

LECTURE 20 and 21: QCD (Part III, IV)

Overview:

- Scaling Violations
- Gluon Pair Production
- 3-Jet Events
- Jet Algorithms

(I used Halzen-Martin and Quigg along with talk by Gavin Salam)

Scaling Violations

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Last lecture we obtained: $\frac{d\sigma}{dp_T^2} \approx e_i^2 \sigma_0 \frac{1}{p_T^2} \frac{\alpha_s}{2\pi} P_{qq}(z)$

$$\text{with } P_{qq} = \frac{4}{3} \left(\frac{1+z^2}{1-z} \right)$$

What is the contribution of gluon emission to the structure functions?

$$\begin{aligned} \hat{\sigma}(y^+ q \rightarrow qg) &= \int_{\mu^2}^{s/4} dp_T^2 \frac{d\sigma}{dp_T^2} \\ &= e_i^2 \sigma_0 \int_{\mu^2}^{s/4} \frac{dp_T^2}{p_T^2} \frac{\alpha_s}{2\pi} P_{qq}(z) \\ &= e_i^2 \sigma_0 \left(\frac{\alpha_s}{2\pi} P_{qq}(z) \log \frac{Q^2}{\mu^2} \right) \end{aligned}$$

$$\rightarrow P_{qq}^2 \text{ max} = \frac{1}{4} = \alpha^2 \frac{(1-z)}{4z} \rightarrow \log \frac{1}{4} \approx \log \alpha^2, \alpha^2 \text{ large}$$

Scaling Violations

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→ μ introduced To regularize $\beta_1^2 \rightarrow 0$ divergence

$$\frac{F_2(x, Q^2)}{x} = \left| \overbrace{}^2 + \left| \overbrace{}^2 \right. \right|^2$$

$$= \sum_q e_q^2 \int_x^1 \frac{dy}{y} q(y) \left(S\left(1 - \frac{x}{y}\right) + \frac{2s}{2\pi} P_{11}\left(\frac{x}{y}\right) \log \frac{Q^2}{\mu^2} \right)$$

$$q(y) \equiv F_1(y)$$

→ $\log Q^2$ Term will introduce scaling violations

→ violation implies gluon emission

Now we will try to obtain quark probability distributions from the above

Scaling Violations

(4)

$$\begin{aligned} F_2(x, Q^2) &= \sum_f e_f^2 \int_x^1 \frac{dy}{y} (f(y) + \Delta f(y, Q^2)) \Delta\left(1 - \frac{x}{y}\right) \\ &= \sum_f e_f^2 (f(x) + \Delta f(x, Q^2)) \end{aligned}$$

with

$$\Delta f(x, Q^2) = \frac{25}{24} \log\left(\frac{Q^2}{\mu^2}\right) \int_x^1 \frac{dy}{y} f(y) P_{ff}\left(\frac{x}{y}\right)$$

→ quark densities depend on Q^2

→ our ability to resolve partons increases with Q^2

we can rewrite expression above as:

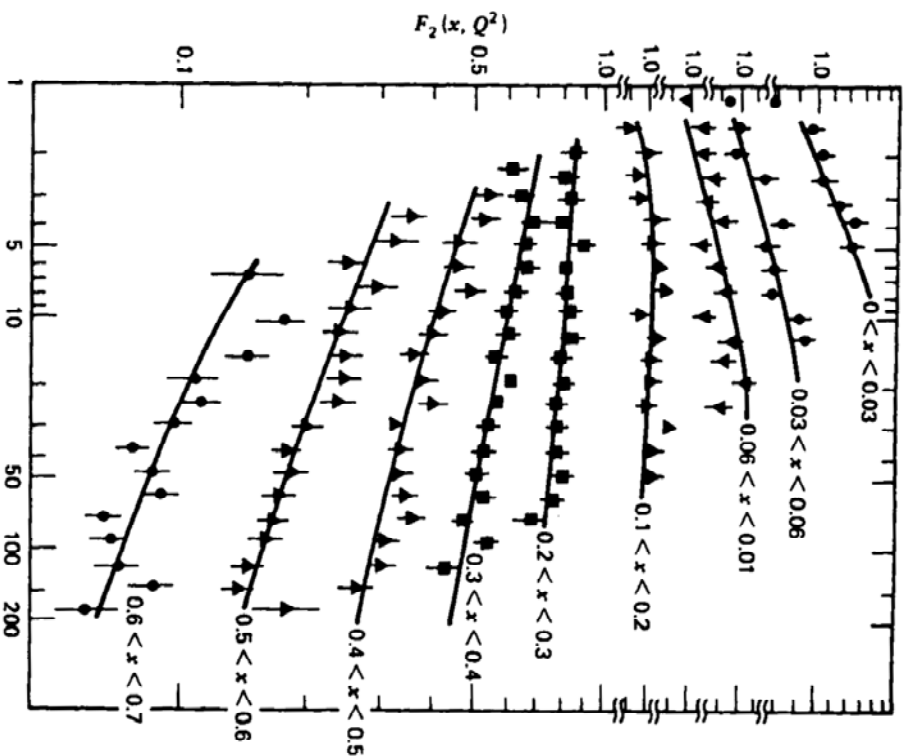
$$\frac{d}{d \log Q^2} f(x, Q^2) = \frac{25}{24} \int_x^1 \frac{dy}{y} f(y, Q^2) P_{ff}\left(\frac{x}{y}\right)$$

Altarelli-Parisi evolution equation
→ New "DGLAP"

Altarelli - Parisi Equation (DGLAP)

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→ a quark with momentum fraction x could have come from parent quark with momentum y (which radiated a gluon). This is associated with prob. $P_{qq}(x/y)$.



Gluon Pair Production

$$\rightarrow \gamma^* \gamma \rightarrow q \bar{q}$$

$$\frac{F_2(x, Q^2)}{x} \Big|_{\gamma^* \gamma \rightarrow q \bar{q}} = \int_0^1 \dots + \dots \Big|^2$$

$$= \sum_q e_q^2 \int_0^1 \frac{dy}{y} g(y) \frac{dx}{z\pi} P_{q\gamma} \left(\frac{x}{y}\right) \log\left(\frac{Q^2}{\mu_z^2}\right)$$

$g(y)$: gluon density in proton

$$P_{q\gamma}(z) = (z^2 + (1-z)^2)$$

↳ prob. that produces $q \bar{q}$ pair with q having z fractional momentum.

Evolution Equations

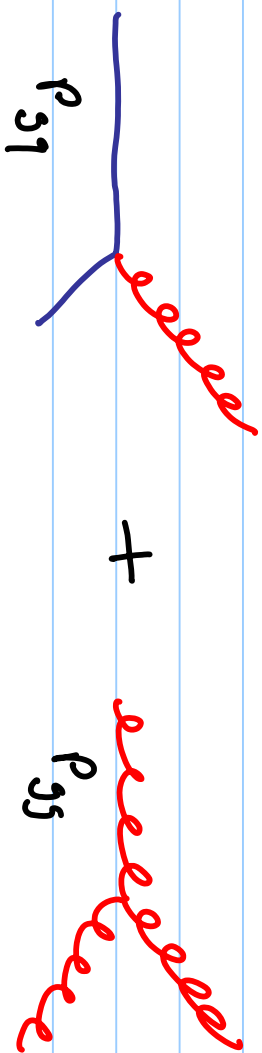
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Putting everything together, we get:

$$\frac{dq_i}{d \log Q^2} = \frac{2^i}{2^i} \int_x^1 \frac{dy}{y} \left(q_i(y, Q^2) P_{q_i} \left(\frac{x}{y} \right) + g(y, Q^2) P_{g_i} \left(\frac{x}{y} \right) \right)$$

$$\frac{d}{d \log Q^2} g(x, Q^2) = \sum_i q_i(y, Q^2) + g(y, Q^2)$$

i.e.



$$\frac{dg(x, Q^2)}{d \log Q^2} = \frac{2^i}{2^i} \int_x^1 \frac{dy}{y} \left(\sum_i q_i(y, Q^2) P_{g_i} \left(\frac{x}{y} \right) + g(x, Q^2) P_{g_j} \left(\frac{x}{y} \right) \right)$$

Evolution Equations (cont.)

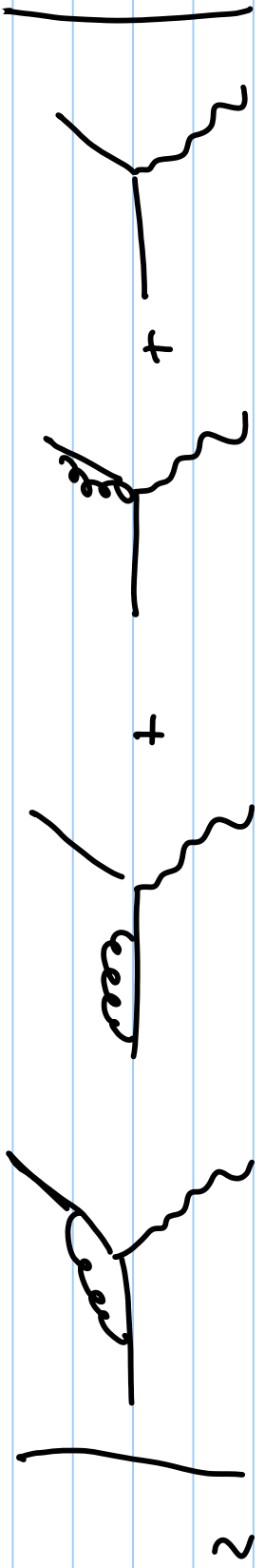
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Recall that: $\varphi(x, Q^2) + \Delta \varphi(x, Q^2) = \int_0^1 dy \int_0^1 dz \varphi(y, Q^2) \varphi_{11}(z, Q^2) \times \delta(x - zy)$

$$\varphi_{11} \equiv \delta(1-z) + \frac{dz}{2z} P_{11}(z) \log\left(\frac{Q^2}{\mu^2}\right)$$

→ prob. of finding a quark "inside" a quark with momentum fraction z of parent quark to first order in dz .

Note that:



Terms cancel divergence at $z=1$!

$$e^+e^- \rightarrow q\bar{q} + X$$

(1)

→ Fragmentation Functions

→ $q\bar{q}$ i.e. 3-jet events

Recall that:

$$\begin{aligned}\sigma(e^+e^- \rightarrow \text{hadrons}) &= \sum_1 \sigma(e^+e^- \rightarrow \text{hadrons}) \\ &= 3 \sum_1 e_q^2 \sigma(e^+e^- \rightarrow \mu^+\mu^-)\end{aligned}$$

$$\Rightarrow R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = 3 \sum_1 e_q^2$$

$$R = \frac{11}{3}$$

$$O(\alpha_s) \quad \text{we'll get} \quad R = 3 \sum_1 e_q^2 \left(1 + \frac{\alpha_s}{4\pi} \left(\frac{16}{3} \right) \right)$$

FRAGMENTATION

(10)

We can write differential cross section as

$$\frac{d\sigma}{dz} (e^+e^- \rightarrow hX) = \sum_q \sigma (e^+e^- \rightarrow q\bar{q}) [D_q^h(z) + D_{\bar{q}}^h(z)]$$

$$z \equiv \frac{E_h}{E_q} = \frac{2Eh}{Q}$$

$$\sum_h \int_0^1 z D_q^2(z) dz = 1, \quad \sum_q \int_{z_{min}}^1 [D_q^h(z) + D_{\bar{q}}^h(z)] dz = N_h$$

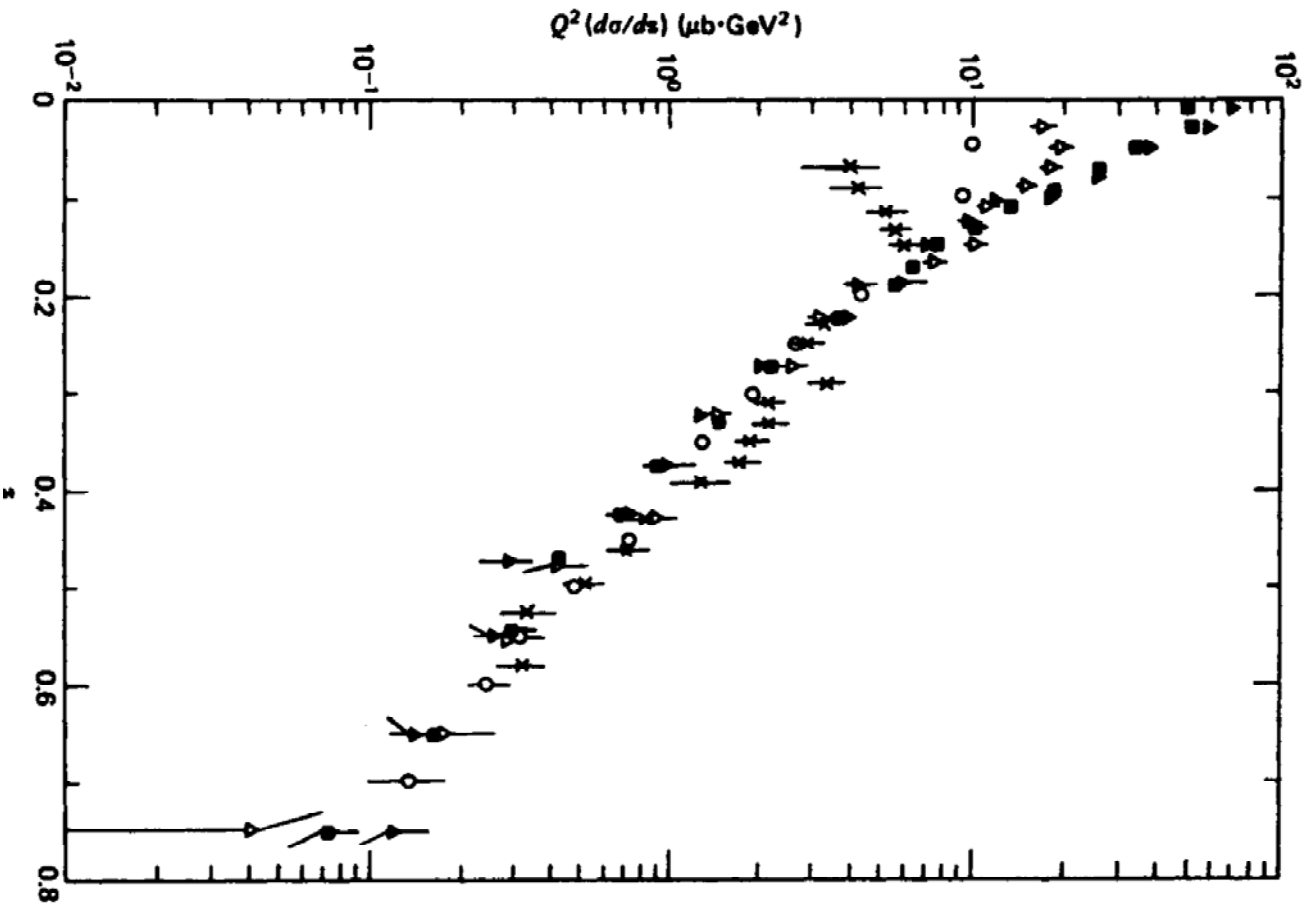
$$z_{min} = z_{min}/Q$$

Using previous results, we get:

$$\frac{1}{\sigma} \frac{d\sigma}{dz} (e^+e^- \rightarrow hX) = \frac{\sum_q e_q^2 [D_q^h(z) + D_{\bar{q}}^h(z)]}{\sum_q e_q^2}$$

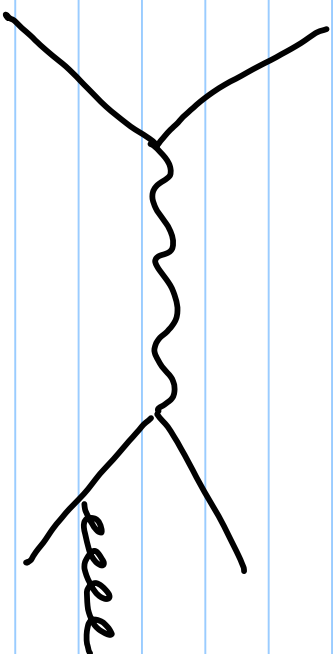
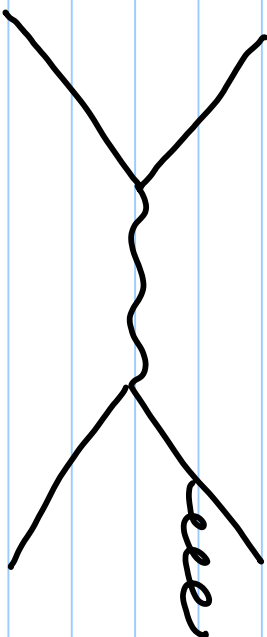
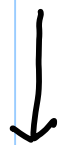
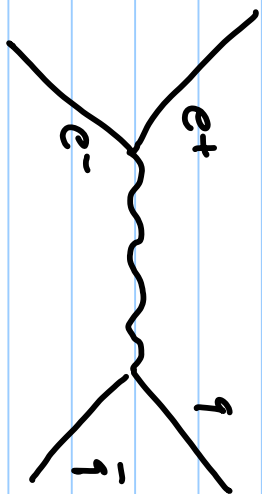
$= \mathcal{F}(z) \rightarrow$ predicted To scale

Fragmentation (cont.)

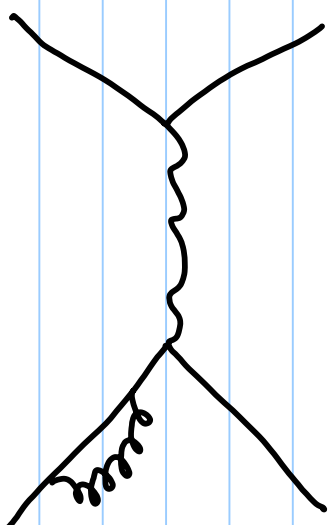
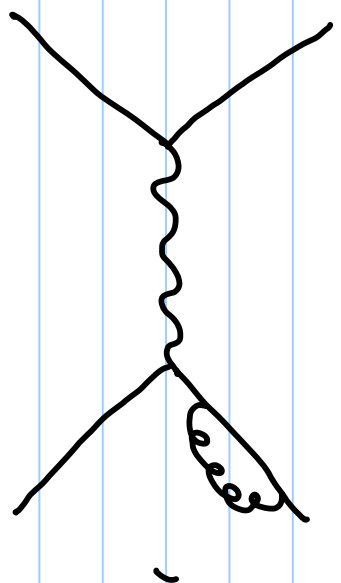
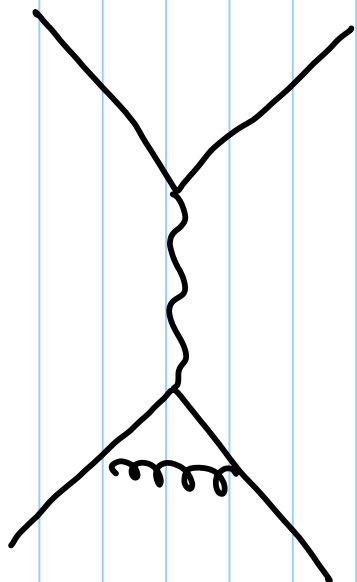


Three-jet Events

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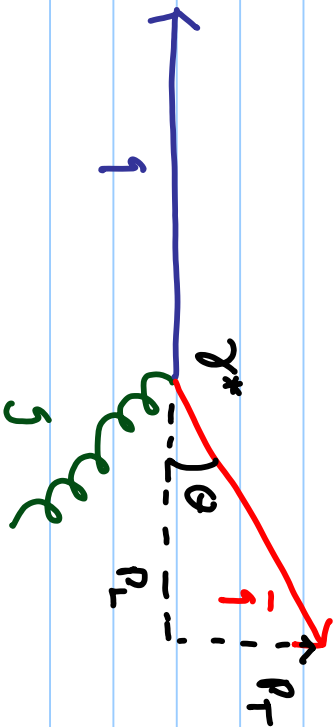


+ other corrections



Three - Jet Events

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$$x_1 = \frac{2E_1}{Q}, \quad \bar{x}_1 = \frac{2E_{\bar{1}}}{Q}, \quad x_3 = \frac{2E_3}{Q}, \quad x_T = \frac{2q_T}{Q}$$

4- momentum fractions:

$$(x_1, 0, 0, -x_1)$$

variables ordered relative
To most energetic jets
"thrust axis"

$$(x_{\bar{1}}, x_T, 0, x_L)$$

$$(x_3, -x_T, 0, x_1 - x_L)$$

Energy conservation $\rightarrow x_1 + x_{\bar{1}} + x_3 = 2$

Three-Jet Events (cont.)

$$x_1^2 - x_f^2 - x_L^2 = 0$$

$$x_3^2 - x_f^2 - (x_L - x_g)^2 = 0$$

$$\rightarrow x_f^2 = \frac{4}{x_1^2} (1 - x_1)(1 - x_1^-)(1 - x_g)$$

We will calculate the cross section for the case where: $x_1 > x_f > x_g$

$$\frac{d\sigma}{dx_1^- dp_1^2} = \sigma(e^+e^- \rightarrow \gamma\bar{\gamma}) \gamma_{f\bar{f}}(x_1^-, p_f^2)$$

↳ Prob. $\bar{\gamma}$ emits g with
momentum fraction $(1 - x_1^-)$

$$\sigma(e^+e^- \rightarrow \gamma\bar{\gamma}) = \frac{4\pi\alpha^2}{Q^2} e_f^2, \quad \gamma_{f\bar{f}} = \gamma_{f_1} = \frac{\alpha_s}{2\pi} \frac{1}{p_2^2} P_{f_1}(x_1^-)$$

$$\rightarrow \frac{1}{\sigma} \frac{d\sigma}{dx_1^- dp_1^2} = \frac{\alpha_s}{2\pi} \frac{1}{x_1^-} P_{f_1}(x_1^-)$$

3-Jet Events (cont.)

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Using previous result for P_{qq} :

$$\frac{1}{\sigma} \frac{d\sigma}{dx_T^2} = \frac{2\alpha_s}{2\pi} \frac{1}{x_T^2} \int_{x_T^-}^{x_T^+} dx \frac{4}{3} \left(\frac{1+x^2}{1-x} \right)$$

largest allowed value for x_T^+ corresponds to $x_T^+ = x_T$

$$x_T^+ = x_T$$

$$x_T^- = x_T^+ \approx 1 - \frac{x_T}{2}, \quad 1+x^2 \approx 2$$

we get:

$$\frac{1}{\sigma} \frac{d\sigma}{dx_T^2} \approx \frac{4\alpha_s}{3\pi} \frac{1}{x_T^2} \int_{x_T^-}^{1-x_T/2} \frac{dx}{1-x}$$

$$\approx \frac{1}{\sigma} \frac{d\sigma}{dx_T^2} \approx \frac{4\alpha_s}{3\pi} \frac{1}{x_T^2} \log \left(\frac{1}{x_T^2} \right)$$

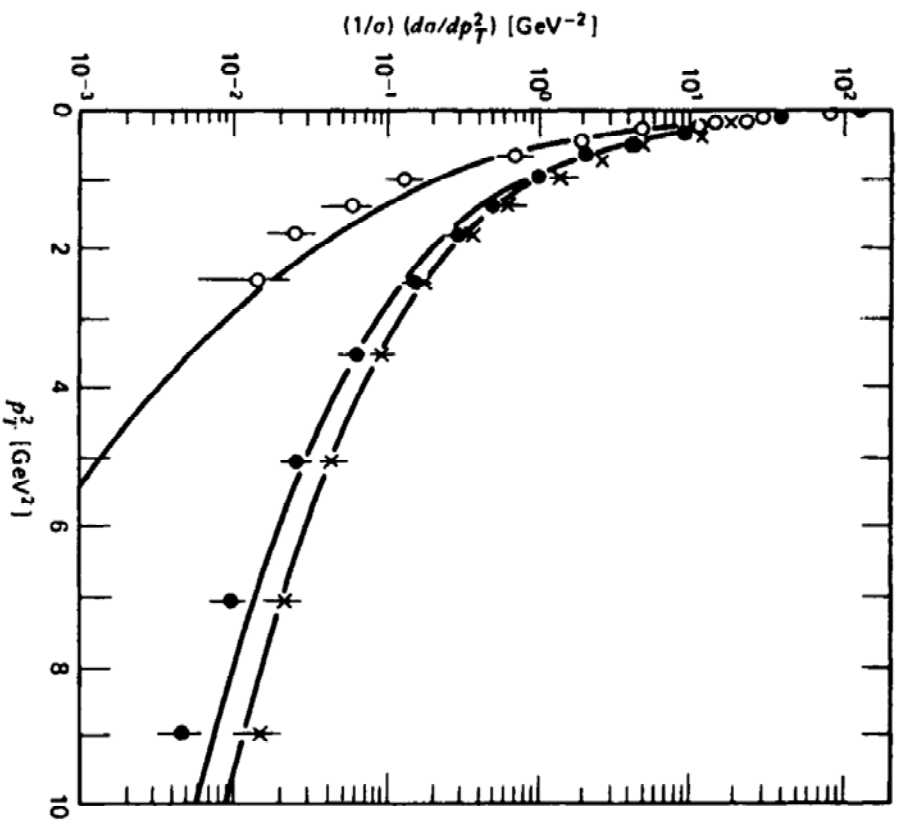
"exact" result: $\frac{1}{\sigma} \frac{d\sigma}{dx_T dx_T^-} = \frac{2\alpha_s}{3\pi} \frac{x_T^2 + x_T'^2}{(1-x_T)(1-x_T')}$

3-Jet Events

$$X_T \equiv \frac{2p_T}{Q}$$

→

$$\frac{1}{\sigma} \frac{d\sigma}{dp_T^2} \sim \alpha_s \frac{1}{p_T^2} \log\left(\frac{Q^2}{4p_T^2}\right)$$



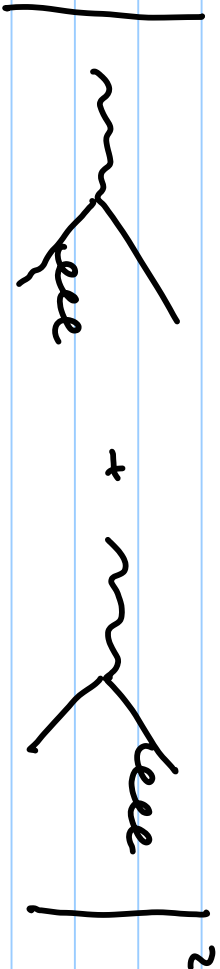
QCD corrections To $e^+e^- \rightarrow$ hadrons

$$\frac{1}{\sigma} \frac{d\sigma}{dx_1 dx_2} = \frac{2\alpha_s}{3\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$1-x_1 = \frac{s}{Q^2} = \frac{2\rho\bar{\rho}}{Q^2} = \frac{2}{Q^2} E\bar{E} (1 - \cos\theta_{\bar{f}f})$$

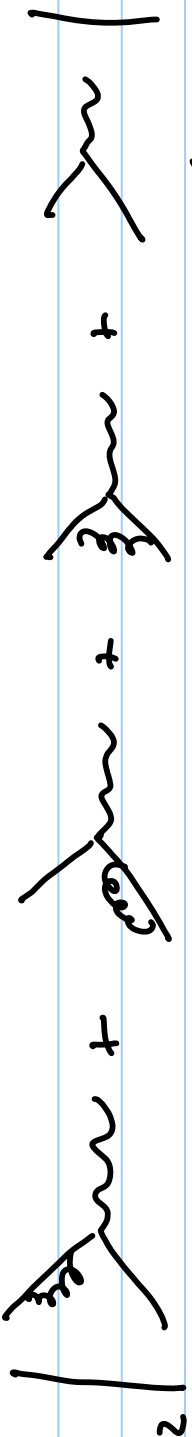
$$R \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = 3 \sum e_i^2 + \text{correction} \rightarrow \text{diverges...}$$

So:



$$\sigma_R = \sigma_0 \frac{\alpha_s}{2\pi} \frac{4}{3} \left[\log^2\left(\frac{m_s}{Q}\right) + 3 \log\left(\frac{m_s}{Q}\right) - \frac{\pi^2}{3} + 5 \right]$$

Other Terms:



$$\sigma_V = \sigma_0 \frac{\alpha_s}{2\pi} \frac{4}{3} \left[-\log^2\left(\frac{m_s}{Q}\right) - 3 \log\left(\frac{m_s}{Q}\right) + \frac{\pi^2}{3} - 7/2 \right]$$

QCD corrections To $e^+e^- \rightarrow$ hadrons

$$\sigma = \sigma_R + \sigma_V = \sigma_f \frac{d\mathcal{L}_f}{d\Omega}$$

$$\Rightarrow R = 3 \sum_f e_f^2 \left[1 + \frac{\alpha_s}{\pi} (Q^2) \right]$$

A word on Jet algorithms (see Sela)