

## LECTURE 6: Spontaneous Symmetry Breaking (Part II)

### Overview:

- Recap of Abelian case
- Ginzburg-Landau
- Higgs Mechanism (non-Abelian case)

(This lecture mostly follows Quigg Chapters 4-5)

## Higgs Mechanism (Abelian case recap)

(2)

We saw that spontaneous breaking of a continuous symmetry leads to massless bosons (Goldstone bosons). We expect one massless boson per broken generator.

We saw however that in the case of a local gauge theory, the massless gauge boson and the massless Goldstone boson conspire to give us a massive gauge boson without the massless Goldstone boson.

In the case we studied, we had before symmetry breaking:

2 scalars: 2 degrees of freedom

1 massless vector boson: 2 degrees of freedom

Total = 4

After breaking we had (explicit in unitary gauge):

1 massive vector boson: 3 degrees of freedom

1 massive Higgs scalar: 1 degree of freedom

Total = 4

# Ginzburg Landau Superconductivity (3)

24: Macroscopic wave function describing condensate  
Free energy of superconductor can be written as:

$$G_{\text{super}}(0) = G_{\text{normal}}(0) + \alpha |\psi|^2 + \beta |\psi|^4$$

$$G_{\text{super}}(B) = G_{\text{super}}(0) + \frac{B^2}{2} + \frac{1}{2\pi^2} ( -i\nabla - e^* A )^2 \psi$$

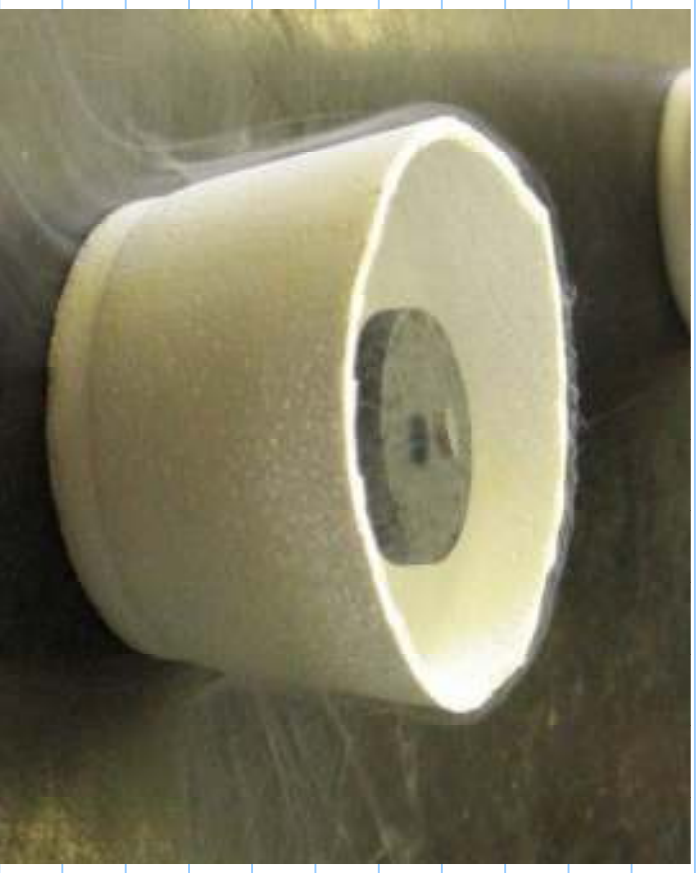
in weak field approx. Field equations derived using  $G_{\text{super}}(B)$  lead to massive photon

Meissner Effect:

- Cooper pairs form BEC condensate below  $T_c \sim 10^0 - 10^2$  K. Condensate disturbed by EM field
- Short range force, attenuation length  $\sim 10^{-6}$  cm
- equivalent to photon acquiring a mass

Electroweak symmetry breaking:

- Higgs condenses below  $T_c \sim 10^{15}$  K. Condensate disturbed by gauge bosons
- Short range force, attenuation length  $\sim 10^{-18}$  cm
- W/Z bosons acquire mass



# Higgs Mechanism (non-Abelian case) (4)

We will study an  $SU(2)$  doublet of complex scalar fields:

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

The Lagrangian is:  $(\partial_\mu \phi)^\dagger (\partial^\mu \phi) - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$   
The covariant derivative:  $D_\mu = \partial_\mu + ig \frac{\gamma_a}{2} B_\mu^a$

Under infinitesimal Transformation:  $\phi(x)' = (1 + \frac{i}{2} \alpha(x) \cdot \gamma) \phi$

$$B_\mu' = B_\mu - \frac{1}{g} \partial_\mu \alpha - \alpha \times B_\mu$$

We obtain

$$\mathcal{L} = (\partial_\mu \phi + ig \frac{\tau}{2} \cdot B_\mu \phi)^\dagger (\partial^\mu \phi + ig \frac{\tau}{2} \cdot B^\mu \phi) - V(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$V(\phi) = \mu^2 (\phi^\dagger \phi) + \lambda (\phi^\dagger \phi)^2$$

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu - g B_\mu \times B_\nu$$

# Higgs Mechanism (non-Abelian case) (5)

minimum of potential at  $\Phi^+ \Phi = \frac{1}{2} (\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) = -\frac{\mu^2}{2\lambda}$

We chose minimum around which To do our expansion:  $\phi_3^2 = -\frac{\mu^2}{\lambda} \equiv v^2$   $\phi_1 = \phi_2 = \phi_4 = 0$

We parametrize Fluctuations from the vacuum  $\phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$  in terms of 4 real scalar fields  $\xi_1, \xi_2, \xi_3, \eta$

$$|\phi_k| = e^{i\gamma \cdot \xi_k / v} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \eta \end{pmatrix} \approx \frac{1}{\sqrt{2}} \begin{pmatrix} 1 + i\xi_3 / v & i(\xi_1 - \xi_2) / v \\ i(\xi_1 + \xi_2) / v & 1 - i\xi_3 / v \end{pmatrix} \begin{pmatrix} 0 \\ v + \eta \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} \xi_2 + i\xi_1 \\ v + \eta - i\xi_3 \end{pmatrix}, \quad \text{so } \Phi^+ \Phi =$$

$$\frac{1}{2} (\xi_1 - i\xi_2, v + \eta + i\xi_3) \frac{1}{\sqrt{2}} \begin{pmatrix} \xi_2 + i\xi_1 \\ v + \eta - i\xi_3 \end{pmatrix} = \frac{\xi_1^2 + \xi_2^2 + \xi_3^2 + v^2 + \eta^2}{2} + 2v\eta$$

We know all terms from  $V$  will cancel save  $\mu^2 \eta$   
 $\rightarrow$  For small oscillations  $\rightarrow$  massive scalar

# Higgs Mechanism (non-Abelian case)

(5)

Let's move to the unitary gauge right away:

$$\varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \eta \end{pmatrix}$$

$$D_\mu \varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} 2m \begin{pmatrix} 0 \\ \eta \end{pmatrix} + \frac{ig}{2} \begin{pmatrix} b_n^3 & \sqrt{2} b_n^- \\ \sqrt{2} b_n^+ & -b_n^3 \end{pmatrix} \begin{pmatrix} 0 \\ v + \eta \end{pmatrix} \end{pmatrix}$$

$$\text{where } b_n^\pm = \frac{1}{\sqrt{2}} (b_n^1 \pm i b_n^2)$$

$$|D_\mu \varphi|^2 = \frac{1}{2} \lambda_m \eta^2 \lambda^{\prime 2} \eta^2 + \frac{1}{4} g^2 v^2 (b_n^+ b_n^- + \frac{1}{2} b_n^3 b_n^3)$$

$$+ \frac{1}{4} g^2 \eta^2 (b_n^+ b_n^- + \frac{1}{2} b_n^3 b_n^3) + \frac{1}{2} g^2 v \eta (b_n^+ b_n^- + \frac{1}{2} b_n^3 b_n^3)$$

→ 3 bosons with mass  $\frac{g v}{2}$

# Higgs Mechanism (non-Abelian case) (7)

Summary:

we started with: 4 scalars : 4 dof  
3 massless bosons:  $3 \times 2 = 6$  dof

$$\boxed{\text{Total} = 10 \text{ dof}}$$

we end up with: 1 scalar (massive) : 1 dof  
3 massive bosons:  $3 \times 3 = 9$  dof

$$\boxed{\text{Total} = 10 \text{ dof}}$$

In the Standard Model we have 3 massive vector bosons (2 charged, one neutral) and one massless boson (neutral).

→ see problem set #1

Problem set 1

Due Thursday  
at noon

FEB 10<sup>th</sup>

⑧

Problem 1

Analyse the spontaneous breaking of a global SU(2) symmetry for the case of 3 real scalar fields in an SU(2) Triplet:

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$$

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi) \cdot (\partial^\mu \phi) - V(\phi)$$

$$V(\phi) = \frac{1}{2} \mu^2 \phi \cdot \phi + \frac{1}{4} \lambda (\phi \cdot \phi)^2$$

Problem 2

Analyse the spontaneous breaking of a local SU(2) symmetry for the case of 3 real scalar fields in an SU(2) Triplet:

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$$

→ use  $\mathcal{L}$  and  $V$  from page 4



Problem set # 1 (cont)

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Problem 3:

We now turn to the following SU(2) x U(1)

Lagrangian:

$$\mathcal{L} = \left| (iD_\mu - g \frac{\tau}{2} \cdot W_\mu - \frac{g'}{2} B_\mu) \phi \right|^2 - V(\phi)$$

We use the following doublet of complex scalar fields

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad \phi^+ \equiv (\phi_1 + i\phi_2) / \sqrt{2}$$

$$\phi^0 \equiv (\phi_3 + i\phi_4) / \sqrt{2}$$

Obtain the mass of the vector bosons using the relevant

Term:  $\left| \left( -i\frac{\tau}{2} \cdot W_\mu - i\frac{g'}{2} B_\mu \right) \phi \right|^2$

you will get an off-diagonal term for  $W_\mu^3$  and  $B_\mu$

Express your result in terms of physical fields that diagonalize the mass matrix.

you can use  $\frac{g'}{g} = \tan \theta_w$

