# *Beyond the Standard Model (in Many Directions)*

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## The Electroweak Theory

Much more detail in my Pylos Lectures

http://lutece.fnal.gov/Talks/CQPylos.pdf

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## SYMMETRIES $\Longrightarrow$ INTERACTIONS

### Phase Invariance (Symmetry) in Quantum Mechanics

QM STATE: COMPLEX SCHRÖDINGER WAVE FUNCTION $\psi(x)$

OBSERVABLES

$$\langle O \rangle = \int d^n x \psi^* O \psi$$

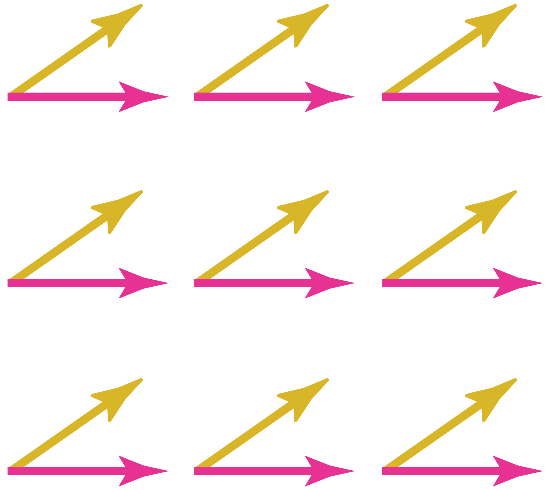
ARE UNCHANGED

UNDER A GLOBAL PHASE ROTATION

$$\psi(x) \to e^{i\theta}\psi(x)$$
$$\psi^*(x) \to e^{-i\theta}\psi^*(x)$$

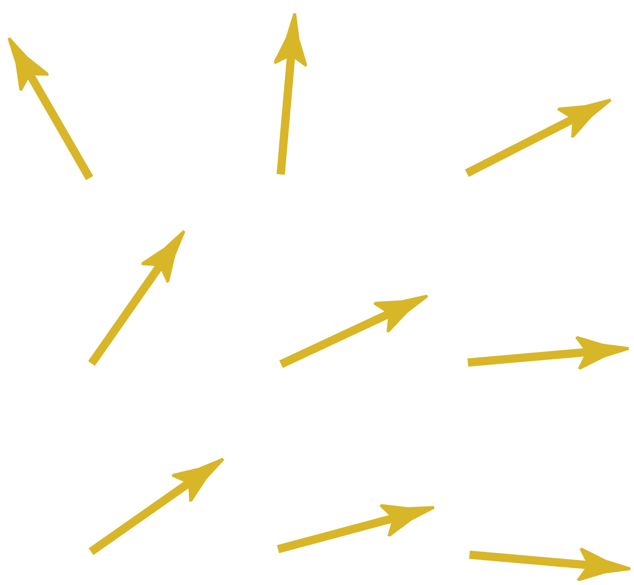
- Absolute phase of the wave function cannot be measured (is a matter of convention).
- Relative phases (interference experiments) are unaffected by a global phase rotation.

## GLOBAL ROTATION — SAME EVERYWHERE



Might we choose one phase convention
in SAN MIGUEL REGLA and another in GENEVA?

## A DIFFERENT CONVENTION AT EACH POINT?



$$\psi(x) \rightarrow e^{iq\alpha(x)}\psi(x)$$

## THERE IS A PRICE.

Some variables (e.g., momentum) and the Schrödinger equation itself contain derivatives. Under the transformation

$$\psi(x) \to e^{iq\alpha(x)}\psi(x)$$

the gradient of the wave function transforms as

$$\nabla\psi(x) \to e^{iq\alpha(x)}[\nabla\psi(x) + iq(\nabla\alpha(x))\psi(x)]$$

The $\nabla\alpha(x)$ term spoils local phase invariance.

TO RESTORE LOCAL PHASE INVARIANCE . . .

Modify the equations of motion and observables.

$$\boxed{\text{Replace } \nabla \text{ by } \nabla + iq\vec{A}}$$

"Gauge-covariant derivative"

If the vector potential $\vec{A}$ transforms under local phase rotations as

$$\vec{A}(x) \to \vec{A}'(x) \equiv \vec{A}(x) - \nabla\alpha(x),$$

then $(\nabla + iq\vec{A})\psi \to e^{iq\alpha(x)}(\nabla + iq\vec{A})\psi$ and $\psi^*(\nabla + iq\vec{A})\psi$ is invariant under local rotations.

NOTE . . .

- $\vec{A}(x) \to \vec{A}'(x) \equiv \vec{A}(x) - \nabla\alpha(x)$ has the form of a gauge transformation in electrodynamics.
- The replacement $\nabla \to (\nabla + iq\vec{A})$ corresponds to $\vec{p} \to \vec{p} - q\vec{A}$

FORM OF INTERACTION IS DEDUCED
FROM LOCAL PHASE INVARIANCE

$\Longrightarrow$ MAXWELL'S EQUATIONS

DERIVED

FROM A SYMMETRY PRINCIPLE

QED is the gauge theory based on
$U(1)$ phase symmetry

# GENERAL PROCEDURE

- Recognize a symmetry of Nature.
- Build it into the laws of physics.
  (Connection with conservation laws)
- Impose symmetry in stricter (local) form.

$\Longrightarrow$ INTERACTIONS

- Massless vector fields (gauge fields)
- Minimal coupling to the conserved current
- Interactions among the gauge fields, if symmetry is non-Abelian

Posed as a problem in mathematics, construction of a gauge theory is always possible (at the level of a classical $\mathcal{L}$; consistent quantum theory may require additional vigilance).

Formalism is no guarantee that the gauge symmetry was chosen wisely.

## Massive Photon? *Hiding Symmetry*

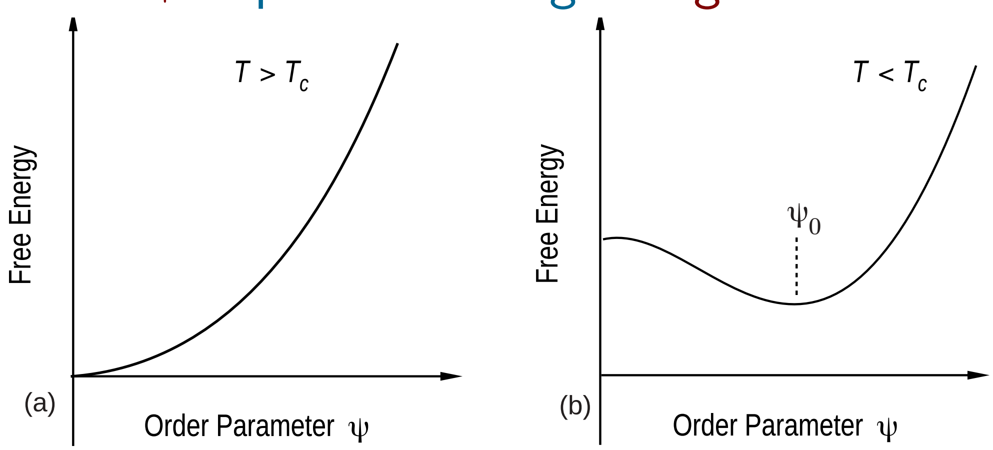
Recall **2** miracles of superconductivity:

- ▷ No resistance
- ▷ Meissner effect (exclusion of $\mathbf{B}$)

Ginzburg–Landau Phenomenology
(not a theory from first principles)

normal, resistive charge carriers . . .

. . . + superconducting charge carriers




$\mathbf{B} = 0$:

$$G_{\text{super}}(0) = G_{\text{normal}}(0) + \alpha\,|\psi|^2 + \beta\,|\psi|^4$$

$$T > T_c: \quad \alpha > 0 \quad \langle|\psi|^2\rangle_0 = 0$$

$$T < T_c: \quad \alpha < 0 \quad \langle|\psi|^2\rangle_0 \neq 0$$

## NONZERO MAGNETIC FIELD

$$G_{\text{super}}(\mathbf{B}) = G_{\text{super}}(0) + \frac{\mathbf{B}^2}{8\pi} + \frac{1}{2m^*}\left|-i\hbar\nabla\psi - \frac{e^*}{c}\mathbf{A}\psi\right|^2$$

$$\left.\begin{array}{c} e^* = -2 \\ m^* \end{array}\right\} \text{ of superconducting carriers}$$

Weak, slowly varying field

$$\psi \approx \psi_0 \neq 0, \; \nabla\psi \approx 0$$

Variational analysis $\Longrightarrow$

$$\boxed{\nabla^2\mathbf{A} - \frac{4\pi e^*}{m^* c^2}\left|\psi_0\right|^2\mathbf{A} = 0}$$

wave equation of a *massive photon*

Photon— *gauge boson* — acquires mass
within superconductor

origin of Meissner effect

# Formulate electroweak theory

three crucial clues from experiment:

▷ Left-handed weak-isospin doublets,

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \qquad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \qquad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

and

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \qquad \begin{pmatrix} c \\ s' \end{pmatrix}_L \qquad \begin{pmatrix} t \\ b' \end{pmatrix}_L ;$$

▷ Universal strength of the (charged-current) weak interactions;

▷ Idealization that neutrinos are massless.

First two clues suggest $SU(2)_L$ gauge symmetry

# A theory of leptons

$$\mathsf{L} = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \qquad \mathsf{R} \equiv e_R$$

weak hypercharges $Y_L = -1$, $Y_R = -2$

Gell-Mann–Nishijima connection, $Q = I_3 + \frac{1}{2}Y$

$\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ gauge group $\Rightarrow$ gauge fields:

⋆ weak isovector $\vec{b}_\mu$, coupling $g$

⋆ weak isoscalar $\mathcal{A}_\mu$, coupling $g'/2$

Field-strength tensors

$$F^\ell_{\mu\nu} = \partial_\nu b^\ell_\mu - \partial_\mu b^\ell_\nu + g\varepsilon_{jk\ell} b^j_\mu b^k_\nu \;, SU(2)_L$$

and

$$f_{\mu\nu} = \partial_\nu \mathcal{A}_\mu - \partial_\mu \mathcal{A}_\nu \;, U(1)_Y$$

# Interaction Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm gauge} + \mathcal{L}_{\rm leptons} \;,$$

with

$$\mathcal{L}_{\rm gauge} = -\tfrac{1}{4}F^{\ell}_{\mu\nu}F^{\ell\mu\nu} - \tfrac{1}{4}f_{\mu\nu}f^{\mu\nu},$$

and

$$\begin{aligned}\mathcal{L}_{\rm leptons} &= \overline{\mathsf{R}}\, i\gamma^{\mu}\left(\partial_{\mu} + i\frac{g'}{2}\mathcal{A}_{\mu}Y\right)\mathsf{R} \\ &+ \overline{\mathsf{L}}\, i\gamma^{\mu}\left(\partial_{\mu} + i\frac{g'}{2}\mathcal{A}_{\mu}Y + i\frac{g}{2}\vec{\tau}\cdot\vec{b}_{\mu}\right)\mathsf{L}.\end{aligned}$$

Electron mass term

$$\mathcal{L}_{e} = -m_{e}(\bar{e}_{\mathsf{R}}e_{\mathsf{L}} + \bar{e}_{\mathsf{L}}e_{\mathsf{R}}) = -m_{e}\bar{e}e$$

would violate local gauge invariance Theory has four massless gauge bosons

$$\mathcal{A}_{\mu} \quad b^{1}_{\mu} \quad b^{2}_{\mu} \quad b^{3}_{\mu}$$

Nature has but one ($\gamma$)

# Hiding EW Symmetry

*Higgs mechanism: relativistic generalization of Ginzburg-Landau superconducting phase transition*

▷ Introduce a complex doublet of scalar fields

$$\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad Y_\phi = +1$$

▷ Add to $\mathcal{L}$ (gauge-invariant) terms for interaction and propagation of the scalars,

$$\mathcal{L}_{\text{scalar}} = (\mathcal{D}^\mu \phi)^\dagger (\mathcal{D}_\mu \phi) - V(\phi^\dagger \phi),$$

where $\mathcal{D}_\mu = \partial_\mu + i\frac{g'}{2}\mathcal{A}_\mu Y + i\frac{g}{2}\vec{\tau} \cdot \vec{b}_\mu$ and

$$\boxed{V(\phi^\dagger \phi) = \mu^2 (\phi^\dagger \phi) + |\lambda| (\phi^\dagger \phi)^2}$$

▷ Add a Yukawa interaction

$$\mathcal{L}_{\text{Yukawa}} = -\zeta_e \left[ \overline{\mathsf{R}}(\phi^\dagger \mathsf{L}) + (\overline{\mathsf{L}}\phi)\mathsf{R} \right]$$

▷ Arrange self-interactions so vacuum corresponds to a broken-symmetry solution: $\mu^2 < 0$
Choose minimum energy (vacuum) state for vacuum expectation value

$$\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \quad v = \sqrt{-\mu^2/|\lambda|}$$

Hides (breaks) $SU(2)_L$ and $U(1)_Y$

but preserves $U(1)_{\rm em}$ invariance

Invariance under $\mathcal{G}$ means $e^{i\alpha\mathcal{G}}\langle\phi\rangle_0 = \langle\phi\rangle_0$, so $\mathcal{G}\langle\phi\rangle_0 = 0$

$$\tau_1\langle\phi\rangle_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \quad \text{broken!}$$

$$\tau_2\langle\phi\rangle_0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} -iv/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \quad \text{broken!}$$

$$\tau_3\langle\phi\rangle_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ -v/\sqrt{2} \end{pmatrix} \neq 0 \quad \text{broken!}$$

$$Y\langle\phi\rangle_0 = Y_\phi\langle\phi\rangle_0 = +1\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \neq 0 \quad \text{broken!}$$

Examine electric charge operator $Q$ on the (electrically neutral) vacuum state

$$
\begin{aligned}
Q\langle\phi\rangle_0 &= \tfrac{1}{2}(\tau_3 + Y)\langle\phi\rangle_0 \\
&= \tfrac{1}{2}\begin{pmatrix} Y_\phi + 1 & 0 \\ 0 & Y_\phi - 1 \end{pmatrix}\langle\phi\rangle_0 \\
&= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \\
&= \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \textit{unbroken!}
\end{aligned}
$$

Four original generators are broken

electric charge is not

▷ $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \to U(1)_{\mathrm{em}}$ (will verify)

▷ Expect massless photon

▷ Expect gauge bosons corresponding to

$$\tau_1,\ \tau_2,\ \tfrac{1}{2}(\tau_3 - Y) \equiv K$$

to acquire masses

# Expand about the vacuum state

Let $\phi = \begin{pmatrix} 0 \\ (v+\eta)/\sqrt{2} \end{pmatrix}$; in *unitary gauge*

$$\begin{aligned} \mathcal{L}_{\rm scalar} &= \tfrac{1}{2}(\partial^\mu \eta)(\partial_\mu \eta) - \mu^2\eta^2 \\ &\quad + \frac{v^2}{8}[g^2 \left|b_1 - ib_2\right|^2 + (g'\mathcal{A}_\mu - gb_\mu^3)^2] \\ &\quad + \text{interaction terms} \end{aligned}$$

Higgs boson $\eta$ has acquired $(\text{mass})^2$ $M_H^2 = -2\mu^2 > 0$

$$\frac{g^2v^2}{8}(\left|W_\mu^+\right|^2 + \left|W_\mu^-\right|^2) \Longleftrightarrow M_{W^\pm} = gv/2$$

Now define othogonal combinations

$$Z_\mu = \frac{-g'\mathcal{A}_\mu + gb_\mu^3}{\sqrt{g^2+g'^2}} \qquad A_\mu = \frac{g\mathcal{A}_\mu + g'b_\mu^3}{\sqrt{g^2+g'^2}}$$

$$M_{Z^0} = \sqrt{g^2+g'^2}\, v/2 = M_W\sqrt{1+g'^2/g^2}$$

$A_\mu$ remains massless

$$\mathcal{L}_{\mathsf{Yukawa}} = -\zeta_e \frac{(v+\eta)}{\sqrt{2}}(\bar{e}_{\mathsf{R}} e_{\mathsf{L}} + \bar{e}_{\mathsf{L}} e_{\mathsf{R}})$$

$$= -\frac{\zeta_e v}{\sqrt{2}}\bar{e}e - \frac{\zeta_e \eta}{\sqrt{2}}\bar{e}e$$

electron acquires $m_e = \zeta_e v/\sqrt{2}$

Higgs coupling to electrons: $m_e/v$ ($\propto$ mass)

Desired particle content . . . + Higgs scalar

Values of couplings, electroweak scale $v$?

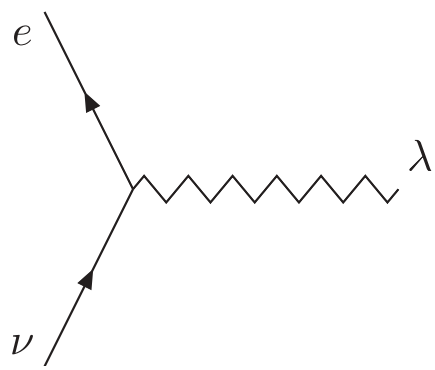
What about interactions?

## Interactions . . .

$$\mathcal{L}_{W\text{-}\ell} = -\frac{g}{2\sqrt{2}}[\bar{\nu}\gamma^{\mu}(1-\gamma_5)eW_{\mu}^{+} + \bar{e}\gamma^{\mu}(1-\gamma_5)\nu W_{\mu}^{-}]$$

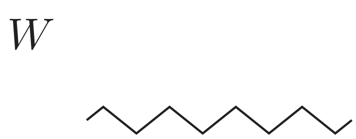
$+$ similar terms for $\mu$ and $\tau$

Feynman rule:




$$\frac{-ig}{2\sqrt{2}}\gamma_{\lambda}(1-\gamma_5)$$

gauge-boson propagator:




$$= \frac{-i(g_{\mu\nu} - k_{\mu}k_{\nu}/M_W^2)}{k^2 - M_W^2} .$$

## Compute $\nu_\mu e \to \mu\nu_e$

$$\sigma(\nu_\mu e \to \mu\nu_e) = \frac{g^4 m_e E_\nu}{16\pi M_W^4} \frac{[1 - (m_\mu^2 - m_e^2)/2m_e E_\nu]^2}{(1 + 2m_e E_\nu/M_W^2)}$$

Reproduces 4-fermion result at low energies if

$$\frac{g^4}{16 M_W^4} = 2G_F^2$$

$$\Rightarrow g^4 = 32(G_F M_W^2)^2 = 64\left(\frac{G_F M_W^2}{\sqrt{2}}\right)^2$$

$$\Rightarrow \frac{g}{2\sqrt{2}} = \left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{\frac{1}{2}}$$

Using $M_W = gv/2$, determine

$$\boxed{v = (G_F\sqrt{2})^{-\frac{1}{2}} \approx 246 \text{ GeV}}$$

the electroweak scale

$$\Rightarrow \langle\phi^0\rangle_0 = (G_F\sqrt{8})^{-\frac{1}{2}} \approx 174 \text{ GeV}$$

# Interactions . . .

$$\mathcal{L}_{A\text{-}\ell} = \frac{gg'}{\sqrt{g^2 + g'^2}} \bar{e}\gamma^\mu e A_\mu$$

. . . vector interaction; $\Rightarrow$ $A_\mu$ as $\gamma$, provided

$$\boxed{gg'/\sqrt{g^2 + g'^2} \equiv e}$$

Define $g' = g \tan\theta_W$ $\qquad$ $\theta_W$: weak mixing angle

$$\begin{aligned} g &= e/\sin\theta_W \geq e \\ g' &= e/\cos\theta_W \geq e \end{aligned}$$

$Z_\mu = b_\mu^3 \cos\theta_W - \mathcal{A}_\mu \sin\theta_W \quad A_\mu = \mathcal{A}_\mu \cos\theta_W + b_\mu^3 \sin\theta_W$

$$\mathcal{L}_{Z-\nu} = \frac{-g}{4\cos\theta_W} \bar{\nu}\gamma^\mu(1-\gamma_5)\nu\, Z_\mu$$

$$\mathcal{L}_{Z-e} = \frac{-g}{4\cos\theta_W} \bar{e}\left[L_e\gamma^\mu(1-\gamma_5) + R_e\gamma^\mu(1+\gamma_5)\right] e Z_\mu$$

$$\begin{aligned} L_e &= 2\sin^2\theta_W - 1 = 2x_W + \tau_3 \\ R_e &= 2\sin^2\theta_W = 2x_W \end{aligned}$$

## $Z$-boson properties

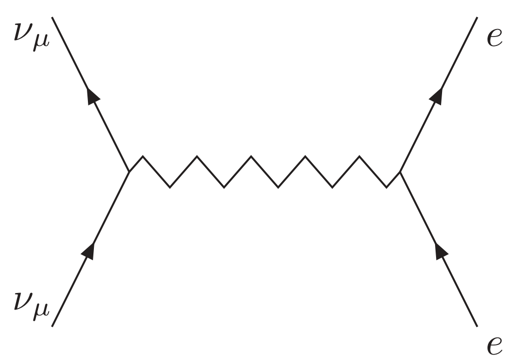
Decay calculation analogous to $W^{\pm}$

$$\Gamma(Z \to \nu\bar{\nu}) = \frac{G_F M_Z^3}{12\pi\sqrt{2}}$$

$$\Gamma(Z \to e^+e^-) = \Gamma(Z \to \nu\bar{\nu}) \left[L_e^2 + R_e^2\right]$$

## Neutral-current interactions

New $\nu e$ reaction, not present in $V - A$



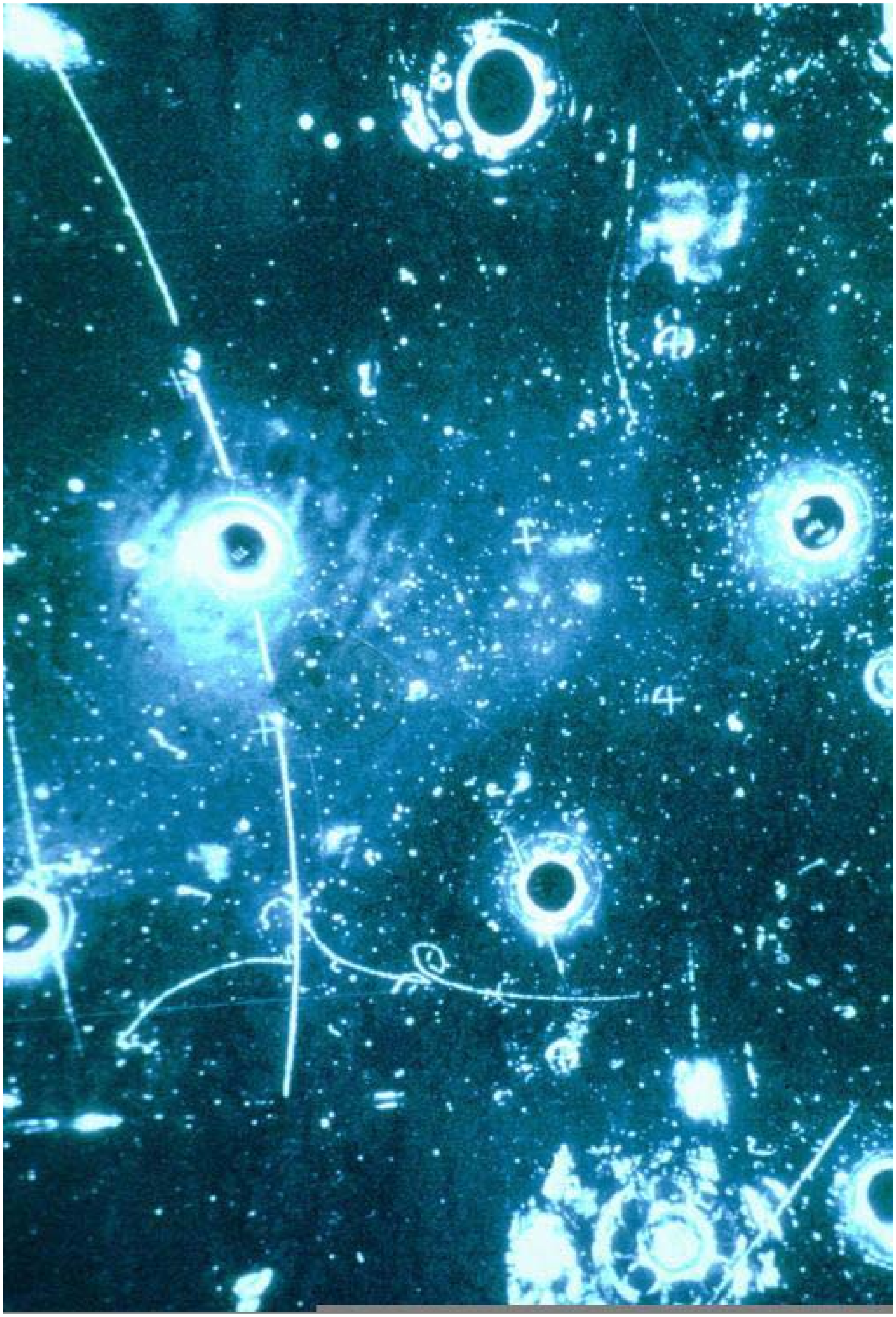

$$\sigma(\nu_\mu e \to \nu_\mu e) = \frac{G_F^2 m_e E_\nu}{2\pi} \left[L_e^2 + R_e^2/3\right]$$

$$\sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e) = \frac{G_F^2 m_e E_\nu}{2\pi} \left[L_e^2/3 + R_e^2\right]$$

$$\sigma(\nu_e e \to \nu_e e) = \frac{G_F^2 m_e E_\nu}{2\pi} \left[(L_e + 2)^2 + R_e^2/3\right]$$

$$\sigma(\bar{\nu}_e e \to \bar{\nu}_e e) = \frac{G_F^2 m_e E_\nu}{2\pi} \left[(L_e + 2)^2/3 + R_e^2\right]$$

# Gargamelle $\nu_\mu e$ Event



# EW interactions of quarks

▷ Left-handed doublet

| | $I_3$ | $Q$ | $Y = 2(Q - I_3)$ |
|---|---|---|---|
| $\mathsf{L}_q = \begin{pmatrix} u \\ d \end{pmatrix}_L$ | $\frac{1}{2}$ | $+\frac{2}{3}$ | $\frac{1}{3}$ |
| | $-\frac{1}{2}$ | $-\frac{1}{3}$ | |

▷ two right-handed singlets

| | $I_3$ | $Q$ | $Y = 2(Q - I_3)$ |
|---|---|---|---|
| $\mathsf{R}_u = u_R$ | $0$ | $+\frac{2}{3}$ | $+\frac{4}{3}$ |
| $\mathsf{R}_d = d_R$ | $0$ | $-\frac{1}{3}$ | $-\frac{2}{3}$ |

▷ CC interaction

$$\mathcal{L}_{W\text{-}q} = \frac{-g}{2\sqrt{2}} \left[ \bar{u}_e \gamma^\mu (1 - \gamma_5) d \, W_\mu^+ + \bar{d} \gamma^\mu (1 - \gamma_5) u \, W_\mu^- \right]$$

identical in form to $\mathcal{L}_{W\text{-}\ell}$: universality ⇔ weak isospin

▷ NC interaction

$$\mathcal{L}_{Z\text{-}q} = \frac{-g}{4 \cos\theta_W} \sum_{i=u,d} \bar{q}_i \gamma^\mu \left[ L_i (1 - \gamma_5) + R_i (1 + \gamma_5) \right] q_i Z_\mu$$

$$L_i = \tau_3 - 2Q_i \sin^2\theta_W \quad R_i = -2Q_i \sin^2\theta_W$$

equivalent in form (not numbers) to $\mathcal{L}_{Z\text{-}\ell}$

# Trouble in Paradise

Universal $u \leftrightarrow d$, $\nu_e \leftrightarrow e$ *not quite right*

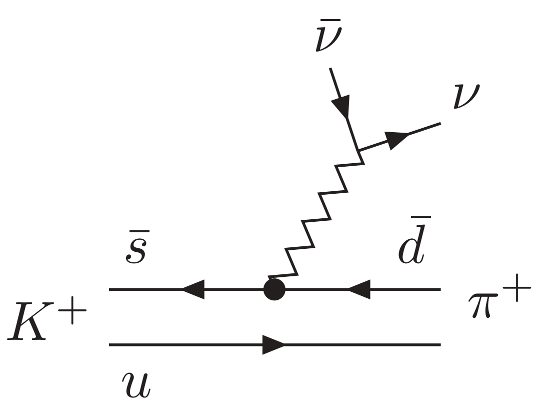
$$\text{Good:} \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \rightarrow \text{Better:} \quad \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L$$

$$d_\theta \equiv d \cos\theta_C + s \sin\theta_C \qquad \cos\theta_C = 0.9736 \pm 0.0010$$

"Cabibbo-rotated" doublet perfects CC interaction (up to small third-generation effects) but $\Rightarrow$ serious trouble for NC

$$\begin{aligned} \mathcal{L}_{Z\text{-}q} \quad = \quad & \frac{-g}{4\cos\theta_W} Z_\mu \{\bar{u}\gamma^\mu [L_u(1-\gamma_5) + R_u(1+\gamma_5)]\, u \\ & + \bar{d}\gamma^\mu [L_d(1-\gamma_5) + R_d(1+\gamma_5)]\, d \cos^2\theta_C \\ & + \bar{s}\gamma^\mu [L_d(1-\gamma_5) + R_d(1+\gamma_5)]\, s \sin^2\theta_C \\ & + \bar{d}\gamma^\mu [L_d(1-\gamma_5) + R_d(1+\gamma_5)]\, s \sin\theta_C \cos\theta_C \\ & + \bar{s}\gamma^\mu [L_d(1-\gamma_5) + R_d(1+\gamma_5)]\, d \sin\theta_C \cos\theta_C \} \end{aligned}$$

Strangeness-changing NC interactions highly suppressed!




BNL E-787 detected two $K^+ \rightarrow \pi^+\nu\bar{\nu}$ candidates, with $\mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu}) = 1.57^{+1.75}_{-0.82} \times 10^{-10}$

(SM: $0.8 \pm 0.3$)

*Phys. Rev. Lett.* **88**, 041803 (2002)

# Glashow–Iliopoulos–Maiani

two left-handed doublets

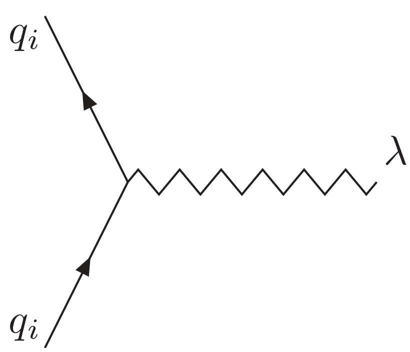
$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L \begin{pmatrix} c \\ s_\theta \end{pmatrix}_L$$

$$(s_\theta = s\cos\theta_C - d\sin\theta_C)$$

$+$ right-handed singlets, $e_R$, $\mu_R$, $u_R$, $d_R$, $c_R$, $s_R$

Required new charmed quark, $c$

Cross terms vanish in $\mathcal{L}_{Z\text{-}q}$,




$$\frac{-ig}{4\cos\theta_W}\gamma_\lambda[(1-\gamma_5)L_i + (1+\gamma_5)R_i] \quad ,$$

$$L_i = \tau_3 - 2Q_i\sin^2\theta_W \quad R_i = -2Q_i\sin^2\theta_W$$

flavor-diagonal interaction!

## Straightforward generalization to $n$ quark doublets

$$\mathcal{L}_{W\text{-}q} = \frac{-g}{2\sqrt{2}}\left[\bar{\Psi}\gamma^{\mu}(1-\gamma_5)\mathcal{O}\Psi\, W_{\mu}^{+} + \text{h.c.}\right]$$

composite $\Psi = \begin{pmatrix} u \\ c \\ \vdots \\ \\ d \\ s \\ \vdots \end{pmatrix}$ flavor structure $\mathcal{O} = \begin{pmatrix} 0 & U \\ 0 & 0 \end{pmatrix}$

$U$: unitary quark mixing matrix

Weak-isospin part: $\mathcal{L}_{Z\text{-}q}^{\text{iso}} = \frac{-g}{4\cos\theta_W}\bar{\Psi}\gamma^{\mu}(1-\gamma_5)\left[\mathcal{O},\mathcal{O}^{\dagger}\right]\Psi$

Since $\left[\mathcal{O},\mathcal{O}^{\dagger}\right] = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \propto \tau_3$

$\Rightarrow$ NC interaction is flavor-diagonal

General $n \times n$ quark-mixing matrix $U$:

$n(n-1)/2$ real $\angle$, $(n-1)(n-2)/2$ complex phases

$3 \times 3$ (Cabibbo–Kobayashi-Maskawa): 3 $\angle$ + 1 phase

$\Rightarrow$ CP violation

Qualitative successes of $SU(2)_L \otimes U(1)_Y$ theory:

- ▷ neutral-current interactions
- ▷ necessity of charm
- ▷ existence and properties of $W^\pm$ and $Z^0$

Decade of precision tests EW (one-per-mille)

| | |
|---|---|
| $M_Z$ | $91\,187.6 \pm 2.1$ MeV/$c^2$ |
| $\Gamma_Z$ | $2495.2 \pm 2.3$ MeV |
| $\sigma^0_{\text{hadronic}}$ | $41.541 \pm 0.037$ nb |
| $\Gamma_{\text{hadronic}}$ | $1744.4 \pm 2.0$ MeV |
| $\Gamma_{\text{leptonic}}$ | $83.984 \pm 0.086$ MeV |
| $\Gamma_{\text{invisible}}$ | $499.0 \pm 1.5$ MeV |

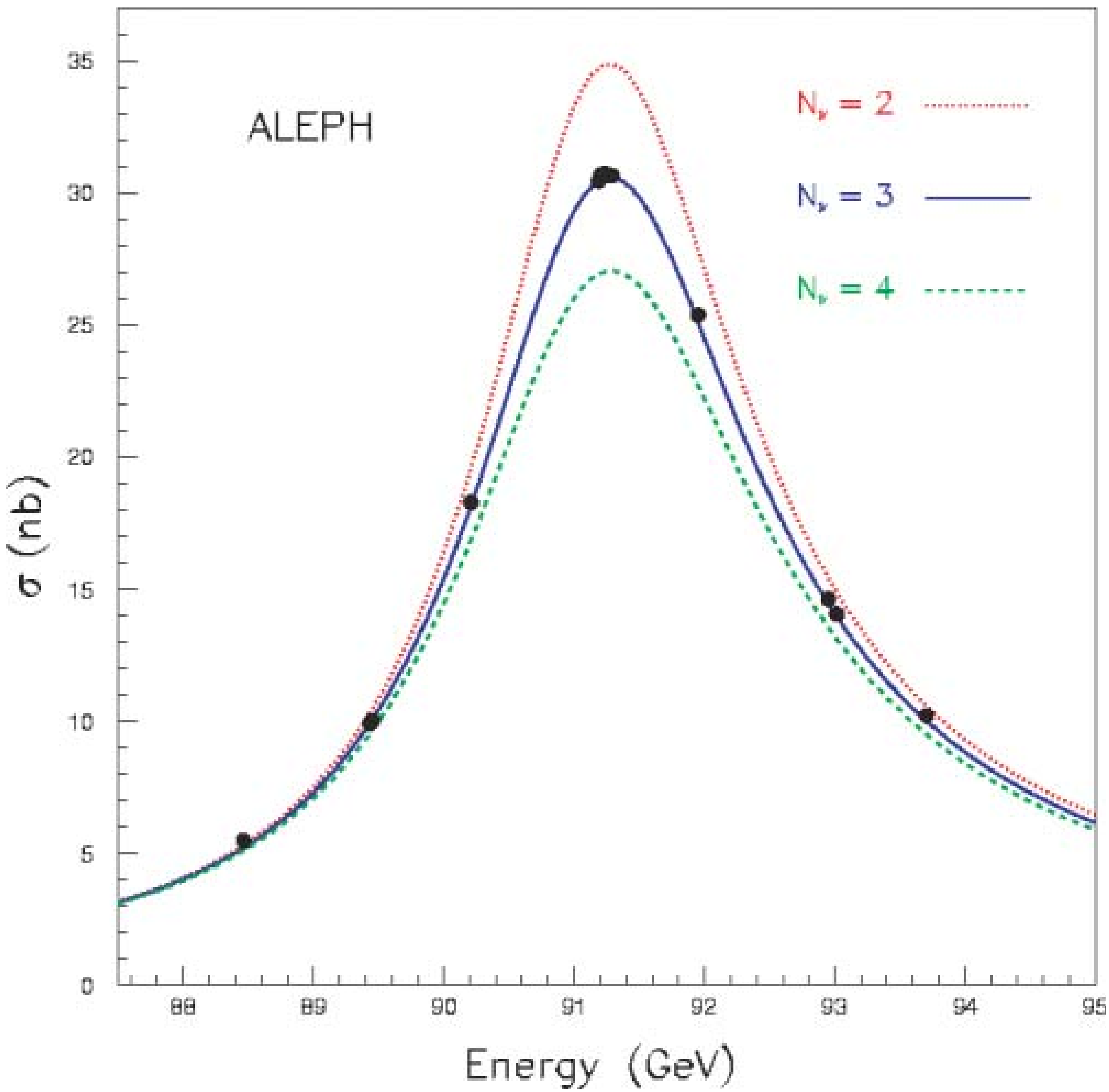
where $\Gamma_{\text{invisible}} \equiv \Gamma_Z - \Gamma_{\text{hadronic}} - 3\Gamma_{\text{leptonic}}$

light neutrinos $N_\nu = \Gamma_{\text{invisible}}/\Gamma^{\text{SM}}(Z \to \nu_i \bar{\nu}_i)$

Current value: $\boxed{N_\nu = 2.994 \pm 0.012}$

. . . excellent agreement with $\nu_e$, $\nu_\mu$, and $\nu_\tau$

# Three light neutrinos

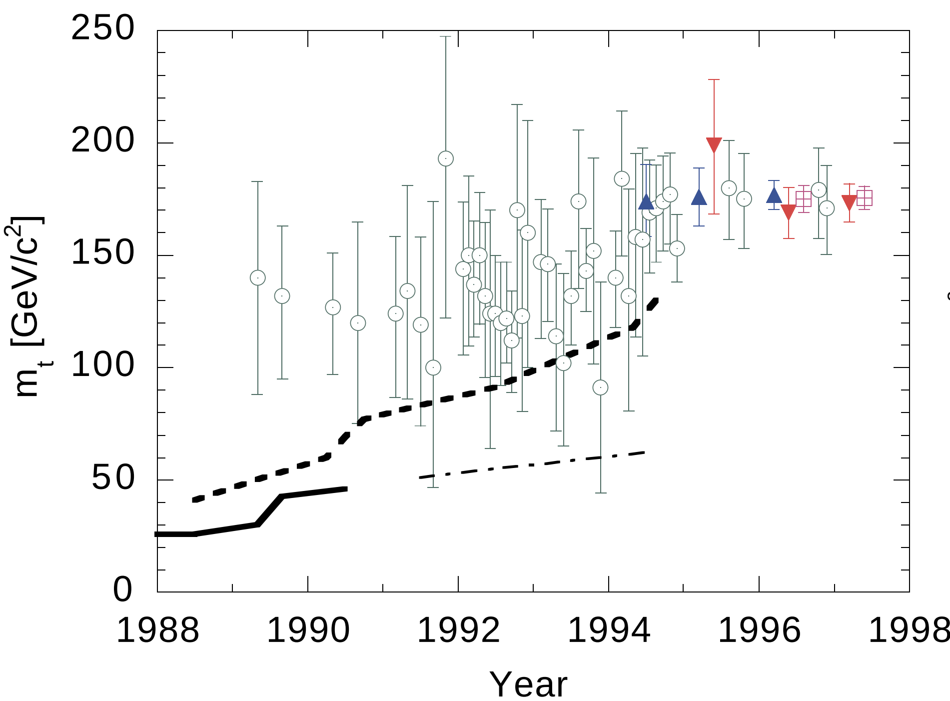


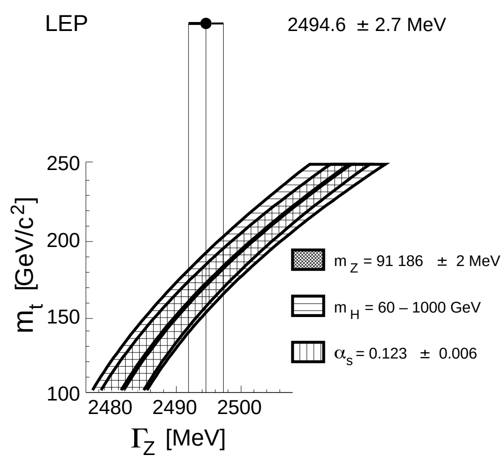
# Global fits . . .

to precision EW measurements:

▷ precision improves with time

▷ calculations improve with time




11.94, LEPEWWG: $m_t = 178 \pm 11^{+18}_{-19}$ GeV/$c^2$

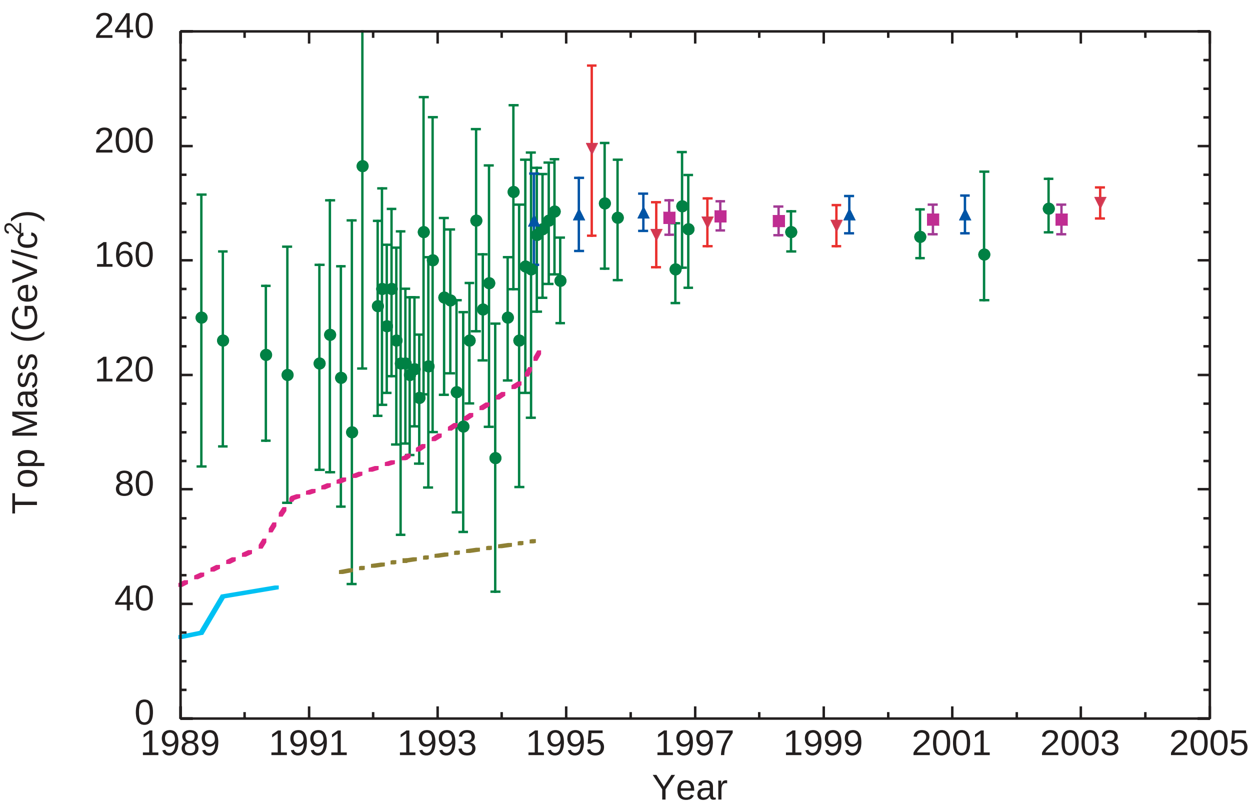
Direct measurements: $m_t = 174.3 \pm 5.1$ GeV/$c^2$

## Global fits . . .

to precision EW measurements:

▷ precision improves with time

▷ calculations improve with time

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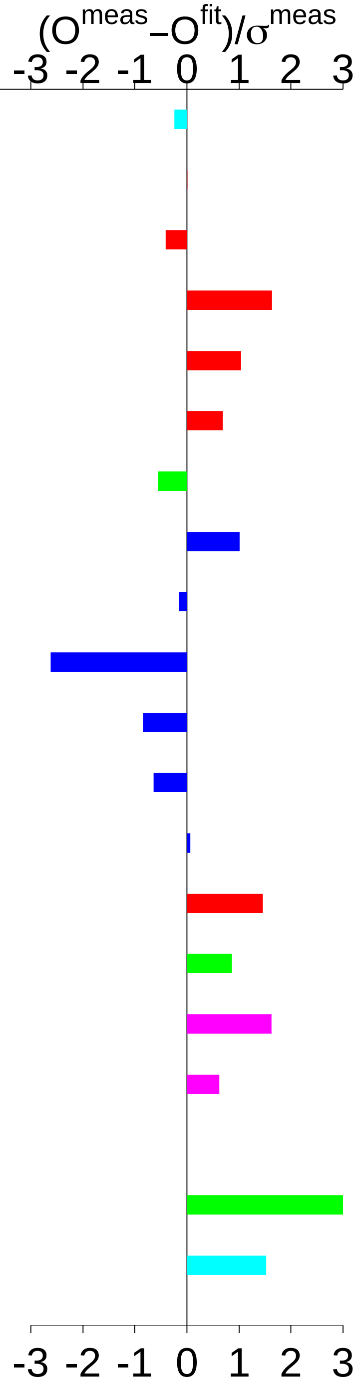
Direct measurements: $m_t = 174.3 \pm 5.1$ GeV/$c^2$

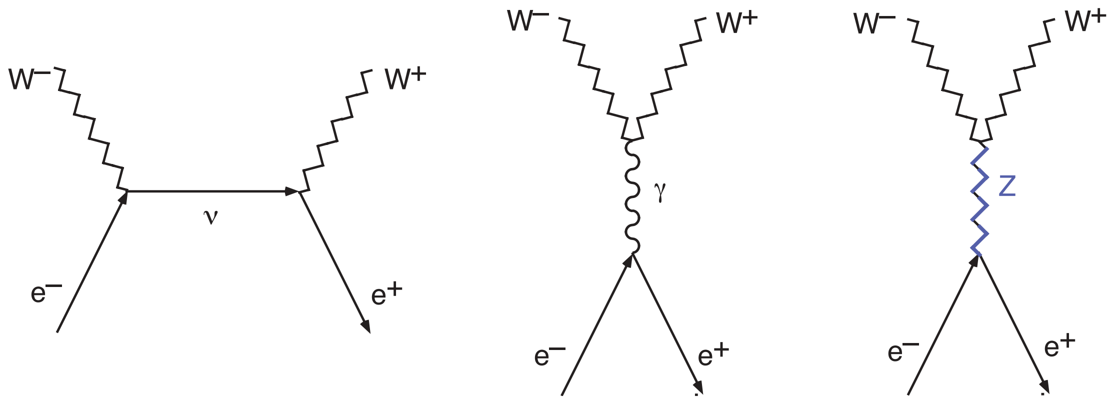
# Summer 2002

| | Measurement | Pull |
|---|---|---|
| $\Delta\alpha^{(5)}_{had}(m_Z)$ | 0.02761 ± 0.00036 | -0.24 |
| $m_Z$ [GeV] | 91.1875 ± 0.0021 | 0.00 |
| $\Gamma_Z$ [GeV] | 2.4952 ± 0.0023 | -0.41 |
| $\sigma^0_{had}$ [nb] | 41.540 ± 0.037 | 1.63 |
| $R_l$ | 20.767 ± 0.025 | 1.04 |
| $A^{0,l}_{fb}$ | 0.01714 ± 0.00095 | 0.68 |
| $A_l(P_\tau)$ | 0.1465 ± 0.0032 | -0.55 |
| $R_b$ | 0.21644 ± 0.00065 | 1.01 |
| $R_c$ | 0.1718 ± 0.0031 | -0.15 |
| $A^{0,b}_{fb}$ | 0.0995 ± 0.0017 | -2.62 |
| $A^{0,c}_{fb}$ | 0.0713 ± 0.0036 | -0.84 |
| $A_b$ | 0.922 ± 0.020 | -0.64 |
| $A_c$ | 0.670 ± 0.026 | 0.06 |
| $A_l$(SLD) | 0.1513 ± 0.0021 | 1.46 |
| $\sin^2\theta^{lept}_{eff}(Q_{fb})$ | 0.2324 ± 0.0012 | 0.87 |
| $m_W$ [GeV] | 80.449 ± 0.034 | 1.62 |
| $\Gamma_W$ [GeV] | 2.136 ± 0.069 | 0.62 |
| $m_t$ [GeV] | 174.3 ± 5.1 | 0.00 |
| $\sin^2\theta_W(\nu N)$ | 0.2277 ± 0.0016 | 3.00 |
| $Q_W$(Cs) | -72.18 ± 0.46 | 1.52 |

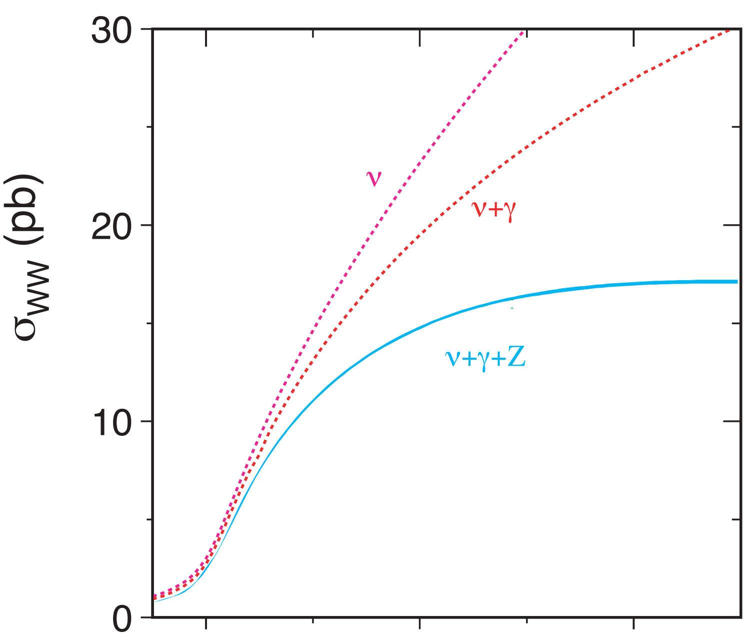


## Gauge symmetry tested in $e^+e^- \to W^+W^-$:

Each diagram grows unacceptably




but the sum is well-behaved



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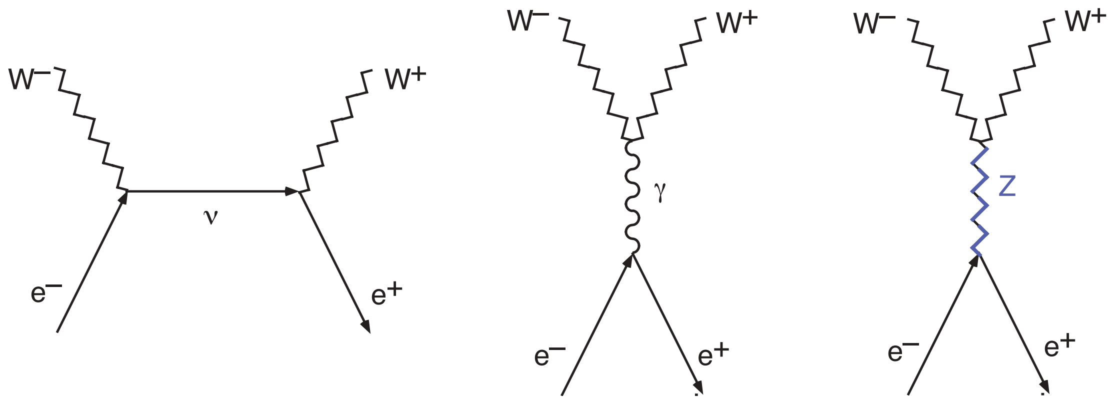


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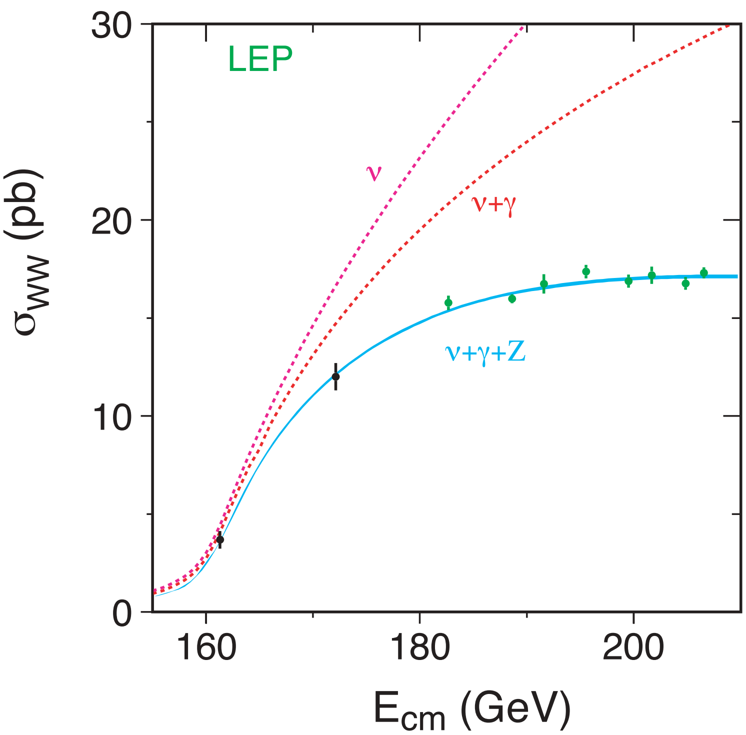



. . . and describes Nature! *New TeV physics*

# The vacuum energy problem

Higgs potential $V(\varphi^\dagger\varphi) = \mu^2(\varphi^\dagger\varphi) + |\lambda|\,(\varphi^\dagger\varphi)^2$

At the minimum,

$$V(\langle\varphi^\dagger\varphi\rangle_0) = \frac{\mu^2 v^2}{4} = -\frac{|\lambda|\,v^4}{4} < 0.$$

Identify $M_H^2 = -2\mu^2$

contributes field-independent vacuum energy density

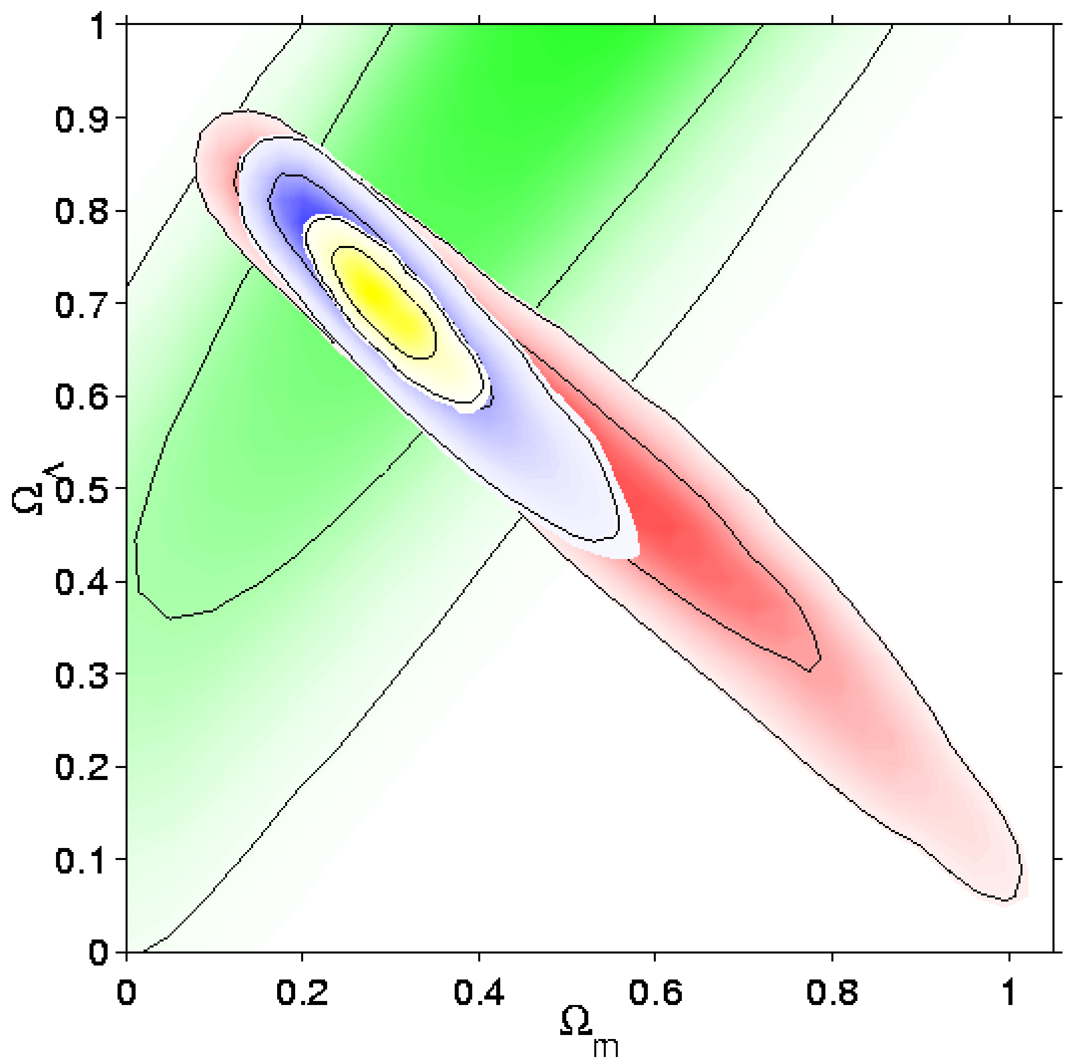
$$\boxed{\varrho_H \equiv \frac{M_H^2 v^2}{8}}$$

Adding vacuum energy density $\varrho_{\text{vac}} \Leftrightarrow$ adding cosmological constant $\Lambda$ to Einstein's equation

$$R_{\mu\nu} - \tfrac{1}{2} R g_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu} + \Lambda g_{\mu\nu}$$

$$\Lambda = \frac{8\pi G_N}{c^4} \varrho_{\text{vac}}$$

observed vacuum energy density $\varrho_{\mathsf{vac}} \lesssim 10^{-46}$ GeV$^4$




Lewis & Bridle, astro-ph/0205436

But $M_H \gtrsim 114$ GeV/$c^2$ $\Rightarrow$

$$\varrho_H \gtrsim 10^8 \text{ GeV}^4$$

MISMATCH BY 54 ORDERS OR MAGNITUDE

## Bounds on $M_H$

EW theory does not predict Higgs-boson mass

Self-consistency $\Rightarrow$ plausible lower and upper bounds

$\triangleright$ Conditional *upper bound* from Unitarity

Compute amplitudes $\mathcal{M}$ for gauge boson scattering at high energies, make a partial-wave decomposition

$$\mathcal{M}(s,t) = 16\pi \sum_J (2J+1) a_J(s) P_J(\cos\theta)$$

Most channels decouple—pw amplitudes are small at all energies (except very near the particle poles, or at exponentially large energies)—for any $M_H$.

Four interesting channels:

$$W_L^+ W_L^- \quad Z_L^0 Z_L^0/\sqrt{2} \quad HH/\sqrt{2} \quad HZ_L^0$$

$L$: longitudinal, $1/\sqrt{2}$ for identical particles

In HE limit,[a] $s$-wave amplitudes $\propto G_F M_H^2$

$$\lim_{s \gg M_H^2} (a_0) \to \frac{-G_F M_H^2}{4\pi\sqrt{2}} \cdot \begin{bmatrix} 1 & 1/\sqrt{8} & 1/\sqrt{8} & 0 \\ 1/\sqrt{8} & 3/4 & 1/4 & 0 \\ 1/\sqrt{8} & 1/4 & 3/4 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$$

Require that largest eigenvalue respect the partial-wave unitarity condition $|a_0| \leq 1$

$$\Longrightarrow \quad M_H \leq \left(\frac{8\pi\sqrt{2}}{3G_F}\right)^{1/2} = 1\ \text{TeV}/c^2$$

condition for perturbative unitarity

[a] Convenient to calculate using *Goldstone-boson equivalence theorem*, which reduces dynamics of longitudinally polarized gauge bosons to scalar field theory with interaction Lagrangian given by $\mathcal{L}_{\text{int}} = -\lambda v h(2w^+w^- + z^2 + h^2) - (\lambda/4)(2w^+w^- + z^2 + h^2)^2$, with $1/v^2 = G_F\sqrt{2}$ and $\lambda = G_F M_H^2/\sqrt{2}$.

▷ If the bound is respected

- ⋆ weak interactions remain weak at all energies
- ⋆ perturbation theory is everywhere reliable

▷ If the bound is violated

- ⋆ perturbation theory breaks down
- ⋆ weak interactions among $W^{\pm}$, $Z$, and $H$ become strong on the 1-TeV scale

⇒ features of *strong* interactions at GeV energies will characterize *electroweak* gauge boson interactions at TeV energies

Threshold behavior of the pw amplitudes $a_{IJ}$ follows from chiral symmetry

| | | |
|---|---|---|
| $a_{00} \approx$ | $G_F s/8\pi\sqrt{2}$ | attractive |
| $a_{11} \approx$ | $G_F s/48\pi\sqrt{2}$ | attractive |
| $a_{20} \approx$ | $-G_F s/16\pi\sqrt{2}$ | repulsive |

New phenomena are to be found in the EW interactions at energies not much larger than 1 TeV

# The EW scale and beyond

EWSB scale, $v = (G_F\sqrt{2})^{-\frac{1}{2}} \approx 246$ GeV, sets

$$M_W^2 = g^2v^2/2 \qquad M_Z^2 = M_W^2/\cos^2\theta_W$$

But it is not the only scale of physical interest

**quasi-certain:** $M_{\text{Planck}} = 1.22 \times 10^{19}$ GeV

**probable:** $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ unification scale $\sim 10^{15-16}$ GeV

**somewhere:** flavor scale

How to keep the distant scales from mixing in the face of quantum corrections?

*OR*

How to stabilize the mass of the Higgs boson on the electroweak scale?

*OR*

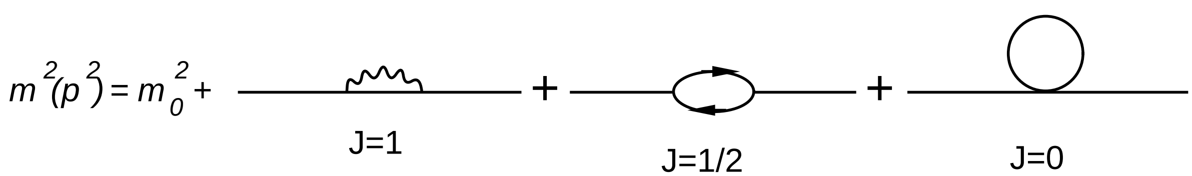
Why is the electroweak scale small?

Higgs potential $V(\phi^\dagger\phi) = \mu^2(\phi^\dagger\phi) + |\lambda|\,(\phi^\dagger\phi)^2$

$\mu^2 < 0$: $\mathrm{SU(2)}_L \otimes \mathrm{U(1)}_Y \to U(1)_{\mathrm{em}}$, as

$$\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ \sqrt{-\mu^2/2|\lambda|} \end{pmatrix} \equiv \begin{pmatrix} 0 \\ \underbrace{(G_F\sqrt{8})^{-1/2}}_{175\ \text{GeV}} \end{pmatrix}$$

*Beyond classical approximation*, quantum corrections to scalar mass parameters:




Loop integrals are potentially divergent.

$$m^2(p^2) = m^2(\Lambda^2) + Cg^2 \int_{p^2}^{\Lambda^2} dk^2 + \cdots$$

$\Lambda$: reference scale at which $m^2$ is known

$g$: coupling constant of the theory

$C$: coefficient calculable in specific theory

$$m^2(p^2) = m^2(\Lambda^2) + Cg^2 \int_{p^2}^{\Lambda^2} dk^2 + \cdots$$

For the mass shifts induced by radiative corrections to remain under control (not greatly exceed the value measured on the laboratory scale), *either*

- ▷ $\Lambda$ must be small, *or*
- ▷ new physics must intervene to cut off the integral

BUT natural reference scale for $\Lambda$ is

$$\Lambda \sim M_{\text{Planck}} = \left(\frac{\hbar c}{G_{\text{Newton}}}\right)^{1/2} \approx 1.22 \times 10^{19}\ \text{GeV}$$

for $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$

OR

$$\Lambda \sim M_U \approx 10^{15}\text{-}10^{16}\ \text{GeV}$$

for unified theory

Both $\gg v/\sqrt{2} \approx 175$ GeV $\Longrightarrow$

**New Physics at $E \lesssim 1$ TeV**

# Second Harvest of Questions

14. What contrives a Higgs potential that hides electroweak symmetry?

15. What separates EW scale from higher scales?

16. What *are* the distinct scales of physical interest?

17. Why is empty space so nearly weightless?

18. What determines the gauge symmetries?

19. What accounts for the range of fermion masses?

20. Why is (strong-interaction) isospin a good symmetry? What does it mean?