\xi^2} + \cancel{\xi^2 v^2} + \cancel{2v^3\eta} + \cancel{2v^3\eta}$$
$$+ \dots\big)$$

For small oscillations around minimum:

$\rightarrow$ only $-\frac{\mu^2}{2}\eta^2$ remains (+ constant + higher order terms)

$\rightarrow$ no mass term for ξ

$$\mathcal{L}_{so} = \frac{1}{2}\big[\partial_\mu\eta\,\partial^\mu\eta - \underbrace{2|\mu^2|\eta^2}_{\text{particle of mass}^2\ 2\mu^2}\big] + \frac{1}{2}\big[\partial_\mu\xi\,\partial^\mu\xi\big]$$

Higgs Mechanism (Abelian case)

Consider the Lagrangian for charged scalars

$$\mathcal{L} = |D^\mu \varphi|^2 - \mu^2 |\varphi|^2 - |\lambda| \varphi^* \varphi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

where $\varphi = \frac{1}{\sqrt{2}} (\varphi_1 \pm i \varphi_2)$

→ complex scalar field

→ $D_\mu \equiv \partial_\mu + iq A_\mu$

→ $F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$

Under local gauge transformations:

$$\varphi(x) \to \varphi(x)' = e^{iq\alpha(x)} \varphi(x)$$

$$A_\mu(x) \to A_\mu'(x) = A_\mu(x) - \partial_\mu \alpha(x)$$

With $\mu^2 > 0$ we have two scalars of mass μ and a massless photon

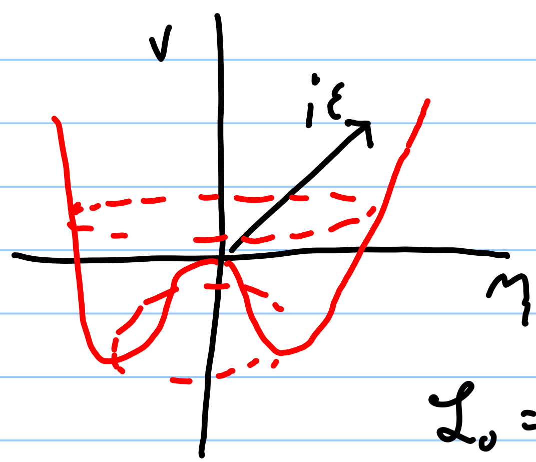
Higgs Mechanism (Abelian case) cont.

For $\mu^2 < 0$ $\rightarrow$ continuum of degenerate vacua at:

$$\langle |\varphi|^2 \rangle_0 = -\frac{\mu^2}{2|\lambda|} \equiv \frac{v^2}{2} \qquad \langle \varphi \rangle_0 = \frac{v}{\sqrt{2}}$$

Define shifted field $\varphi' = \varphi - \langle \varphi \rangle_0$

which we parametrize as $\varphi = \frac{e^{i\xi/v}}{\sqrt{2}}(v+\eta)$



$$\simeq \frac{1}{\sqrt{2}}(v + \eta + i\xi)$$

small oscillations

$$\mathcal{L}_0 = \frac{1}{2}(\partial_\mu \eta \, \partial^\mu \eta) + \frac{1}{2}(\partial_\mu \xi \, \partial^\mu \xi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \underbrace{- \left[-|\mu^2| \, |\varphi|^2 + |\lambda| \, |\varphi^* \varphi|^2 \right]}_{V}$$

Higgs Mechanism (Abelian case) cont.

$$V = -|\mu^2|\left(|\varphi|^2 - \frac{1}{v^2}(\varphi^*\varphi)^2\right)$$

$$= -|\mu^2|\left(\frac{1}{2}(v+\eta+i\xi)(v+\eta-i\xi) - \frac{1}{v^2}(\varphi^*\varphi)^2\right)$$

$$= -|\mu^2|\left(\frac{1}{2}(v^2+2v\eta+\eta^2+\xi^2) - \frac{1}{v^2}\underbrace{(\varphi^*\varphi)^2}\right)$$

$$\frac{1}{4v^2}\left(v^2\xi^2+\xi^2v^2+v^2\eta^2+\eta^2v^2+4v^2\eta^2+2v^3\eta+2\eta v^3+\ldots\right)$$

$$\Rightarrow \quad V = +|\mu^2|\eta^2 + \ldots$$

Now $|D_\mu \varphi|^2$

$$= \frac{1}{2}(\partial_\mu + iqA_\mu)(v+\eta+i\xi)\,(\partial^\mu - iqA^\mu)(v+\eta-i\xi)$$

$$= \frac{1}{2}\partial_\mu\eta\,\partial^\mu\eta + \frac{1}{2}\partial_\mu\xi\,\partial^\mu\xi + \frac{q^2v^2}{2}A_\mu A^\mu + qvA_\mu(\partial^\mu\xi) + \ldots$$

Higgs Mechanism (Abelian case) cont.

we can write $\partial^\mu \xi$ and A^μ terms as:

$$\frac{q^2 v^2}{2}\left(A_\mu + \frac{1}{qv}\partial_\mu \xi\right)\left(A^\mu + \frac{1}{qv}\partial^\mu \xi\right)$$

use gauge transformation: $A_\mu \to A'_\mu = A_\mu + \frac{1}{qv}\partial_\mu \xi$

associated with phase rotation:

$$\varphi \to \varphi' = e^{-i\xi(x)/v}\varphi(x) = \frac{(v+\eta)}{\sqrt{2}}$$

this gives us:

$$\mathcal{L}_{so} = \frac{1}{2}\left[\partial_\mu \eta\, \partial^\mu \eta + 2\mu^2 \eta^2\right] - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \frac{q^2 v^2}{2}A'^\mu A'_\mu$$

- η field with mass^2 $2|\mu^2|$
- massive vector field with mass $= qv$
- no ξ field

Answer to Gabe's question

we used gauge transformation: $A_\mu \to A'_\mu = A_\mu + \frac{1}{qv}\partial_\mu \xi$

associated with phase rotation:

$$\varphi \to \varphi' = e^{-i\xi(x)/v}\varphi(x) = \frac{(v+\eta)}{\sqrt{2}}$$

so our field is now $\varphi(x) = \left(\frac{v+\eta}{\sqrt{2}}\right)$

if we do a local phase rotation $e^{i\xi(x)/v}$

the field becomes $\varphi'(x) = e^{i\xi(x)/v}\left(\frac{v+\eta}{\sqrt{2}}\right) \simeq \frac{1}{\sqrt{2}}(v+\eta+i\xi)$

$$A'_\mu = \left(A_\mu - \frac{1}{qv}\partial_\mu \xi\right)$$

Now $|D_\mu \varphi|^2$

$$= \frac{1}{2}(\partial_\mu + iqA'_\mu)(v+\eta+i\xi)\,(\partial^\mu - iqA'^\mu)(v+\eta-i\xi)$$

$$= \frac{1}{2}\partial_\mu\eta\,\partial^\mu\eta + \underset{1}{\frac{1}{2}\partial_\mu\xi\,\partial^\mu\xi} + \underset{2}{\frac{q^2v^2}{2}A'_\mu A'^\mu} + \overset{3}{\frac{qv}{2}A'_\mu(\partial^\mu\xi)} + \frac{qv}{2}A'^\mu\partial_\mu\xi_{4} + \ldots$$

$$\frac{q^2v^2}{2}\left(A_\mu - \frac{1}{qv}\partial_\mu\xi\right)\left(A^\mu - \frac{1}{qv}\partial^\mu\xi\right) \qquad (2)$$

$$= \frac{q^2v^2}{2}A_\mu A^\mu + \frac{1}{2}\partial^\mu\xi\cancel{\partial_\mu\xi} - \frac{qv}{2}\cancel{\partial^\mu\xi A_\mu} - \frac{qv}{2}\cancel{A^\mu\partial_\mu\xi}$$

$$\frac{qv}{2}A'_\mu\partial^\mu\xi = \frac{qv}{2}\left(A_\mu - \frac{1}{qv}\partial_\mu\xi\right)\partial^\mu\xi \qquad (3)$$

$$= \frac{qv}{2}\cancel{A_\mu\partial^\mu\xi} - \frac{1}{2}\partial_\mu\xi\cancel{\partial^\mu\xi}$$

$$\frac{qv}{2}A'^\mu\partial_\mu\xi = \frac{qv}{2}\cancel{A^\mu\partial_\mu\xi} - \frac{1}{2}\partial^\mu\xi\partial_\mu\xi \qquad (4)$$

(1) + (2) + (3) + (4)

$$= \frac{1}{2}\partial_\mu\xi\cancel{\partial^\mu\xi} - \frac{1}{2}\partial^\mu\xi\cancel{\partial_\mu\xi} + \frac{q^2v^2}{2}A_\mu A^\mu$$