LECTURE 8: Weak Interactions (Part Deux)

Overview:

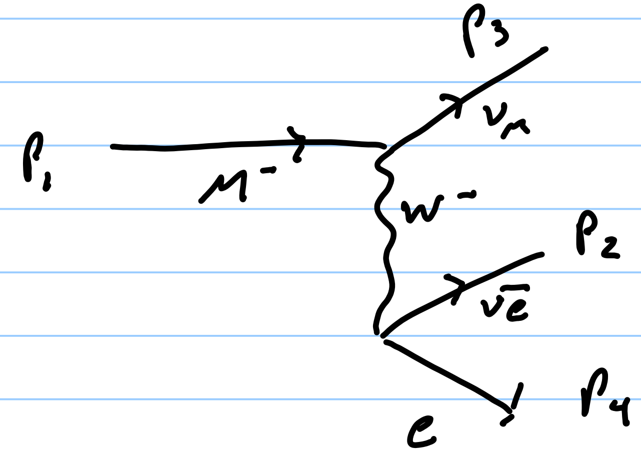
-Muon Decay

-Neutrino scattering (cont.)

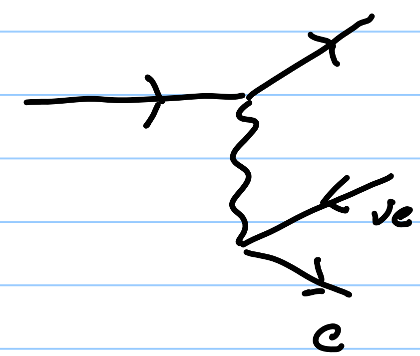
-W decay

(This lecture mostly follows Griffiths Chapter 10
and Quigg Chapter 6)

Muon decay



or



$$\mathcal{M} = \frac{g_w^2}{8M_W^2}\left[\bar{u}(3)\gamma^\mu(1-\gamma^5)u(1)\right]\left[\bar{u}(4)\gamma_\mu(1-\gamma^5)v(2)\right]$$

this is the same $\mathcal{M}$ as we got for inverse muon decay.

$$\Rightarrow \langle|\mathcal{M}|^2\rangle = 2\left(\frac{g_w}{M_W}\right)^4 (p_1\cdot p_2)(p_3\cdot p_4)$$

$p_1 = (m_\mu, 0, 0, 0) \quad \Rightarrow \quad p_1\cdot p_2 = m_\mu E_2$

Muon decay (cont)

$$(p_3 + p_4) = p_3^2 + p_4^2 + 2\, p_3 \cdot p_4 = m_e^2 + 2\, p_3 \cdot p_4$$

$$= (p_1 - p_2) = m_\mu^2 - 2\, p_1 \cdot p_2$$

$$\Rightarrow \quad p_3 \cdot p_4 = \left(\frac{m_\mu^2 - m_e^2}{2} \right) - m_\mu E_2$$

$m_e \sim 200$ less than m_μ $\longrightarrow$ we neglect ...

$$\langle |M|^2 \rangle = \left(\frac{g_w}{m_w} \right)^2 m_\mu^2 E_2 \,(m_\mu - 2E_2)$$

$$d\Gamma = \frac{\langle |M|^2 \rangle}{2 m_\mu} \left(\frac{d^3 \vec{p}_2}{(2\pi)^3 2E_2} \right) \left(\frac{d^3 \vec{p}_3}{(2\pi)^3 2E_3} \right) \left(\frac{d^3 \vec{p}_4}{(2\pi)^3 2E_4} \right) (2\pi)^4 \, \delta^4 (p_1 - p_2 - p_3 - p_4)$$

$$E_2 = p_2 \;, \quad E_3 = p_3 \;, \quad E_4 = p_4$$

$$\delta^4 (\quad) = \delta (m_\mu - E_2 - E_3 - E_4) \, \delta^3 (p_2 + p_3 + p_4)$$

integrate over p_3:

Muon decay (cont)

$$d\Gamma = \frac{\langle |M|^2 \rangle}{16(2\pi)^5 m_\mu} \frac{d^3\vec{p}_2 \, d^3\vec{p}_4}{E_2 E_3 E_4} \delta(m_\mu - E_2 - E_3 - E_4)$$

E_3 now is $= |p_2 + p_4|$

$$|p_2 + p_4|^2 = E_3^2 = p_2^2 + p_4^2 + 2\, p_2 \cdot p_4$$

$$= (E_2^2 + E_4^2 + 2E_2E_4 \cos\theta) \quad \to \text{set polar axis along } \vec{p}_4$$

$$d^3\vec{p}_2 = E_2^2 \, dE_2 \sin\theta \, d\theta \, d\varphi$$

φ integral $= 2\pi$

θ integral we set $x = \sqrt{E_2^2 + E_4^2 + 2E_2E_4\cos\theta} = E_3$

$$dx = \frac{-E_2 E_4 \sin\theta \, d\theta}{E_3}$$

$$\int_0^\pi \frac{\sin\theta}{E_3} d\theta \, \delta(m_\mu - E_2 - E_3 - E_4)$$

becomes: $\displaystyle \frac{1}{E_2 E_4} \int_{x_-}^{x_+} \delta(m_\mu - x - E_2 - E_4) \, dx$

Muon decay (cont.)

$= \dfrac{1}{E_2 E_4}$ if $x_- < \delta(m_\mu - E_2 - E_4) < x_+$

$= 0$ otherwise

now $x_\pm = \sqrt{E_2^2 + E_4^2 \pm 2E_2E_4} = |E_2 \pm E_4|$

so: $|E_2 - E_4| < (m_\mu - E_2 - E_4) < E_2 + E_4$

$= \frac{1}{2}\left[\,|E_2 - E_4| + E_2 + E_4\right] < \frac{m_\mu}{2} < E_2 + E_4$

$\longrightarrow$ $E_2 < \frac{m_\mu}{2}$, $(E_2 + E_4) > \frac{m_\mu}{2}$

$E_4 < \frac{m_\mu}{2}$

Muon Decay (cont.)

Putting everything back together we get:

$$d\Gamma = \frac{\langle |M|^2 \rangle}{(4\pi)^4 m_\mu} \, dE_2 \, \frac{d^3 p_4}{E_4^2}$$

$$\langle |M|^2 \rangle = \left(\frac{g_w}{M_W}\right)^4 m_\mu^2 E_2 (m_\mu - 2E_2)$$

$$d\Gamma = \frac{m_\mu}{(4\pi)^4} \left(\frac{g_w}{M_W}\right)^4 \frac{dE_2}{E_4^2} \, E_2 (m_\mu - 2E_2) \, d^3 p_4$$

integrate over E_2

$$\left(\frac{g_w}{4\pi M_W}\right)^4 m_\mu \frac{d^3 p_4}{E_4^2} \int_{\frac{1}{2} m_\mu - E_4}^{\frac{1}{2} m_\mu} E_2 (m_\mu - 2E_2) \, dE_2$$

$$= \left(\frac{g_w}{4\pi M_W}\right)^4 m_\mu \left(\frac{m_\mu}{2} - \frac{2}{3} E_4\right) d^3 p_4$$

Muon decay

using : $d^3p_4 = 4\pi E_4^2 \, dE_4$

we get $\Gamma = \left(\frac{g_w}{m_w}\right)^4 \frac{m_\mu^2 E_4^2}{2(4\pi)^3}\left(1 - \frac{4E_4}{3m_\mu}\right) dE_4$

$$\Gamma = \left(\frac{g_w}{m_w}\right)^4 \frac{m_\mu^2}{2(4\pi)^3} \int_0^{1/2 m_\mu} \left(1 - \frac{4E_4}{3m_\mu}\right) E_4^2 \, dE_4$$

int : $\frac{E_4^3}{3} \to \frac{m_\mu^3}{24}$, $\frac{-4E_4^4}{3 \cdot 4 m_\mu} = \frac{-4m_\mu^3}{4 \cdot 3 \cdot 16} = \frac{m_\mu^3}{48}$

$\to \frac{m_\mu^3}{48}$

$\Rightarrow \Gamma = \left(\frac{g_w m_\mu}{m_w}\right)^4 \frac{m_\mu}{96(4\pi)^3}$, $\tau = \frac{1}{\Gamma}$

$G_F = \frac{\sqrt{2}}{8}\left(\frac{g_w}{m_w}\right)^2 \Rightarrow \tau = \frac{192\pi^3}{G^2 m_\mu^5}$ $\left(\frac{192\pi^3 \hbar^7}{G^2 m_\mu^5 c^4}\right)$

$\sim 2\,\mu s$

Neutrino scattering (cont.)

We saw that the cross section for inverse muon decay was given by:

$$\sigma_{\nu e \to \mu \nu} = \frac{G_F^2 s}{\pi}$$

This process will violate unitarity at ~ 300 GeV

→ note that this is in the context of Fermi's theory with 4-particle coupling

→ we used $q \ll M_W$

What do we get if we use the full propagator?

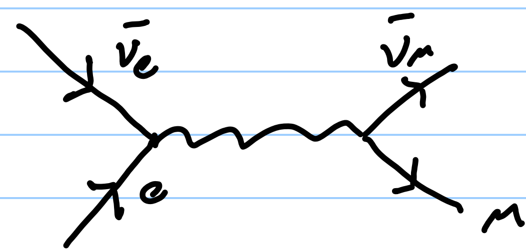
$$\sigma(\nu_\mu e \to \mu \nu_e) \simeq \frac{2G_F^2}{\pi} m_e E \frac{\left[1 - (m_\mu^2 - m_e^2)/2mE\right]^2}{(1 + 2mE/m_W^2)}$$

at high energies: $\sigma \sim \frac{G_F^2}{\pi} M_W^2$

(Still violates unitarity but at very, very high energies)

Neutrino scattering (cont.)

For the s-channel process $\bar{\nu}_e e \rightarrow \bar{\nu}_\mu \mu$:



$$\sigma = \frac{2mE\, G_F^2 \left[1 - (m_\mu^2 - m_e^2)/2mE \right]^2}{3\pi (1 - 2mE/M_W^2)^2}$$

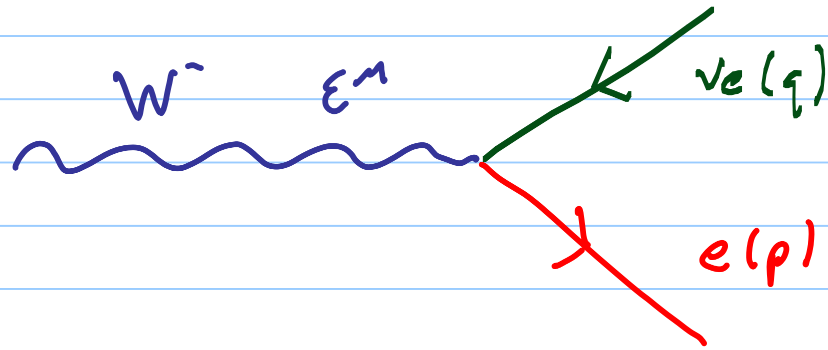
For very high energies $\sigma \approx \dfrac{G_F^2 M_W^4}{5\pi m E}$

→ Tends towards 0 as $E \rightarrow \infty$

→ adding the boson propagator fixed unitarity problems*

→ $(1 - 2mE/M_W^2)$ will make $\sigma \rightarrow \infty$ if $2mE = M_W^2$
we need to include the W width

W decay



$\nu_e(q) \to \bar{\nu}_e$ going forward, ν_e going backward

$$\mathcal{M} = -i\left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{1/2} \bar{u}(p)\,\gamma_\mu(1-\gamma_5)\,v(q)\,\varepsilon^\mu$$

$\varepsilon^\mu \equiv (0, \hat{\varepsilon})$ is the polarization vector of the W

→ we neglect the electron mass

$$|\mathcal{M}|^2 = \frac{G_F M_W^2}{\sqrt{2}} \operatorname{Tr}\left[\not{\varepsilon}(1-\gamma_5)\not{q}(1+\gamma_5)\not{\varepsilon}^*\not{p}\right]$$

$$= \frac{G_F M_W^2}{\sqrt{2}}\, 2\operatorname{Tr}\left[(1+\gamma_5)\not{\varepsilon}\not{q}\not{\varepsilon}^*\not{p}\right]$$

W decay

$$|M|^2 = \frac{8 G_F M_W^2}{\sqrt{2}} \Big((\varepsilon \cdot q)(\varepsilon^* \cdot p) - (\varepsilon \cdot \varepsilon^*)(p \cdot q) + (\varepsilon \cdot p)(\varepsilon^* \cdot q)$$
$$+ i \epsilon_{\mu\nu\rho\sigma} \varepsilon^\mu q^\nu \varepsilon^{*\rho} p^\sigma \Big]$$

let's pick the longitudinal polarization for the W: $\varepsilon^\mu = (0;0,0,1) = \varepsilon^{\mu *}$ (helicity 0)

$\rightarrow$ the $\epsilon_{\mu\nu\rho\sigma}$ term vanishes

$$p = \frac{M_W}{2}(1, \sin\theta, 0, \cos\theta)$$
$$q = \frac{M_W}{2}(1, -\sin\theta, 0, -\cos\theta)$$

$$|M|^2 = \frac{8 G_F M_W^2}{\sqrt{2}} \cdot \frac{M_W^2}{4} \left(-\cos^2\theta - -1 \cdot [1 + \sin^2\theta + \cos^2\theta] - \cos^2\theta \right.$$
$$= \frac{4 G_F M_W^4}{\sqrt{2}} \sin^2\theta$$

W Decay

$$\frac{d\Gamma}{d\Omega} = \frac{|M|^2}{64\pi^2 M_W} = \frac{G_F M_W^3}{16\pi^2\sqrt{2}} \sin^2\theta$$

$$d\Gamma = \cdot \frac{G_F M_W^3}{16\pi^2\sqrt{2}} \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta \, \sin^2\theta \, d\theta$$

$$\text{int} = -\cos\theta + \frac{\cos^3\theta}{3} \Big|_0^{\pi} = 1 - \frac{1}{3} - (-1 + 1/3)$$

$$= \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$$

$$\boxed{\Gamma = \frac{G_F M_W^3}{16\pi^2\sqrt{2}} \cdot 2\pi \cdot \frac{4}{3} = \frac{G_F M_W^3}{6\pi\sqrt{2}}}$$

$= 227$ MeV (for $M_W = 80.4$)

$G_F = 1.166 \times 10^{-5}$ GeV^{-2}

Total width: 2.06 GeV

W decay

You can check that you get the same result for helicity +1:

$$\varepsilon_\mu = (0, -1, -i, 0)/\sqrt{2}$$

→ note that $$\frac{d\Gamma}{d\Omega} = \frac{G_F M_W^3}{32\pi^2\sqrt{2}}(1-\cos\theta)^2$$

helicity −1 will give $$\frac{G_F M_W^3}{32\pi^2\sqrt{2}}(1+\cos\theta)^2$$